SECTION A1

ITEM 1

$$\frac{\left(\alpha\beta^{-4}\right)^{-2}}{\alpha^2\beta}$$

$=\dfrac{\alpha^{-2}\beta^{-4\times(-2)}}{\alpha^2\beta}$ 1M $(ab)^\ell = a^\ell b^\ell$ or $(a^h)^k = a^{hk}$

$=\dfrac{\alpha^{-2}\beta^{8}}{\alpha^2\beta}$

$=\alpha^{-2-2}\beta^{8-1}$ 1M $c^p c^q = c^{p+q}$ or $\dfrac{1}{d^{-r}} = d^r$

$=\alpha^{-4}\beta^{7}$

$=\dfrac{\beta^7}{\alpha^4}$ 1A

ITEM TOTAL 3

ITEM 2

$$\frac{1}{u}+\frac{2}{v}=\frac{3}{f}$$

$$\frac{2}{v}=\frac{3}{f}-\frac{1}{u}$$ 1M put v on one side

$$\frac{2}{v}=\frac{3u-f}{fu}$$ 1M get common denominator

$$v=\frac{2fu}{3u-f}$$ 1A

ITEM TOTAL 3

ITEM 3A

$m^2 + 3mn - 28n^2 = (m+7n)(m-4n)$	1A

ITEM 3B

$3m + 21n - m^2 - 3mn + 28n^2$	
$= 3(m+7n) - (m^2 + 3mn - 28n^2)$	
$= 3(m+7n) - (m+7n)(m-4n)$	1M using (a)
$= (m+7n)(3 - m + 4n)$	1A
	ITEM TOTAL 3

ITEM 4A

First inequality:

$2-6(y+2)>1$

$-6(y+2)>-1$

$y+2<\frac{1}{6}$

$y<\frac{-11}{6}$ 1A

Second inequality:

$-2y-10\geq\frac{y}{2}$

$\frac{5y}{2}\leq-10$ 1M put y on one side

$y\leq-4$

The required range is $y<\frac{-11}{6}$. 1A

ITEM 4B

-2 1A

ITEM TOTAL 4

ITEM 5

Let $18k$ and $11k$ be the original number of men and women in the company respectively.	
$\frac{18k-30}{11k-15}=\frac{3}{2}$	1M k-method + 1M $\frac{x-30}{y-15}=\frac{3}{2}$ + 1A
$36k-33k=60-45$	
$k=5$	
The original number of women is 55.	1A

ALTERNATIVE	
Let x and y be the original number of men and women in the company respectively.	
$\frac{x}{y}=\frac{18}{11}$	1M
$\frac{x-30}{y-15}=\frac{3}{2}$	1M
$11x-18y=0$	
$2x-60=3y-45$	
$2x-60=3\left(\frac{11x}{18}\right)-45$	1A get one equation in one unknown
Solving the equations to get $y=55$.	1A

ITEM TOTAL 4

ITEM 6

Let \$$x$ be the marked price of A.	
Then marked price of B is \$$\dfrac{x}{1-24\%}$.	1M A is 24% less of B
Selling price of A is \$$(1-10\%)x$,	1M*EITHER discount 10% to marked price of A
and selling price of B is \$$\dfrac{(1-10\%)x}{1-24\%}$.	1M*EITHER discount 10% to marked price of B
$\dfrac{(1-10\%)x}{1-24\%}-(1-10\%)x=64.8$	1M difference of selling prices
$x\left(\dfrac{0.9}{0.76}-0.9\right)=64.8$	
$x=228$	1A
	ITEM TOTAL 4

NOTE: No mark can be given if definition(s) of unknown(s) is/are unclear.

ITEM 7A

$AB=\sqrt{[-4-(-9)]^2+(-1-11)^2}$	1M*EITHER
$=13$	
$BC=\sqrt{(-9-9)^2+[11-(-1)]^2}$	1M*EITHER
$=\sqrt{468}$	
$AC=9-(-4)=13$	
$AB = AC$	
$\triangle ABC$ is an isosceles triangle.	1 f.t.

ITEM 7B

By (a), $\triangle ABC$ is an isosceles triangle with $AB = AC$.

