2021-2022
Math CP
PAPER 1

Bishop Hall Jubilee School
2021-2022 Mock HKDSE Examination

F.6 MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

Date: 11/2/2022
Time: 8:20 – 10:35 A.M.
Duration: 2 hours 15 minutes
Total page no.: 24 (including cover page)
Maximum possible marks: 105
This paper must be answered in English.

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on Page 1.
2. This paper consists of THREE sections, A(1), A(2) and B.
3. Attempt ALL question in this paper. Write your answers in the spaces provided in this Question-Answer Book.
4. Graph paper, rough work sheets and supplementary answer sheets will be supplied on request. Write down your name, class and class number if necessary.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

Name	
Class	
Class Number	

Q.1	
Q.2	
Q.3	
Q.4	
Q.5	
Q.6	
Q.7	
Q.8	
Q.9	
Q.10	
Q.11	
Q.12	
Q.13	
Q.14	
Q.15	
Q.16	
Q.17	
Q.18	
Q.19	
Total	

SECTION A(1) (35 marks)

1. Simplify $\frac{\left(\alpha\beta^{-4}\right)^{-2}}{\alpha^2\beta}$ and express your answer with positive indices. (3 marks)

2. Make *v* the subject of the formula $\frac{1}{u}+\frac{2}{v}=\frac{3}{f}$. (3 marks)

3. Factorize

   (a) $m^2+3mn-28n^2$,

   (b) $3m+21n-m^2-3mn+28n^2$.

   (3 marks)

4. (a) Find the range of values of y which satisfy $2-6(y+2)>1$ or $-2y-10\geq\frac{y}{2}$.

   (b) Write down the greatest integer satisfying the inequalities in (a).

(4 marks)

5. In a company, the ratio of the number of men to the number of women is $18:11$. If 30 men and 15 women leave the company, the ratio of the number of men to the number of women is $3:2$. Find the original number of women in the company. (4 marks)

6. The marked price of textbook *A* is 24% less than the marked price of textbook *B*. Both textbook *A* and textbook *B* are sold at a discount of 10% on its marked price. The difference in the selling prices of textbook *A* and textbook *B* is \$64.8. Find the marked price of textbook *A*. (4 marks)

7. The coordinates of the points A, B and C are (–4, –1), (–9, 11) and (9, –1) respectively.
   (a) Show that $\triangle ABC$ is isosceles.
   (b) Find the equation of the axis of symmetry of $\triangle ABC$.

(4 marks)

8. In Figure 1, D and E are points lying on AB and AC respectively. BE and CD intersect at the point F. It is given that $AD = DC$, $BC // DE$, $\angle BAC = 50°$ and $\angle AED = 55°$.

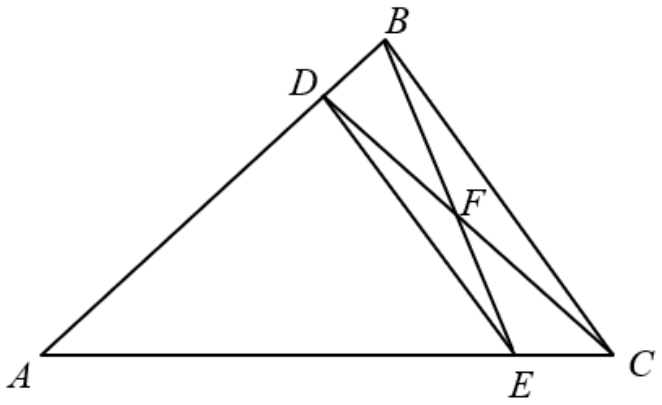


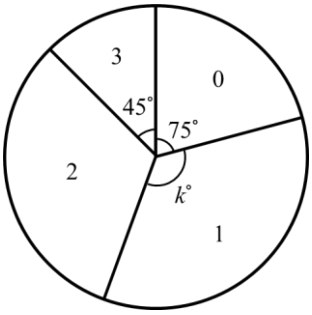
Figure 1

(a) Find $\angle BCD$.

(b) Let $\angle BED = \alpha$. Express $\angle BFC$ in terms of α.

(5 marks)

9. The pie chart below shows the distribution of the numbers of siblings of a group of students. It is known that the probability of a randomly selected student from the group has less than 2 siblings is equal to $\frac{7}{12}$.



Distribution of the numbers of siblings of a group of students

(a) Find k.

(b) Find the minimum number of students in the group.

(c) Write down the median of the distribution.

(5 marks)

SECTION A(2) (35 marks)

10. The cost for a company to produce a batch of n electronic components is \$$C$. C is partly constant and partly varies as the cube root of n. When $n = 8000$, $C = 20\,000$ and when $n = 27\,000$, $C = 23\,000$.
    (a) Find the cost if the company wishes to produce a batch of 125 000 electronic component. (4 marks)
    (b) Someone claims that the total cost of producing three batches of 125 000 components is at least double the cost of producing a batch of 729 000 components. Is the claim correct? Explain your answer. (2 marks)

11. The stem-and-leaf diagram below shows the distribution of the time (in minutes) spent on lunch by 6A students on a specific day.

Stem (10 minutes)	Leaf (1 minute)
0	8 9
1	2 *a* *a* *a* 7
2	0 1 3 5 6 8
3	0 2 *b* 8
4	3 8 9

It is given that the median of the above distribution is greater than the mode by 9 minutes.

(a) Find a. (2 marks)

(b) It is given that the difference between the range and the inter-quartile range of the distribution is less than 22 minutes. Find

(i) b,

(ii) the greatest possible standard deviation of the distribution.

(5 marks)

12. In Figure 2, an hourglass consisting of two similar right circular cones is held vertically on a table. The upper cone is larger than the lower cone. The base radii of the larger cone and the smaller cone are 9 cm and 6 cm respectively. The height of the hourglass is 20 cm. Initially, the smaller cone is filled with liquid.

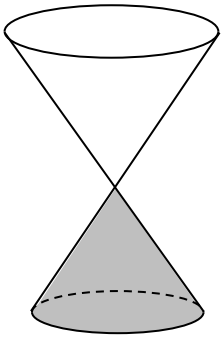


Figure 2

(a) Find the volume of liquid in terms of π. (2 marks)

(b) The hourglass is inverted such that the base of the larger cone lies on the table. The liquid starts falling into the larger cone and forms a frustum. Find the final area of the curved surface of the larger cone in contact with the liquid. (5 marks)

13. The cubic polynomial $\mathrm{f}(x)$ is divisible by x^2+x-1. When $\mathrm{f}(x)$ is divided by $x+7$, the remainder is -123. When $\mathrm{f}(x)$ is divided by $x-3$, the remainder is 297.

(a) Find the quotient when $\mathrm{f}(x)$ is divided by x^2+x-1. (3 marks)

(b) Someone claims that all the roots of the equation $\mathrm{f}(x)=0$ are rational. Is the claim correct? Explain your answer (3 marks)

14. The equation of the circle C is $x^2 + y^2 - 10x - 38y + 350 = 0$. Denote the centre of C by G.

(a) Write down the coordinates of G. (1 mark)

(b) The equation of the straight line L is $12x + 5y + 183 = 0$. It is found that C and L do not intersect. Let P be a point on L such that P is nearest to G.

(i) Find the distance between P and G.

(ii) Let Q be a moving point on C. When Q is nearest to P,

(1) describe the geometric relationship between P, Q and G;

(2) find the ratio of the area of $\triangle OPQ$ to the area of $\triangle OGQ$, where O is the origin.

(8 marks)

SECTION B (35 marks)

15. There are 11 men and 9 women in a team. A committee is formed by randomly selecting 5 persons from the team.
    (a) How many different committees can be formed? (1 mark)
    (b) Find the probability that there are more men than women in the committee. (2 marks)

16. The 6th term and the 7th term of a geometric sequence are 2720 and 5440 respectively.

(a) Find the common ratio of the sequence. (2 marks)

(b) Find the greatest value of m such that the difference of the $(2m + 3)$th term and the $(m - 1)$th term is less than 2×10^6. (3 marks)

17. In Figure 3, A, B and C are points lying on a circle. AB is a diameter of the circle. AD is the tangent to the circle at A such that BCD is a straight line.