Axis of symmetry of $\triangle ABC$ is the perpendicular bisector of BC which passes through A.

Slope of $BC=\frac{11-(-1)}{-9-9}=\frac{-2}{3}$	
Slope of the axis of symmetry $=\frac{-1}{\frac{-2}{3}}=\frac{3}{2}$	
The equation of the axis of symmetry is	
$y-(-1)=\frac{3}{2}[x-(-4)]$	1M using vertex A and slope of perpendicular of BC
$2(y+1)=3(x+4)$	
$3x-2y+10=0$ or $y=\frac{3}{2}x+5$	1A

ALTERNATIVE

Midpoint of BC is $\left(\frac{-9+9}{2},\frac{11+(-1)}{2}\right)=(0,5)$.	
The equation of the axis of symmetry is	
$y-(-1)=\frac{5-(-1)}{0-(-4)}[x-(-4)]$	1M using vertex A and midpoint of BC
$3x-2y+10=0$ or $y=\frac{3}{2}x+5$	1A

ALTERNATIVE

Midpoint of BC is $\left(\frac{-9+9}{2},\frac{11+(-1)}{2}\right)=(0,5)$.

Slope of $BC=\frac{11-(-1)}{-9-9}=\frac{-2}{3}$

Slope of the axis of symmetry $= \dfrac{-1}{\frac{-2}{3}} = \dfrac{3}{2}$

Let the equation of axis of symmetry be $y = \dfrac{3}{2}x + c$. 1M using midpoint of *BC* and slope of perpendicular of *BC*

Midpoint of *BC* lies on the axis of symmetry.

The equation of axis of symmetry is $y = \dfrac{3}{2}x + 5$. 1A

ITEM TOTAL 4

ITEM 8A

In ΔACD,		
$AD = DC$		
$\angle ACD = \angle CAD = 50°$	by isosceles triangle	1M CAN BE ABSORBED
$\angle ACB = \angle AED = 55°$	by parallel lines	1M CAN BE ABSORBED
$\angle BCD$		
$= \angle ACB - \angle ACD$		
$= 55° - 50°$		
$= 5°$		1A

ITEM 8B

$\angle CBE = \angle BED = \alpha$	by parallel lines	1M
In ΔBCF,		
$\angle BFC$		
$= 180° - \angle BCD - \angle CBE$		
$= 180° - 5° - \alpha$		
$=175° - \alpha$		1A
		ITEM TOAL 5

ITEM 9A

$\frac{k}{360}+\frac{75}{360}=\frac{7}{12}$	1M
$k=135$	1A disregard unit

ITEM 9B

The probabilities are $\frac{5}{24}$, $\frac{3}{8}$, $\frac{7}{24}$, $\frac{1}{8}$.	1M mentioning all the probabilities as fraction.
The lowest common denominator is 24.	
The number of students must be integer.	
The lowest such possible number is 24.	1A

ALTERNATIVE	
The angles of sectors are $45°, 75°, 135°$, and $105°$.	1M mentioning all the sizes of angles.
The HCF of these angles is $15°$	
The number of students must be integer.	
The lowest such number is $\frac{360}{15}=24$.	1A

ITEM 9C

1	1A
	ITEM TOTAL 5
	SECTION TOTAL 35

SECTION A2

ITEM 10A

Let $C = a + b\sqrt[3]{n}$, where *a* and *b* are non-zero constants. 1M

$20000 = a + b\sqrt[3]{8000}$ 1M*EITHER

$a + 20b = 20000$

$23000 = a + b\sqrt[3]{27000}$ 1M*EITHER

$a + 30b = 23000$

Solving the equations to get

$b = 300$, $a = 14000$ 1A

$C = 14000 + 300\sqrt[3]{n}$

When $n = 125000$,

$C = 14000 + 300\sqrt[3]{125000} = 29000$. 1A

The required cost is $29000.

ITEM 10B

When $n = 729000$,

$C = 14\ 000 + 300\sqrt[3]{729\ 000}$ 1M

$= 41000$

By (a), total cost of producing three batches of 125000 components

$= \$(29\ 000 \times 3)$

$= \$87000$

$> \$41000 \times 2$

The claim is correct. 1A f.t.