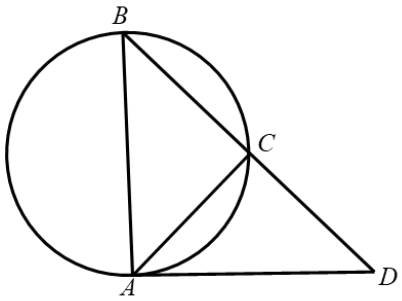


Figure 3

(a) Show that $\triangle ABC \sim \triangle DBA$. (2 marks)

(b) Suppose that $AB = 20$ cm and $CD = 9$ cm.

(i) Find the length of BC.

(ii) Let T and U be points on AD and AB respectively such that $TU // DB$. If area $\triangle ATU$: area of $TDBU = 1:8$, find the perimeter of $\triangle ATU$.

(4 marks)

18. In Figure 4(a), the diagonals of cardboard $ABCD$ intersect at E. It is given that $AB = AD = 6$ cm, $BC = DC = 10$ cm and $\angle ABD = 35°$.

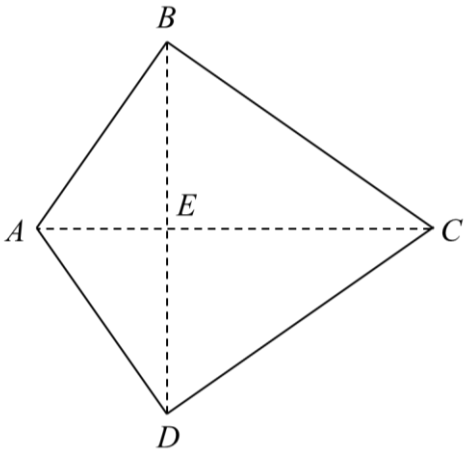


Figure 4(a)

(a) Find $\angle BCD$. (3 marks)

(b) The cardboard $ABCD$ in Figure 4(a) is folded along BD such that A is vertically above the line EC. Two extra triangular cardboards ACD and ABC are placed to form the tetrahedron $ABCD$ as shown in Figure 4(b). It is given that the volume of the tetrahedron $ABCD$ is 45 cm^3.

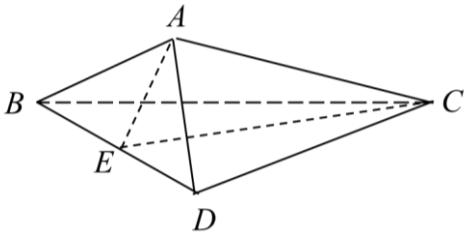


Figure 4(b)

(i) Someone claims that the angle between $\triangle ABD$ and $\triangle BCD$ exceed 60°. Do you agree? Explain your answer.

(ii) Find the total surface area of the tetrahedron $ABCD$.

(6 marks)

19. Let $\mathrm{f}(x)=x^2+(k-4)x-4k$, where $k>-4$ is a constant. Let M be the vertex of the graph of $y=\mathrm{f}(x)$. It is given that the graph of $y=\mathrm{f}(x)$ passes through the same point P on the x-axis for all values of k. Denote the origin by O.

(a) (i) Find the coordinates of P.

(ii) Using the method of completing the square, express the coordinates of M in terms of k.

(4 marks)

(b) The graph of $y=\mathrm{f}(x)$ intersects the x-axis at points P and Q and intersects the y-axis at point R. It is given that M lies in the third quadrant. Someone claims that the area of $\triangle OMQ$ is at least 4 times the area of $\triangle OMR$ for any value of k. Do you agree? Explain your answer. (5 marks)

(c) Find the range of values of k such that the circumcentre of $\triangle PQM$ lies inside the triangle itself. (3 marks)

END OF PAPER