ITEM TOTAL 6

ITEM 11A

$\text{Median} = \frac{23+25}{2} = 24$ minutes	1M*EITHER
Mode = (10 + a) minutes	1M*EITHER
$24-(10+a)=9$	
$a=5$	1A

ALTERNATIVE

$\text{Mode} = 24-9=15$ minutes	1M*EITHER
$10+a=15$	
$a=5$	1A

ITEM 11BI

$\text{Range} = 49-8=41$ minutes	1M
Inter-quartile range	
$= \frac{32+(30+b)}{2} - \frac{15+15}{2}$	1M
$= \left(16+\frac{b}{2}\right)$ minutes	
$41-\left(16+\frac{b}{2}\right) < 22$	
$b > 6$	

ALTERNATIVE

$41-\text{IQR} < 22$	
$\text{IQR} > 19$	
$\frac{32+(30+b)}{2} - \frac{15+15}{2} > 19$	1M
$b > 6$	

From the stem-and-leaf diagram, $b \le 8$.	
The possible values of b are 7 and 8.	1A

ITEM 11BII

When b = 7, the standard deviation of the distribution	
≈ 12.13043693 minutes	1M*EITHER (at least 3 s.f.)
When b = 8, the standard deviation of the distribution	
≈ 12.17949096 minutes	1M*EITHER (at least 3 s.f.)
The greatest possible standard deviation of the distribution is 12.2 minutes.	
	1A f.t.

ITEM TOTAL 7

ITEM 12A

Height of the smaller cone

$=\left(20\times\frac{6}{6+9}\right)$ cm 1M

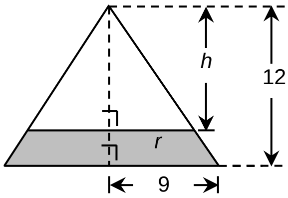
= 8 cm

Volume of the liquid

$=\frac{1}{3}\pi(6^2)(8)$ cm^3

$=96\pi$ cm^3 1A with unit (301.59289 cm^3)

ITEM 12B



Let r cm be the radius of the upper base of the frustum.

Let h cm be the distance between the apex and the upper base of the frustum.

Height of the larger cone = $(20-8)$ cm = 12 cm

Refer to the figure.

$\frac{h}{12}=\frac{r}{9}$ 1M relating height and radius of empty cone

$h=\frac{4r}{3}$

$\frac{1}{3}\pi r^2\left(\frac{4r}{3}\right)+96\pi=\frac{1}{3}\pi\left(9^2\right)(12)$ 1M empty cone + liquid = whole cone

$\frac{4}{9}\pi r^3=228\pi$

$r=\sqrt[3]{513}$

Required area

$=\left[\pi(9)\sqrt{9^2+12^2}-\pi(\sqrt[3]{513})\sqrt{(\sqrt[3]{513})^2+\left(\frac{4}{3}\sqrt[3]{513}\right)^2}\right]$ cm^2

1M cur surf area + 1M lateral height of empty cone

≈ 88.6 cm^2 (3 s.f.) 1A

ALTERNATIVE

$\frac{\text{volume of empty cone}}{\text{volume of whole cone}}$

$=\frac{324\pi-96\pi}{324\pi}$ 1M

$=\frac{19}{27}$

$=\left(\dfrac{\text{curved surface area of empty cone}}{\text{curved surface area of whole cone}}\right)^{\frac{3}{2}}$	1M
$\dfrac{\text{curved surface area of empty cone}}{\text{curved surface area of whole cone}}$	
$=\left(\dfrac{19}{27}\right)^{\frac{2}{3}}$	
Curved surface area of whole cone is	
$\pi(9)\sqrt{9^2+12^2}$	
Curved surface area required is	
$\left(\pi(9)\sqrt{9^2+12^2}\right)\left(1-\left(\dfrac{19}{27}\right)^{\frac{2}{3}}\right)$	1M curved surf area + 1M area ratio ($\times\left(\dfrac{19}{27}\right)^{\frac{2}{3}}$)
$\approx 88.6\text{ cm}^2$ (3 s.f.)	1A

ITEM TOTAL 7

ITEM 13A

Let $f(x)=(x^2+x-1)(ax+b)$, where a and b are constants.	1M
$f(-7)=-123$	1M*EITHER
$[(-7)^2+(-7)-1][a(-7)+b]=-123$	
$-7a+b=-3$	
$f(3)=297$	1M*EITHER
$[(3)^2+(3)-1][a(3)+b]=297$	
$3a+b=27$	
Solving to get $a=3$, $b=18$	1A
The quotient is $3x+18$.	

ITEM 13B

$f(x)=0$	
$(x^2+x-1)(3x+18)=0$	1M
$x^2+x-1=0$ or $3x+18=0$	
$x=\frac{-1+\sqrt{5}}{2}$ or $x=\frac{-1-\sqrt{5}}{2}$ or $x=-6$	1M
$\frac{-1+\sqrt{5}}{2}$ and $\frac{-1-\sqrt{5}}{2}$ are not rational.	(either)
Thus, the claim is not correct.	1A f.t.
NOTE: Must state clearly which root(s) is/are not rational.	
	ITEM TOTAL 6

ITEM 14A

The coordinates of G are $(5,19)$. 1A

ITEM 14BI

Let (h, k) be the coordinates of P.

Since P lies on L, we have $12h + 5k + 183 = 0$.

Note that GP is perpendicular to the straight line $12x + 5y + 183 = 0$.

Also note that the slope of GP is $\frac{k-19}{h-5}$.

Hence, we have $\left(\frac{k-19}{h-5}\right)\left(\frac{-12}{5}\right) = -1$. 1M

So, we have $5h - 12k + 203 = 0$.

Solving the equations $12h + 5k + 183 = 0$ and $5h - 12k + 203 = 0$, 1M

we have $h = -19$ and $k = 9$. 1A

ALTERNATIVE

Note that GP is perpendicular to the straight line $12x + 5y + 183 = 0$.

The slope of GP is $\frac{-1}{\frac{-12}{5}} = \frac{5}{12}$.

The equation of GP is

$y - 19 = \frac{5}{12}(x - 5)$. 1M

$5x - 12y + 203 = 0$

P is the intersection of GP and L.

Solving the equations $5x - 12y + 203 = 0$ and $12x + 5y + 183 = 0$, 1M

$P = (-19, 9)$ 1A

The distance between P and G

$= \sqrt{[5-(-19)]^2 + (19-9)^2}$

$= 26$ 1A

ITEM 14BII1

P, Q and G are collinear. 1A

P, Q, R lie on a straight line. 1A

ITEM 14BII2

Note that the radius of the circle is 6.

The distance between G and Q is 6. 1M

The distance between P and Q is $26 - 6 = 20$. 1M

The required ratio is $PQ : GQ = 10 : 3$. 1A

ITEM TOTAL 9

SECTION TOTAL 35

SECTION B

ITEM 15A

Number of teams can be formed is

C_5^{20}

$=15504$ 1A

ITEM 15B

The required probability is

$\frac{C_3^{11}C_2^9 + C_4^{11}C_1^9 + C_5^{11}C_0^9}{15504}$ 1M numerator in fraction

$= \frac{781}{1292}$ 1A

ITEM TOTAL 3

ITEM 16A

Let r be the common ratio.

$r = \dfrac{5440}{2770}$	1M
$r = 2$	1A

ITEM 16B

Let a be the first term of the sequence.

$ar^5 = 2720$	
$a = 85$	
$85\left(2^{2m+2}\right) - 85\left(2^{m-2}\right) < 2\left(10^6\right)$	
$1360\left(2^m\right)^2 - 85\left(2^m\right) - 8\left(10^6\right) < 0$	1M in the form of a quadratic inequality
$\dfrac{85 - \sqrt{85^2 - 4(1360)(-8\times10^6)}}{2(1360)} < 2^m < \dfrac{85 + \sqrt{85^2 - 4(1360)(-8\times10^6)}}{2(1360)}$	
$\log 2^m < \log \dfrac{85 + \sqrt{85^2 - 4(1360)(-8\times10^6)}}{2(1360)}$	1M
$m \log 2 < \log \dfrac{85 + \sqrt{85^2 - 4(1360)(-8\times10^6)}}{2(1360)}$	
$m < 6.261676643$	
The greatest value of m is 6.	1A
	ITEM TOTAL 5

ITEM 17A

$\angle ACB = 90°$ (∠ in semi-circle)

$\angle DAB = 90°$ (tangent ⊥ radius)

$\therefore \angle ACB = \angle DAB$

$\angle ABC = \angle DBA$ (common angle)

$\angle CAB = \angle ADB$ (∠ sum of △)

$\therefore \triangle ABC \sim \triangle DBA$ (AA) (AAA)

2 CORRECT PROOF WITH REASONS

1 CORRECT PROOF

ITEM 17BI

Let $BC = x$ cm.

$\frac{AB}{DB} = \frac{BC}{BA}$ 1M using the similar triangles

$\frac{AB}{BC + CD} = \frac{BC}{AB}$

$\frac{20}{x+9} = \frac{x}{20}$

$x^2 + 9x - 400 = 0$

$x = 16$ 1A

ITEM 17BII

$DB = 16 + 9 = 25$ cm

$AD = \sqrt{DB^2 - AB^2}$

$= \sqrt{25^2 - 20^2}$

$= 15$ cm

Perimeter of $\triangle ADB$ is $15 + 20 + 25 = 60$ cm.

$\frac{\text{perimeter of } \triangle ATU}{\text{perimeter of } \triangle ADB}$

$= \sqrt{\frac{\text{area of } \triangle ATU}{\text{area of } \triangle ADB}}$ 1M

$= \sqrt{\frac{1}{8+1}}$

$= \frac{1}{3}$

Perimeter of $\triangle ATU$ is 20 cm. 1A

ITEM TOTAL 6

ITEM 18A

Consider $\triangle ABD$.

$\angle ADB = \angle ABD = 35°$

$\angle BAD$

$= 180° - 35° - 35°$

$= 110°$

By the sine formula, we have

$\dfrac{BD}{\sin \angle BAD} = \dfrac{AD}{\sin \angle ABD}$

$\dfrac{BD}{\sin 110°} = \dfrac{6}{\sin 35°}$ — 1M using left side triangle to get *BD*

$BD \approx 9.829824531$ cm

Consider $\triangle BCD$.

By the cosine formula, we have

$\cos \angle BCD = \dfrac{BC^2 + CD^2 - BD^2}{2(BC)(CD)}$ — 1M using SSS to get the angle

$\approx \dfrac{10^2 + 10^2 - 9.829824531^2}{2(10)(10)}$

≈ 0.516872748

$\angle BCD$

$\approx 58.87728555°$ — 1A

$= 58.9°$ (3 s.f.)

ITEM 18BI

The area of $\triangle BCD$

$\approx \dfrac{1}{2}(10)(10)\sin 58.87728555°$

$\approx 42.80311209 \text{ cm}^2$

Let *G* be the foot of perpendicular of *A* onto *EC*.

The angle between $\triangle ABD$ and $\triangle BCD$ is $\angle AEG$. (or $\angle AEC$)

1A identify angle between planes

$45 \approx \dfrac{1}{3}(42.80311209)(AG)$ — 1M height of tetrahedron

$AG \approx 3.153976274$ cm

Consider $\triangle ABE$.

$AE = 6\sin 35°$

≈ 3.441458618 cm

Consider $\triangle AEG$.

$\sin\angle AEG = \dfrac{AG}{AE}$

$\approx \dfrac{3.153976274}{3.441458618}$

$\angle AEG \approx 66.414\,636\,57° > 60°$

The angle between $\triangle ABD$ and $\triangle BCD$ exceeds 60°.

Agreed. 1A f.t.

ITEM 18BII

Consider $\triangle BEC$.

$EC = \sqrt{BC^2 - BE^2}$

$\approx \sqrt{10^2 - \left(\dfrac{9.829824531}{2}\right)^2}$

≈ 8.708825261 cm

Consider $\triangle AEC$.

By the cosine formula, we have

$AC^2 = AE^2 + EC^2 - 2(AE)(EC)\cos\angle AEC$

$\approx 3.441458618^2 + 8.708825261^2 - 2(3.441458618)(8.708825261)\cos 66.41463657°$

$AC \approx 7.981449516$ cm 1M

Note that $\triangle ADC \cong \triangle ABC$.

Consider $\triangle ADC$.

Let $s = \dfrac{AD + DC + AC}{2} \approx 11.99072476$ cm.

By the Heron's formula,

area of $\triangle ADC$

$= \sqrt{s(s-AD)(s-DC)(s-AC)}$

$= \sqrt{s(s-6)(s-10)(s-7.981449516)}$

≈ 23.94423384 cm^2 1M

Total surface area of the tetrahedron

= area of $\triangle ABD$ + area of $\triangle BCD$ + area of $\triangle ADC$ + area of $\triangle ABC$

$\approx \dfrac{1}{2}(6)(6)\sin 110° + 42.80311209 + 23.94423384 + 23.94423384$

$= 108$ cm^2 (3 s.f.) 1A

ITEM TOTAL 9

ITEM 19AI

Let $\mathrm{f}(x)=x^2+(k-4)x-4k=0$	1M
$x=\dfrac{-(k-4)\pm\sqrt{(k-4)^2+4(4k)}}{2}$	
$x=\dfrac{-k+4\pm\sqrt{k^2-8k+16+16k}}{2}$	
$x=\dfrac{-k+4\pm\sqrt{k^2+8k+16}}{2}$	
$x=\dfrac{-k+4\pm(k+4)}{2}$	
$x=4$ or $x=-k$	
The point P is $(4,0)$.	1A

ALTERNATIVE	
$\mathrm{f}(x)=x^2+(k-4)x-4k=0$	1M
$x^2+kx-4x-4k=0$	
$x(x+k)-4(x+k)=0$	
$(x+k)(x-4)=0$	
$x=4$ or $x=-k$	
The point P is $(4,0)$.	1A

ITEM 19AII

$\mathrm{f}(x)=x^2+(k-4)x-4k$

$=x^2+(k-4)x+\dfrac{(k-4)^2}{4}-\dfrac{(k-4)^2}{4}-4k$ 1M $+\left(\dfrac{B}{2}\right)^2-\left(\dfrac{B}{2}\right)^2$ in x^2+Bx+C

$=\left(x+\dfrac{k-4}{2}\right)^2-\dfrac{(k-4)^2}{4}-4k$

$=\left(x+\dfrac{k-4}{2}\right)^2-\dfrac{k^2}{4}-2k-4$

The coordinates of M are $\left(\dfrac{4-k}{2},\dfrac{-k^2}{4}-2k-4\right)$. 1A

ITEM 19B

The coordinates of Q are $(-k,0)$. 1A*EITHER

The coordinates of R are $(0,-4k)$. 1A*EITHER

The area of $\triangle OMQ$

$$=\frac{k\left(\frac{k^2}{4}+2k+4\right)}{2}$$ 1M

The area of $\triangle OMR$

$$=\frac{4k\left(\frac{k-4}{2}\right)}{2}$$ 1M

Area of $\triangle OMQ$ – 4 × area of $\triangle OMR$

$$=\frac{k\left(\frac{k^2}{4}+2k+4\right)}{2}-2\left(4k\left(\frac{k-4}{2}\right)\right)$$

$$=\frac{k^3}{8}+k^2+2k-4k^2+16k$$

$$=\frac{k}{8}\left(k^2-24k+144\right)$$ 1A

$$=\frac{k\left(k-12\right)^2}{8}\geq 0$$

Agreed. 1A f.t.

ITEM 19C

The circumcentre lies inside the triangle if the triangle is acute.

Since $PM=QM$, it is enough to show that $\tan\angle PQM>1$ for all $k>-4$.

1M

$$\tan\angle PQM=\frac{\text{perpendicular distance of } M \text{ from } x\text{-axis}}{\frac{PQ}{2}}$$

$$=\frac{\frac{k^2}{4}+2k+4}{\frac{4-k}{2}+k}$$ 1M

$$=\frac{\frac{k^2+8k+16}{4}}{\frac{4+k}{2}}>1$$

$$k^2+8k+16>8+2k$$

$$k^2+6k+8>0$$

$$\left(k+4\right)\left(k+2\right)>0$$

The required range is $k>-2$. 1A f.t.

ITEM TOTAL 12

SECTION TOTAL 35

PAPER TOTAL 105