2021-2022 S6 Mock Paper II

1. D

$$\frac{(2y^2)^{-4}}{4y^{-3}} = \frac{y^3}{4(2y^2)^4} = \frac{y^3}{64y^8} = \underline{\underline{\frac{1}{64y^5}}}$$

2. C

$(3a + 2b)^2 - (2a - 3b)^2$

$= [(3a + 2b) + (2a - 3b)][(3a + 2b) - (2a - 3b)]$

$= (3a + 2b + 2a - 3b)(3a + 2b - 2a + 3b)$

$= \underline{\underline{(5a - b)(a + 5b)}}$

3. A

$$x = 2 - \frac{y+1}{y}$$

$$x - 2 = -\frac{y+1}{y}$$

$$x - 2 = -1 - \frac{1}{y}$$

$$\frac{1}{y} = 1 - x$$

$$y = \underline{\underline{\frac{1}{1-x}}}$$

4. A

L.H.S. $= (x - m)(x + 3)$

$= x^2 - mx + 3x - 3m$

$= x^2 + (3 - m)x - 3m$

$\therefore \quad x^2 + (3-m)x - 3m \equiv x^2 - 4x + n$

By comparing the coefficients of x and the constant term, we have

$$\begin{cases} 3 - m = -4 & \text{......(1)} \\ -3m = n & \text{......(2)} \end{cases}$$

From (1), we have

$m = 7$

By substituting $m = 7$ into (2), we have

$-3(7) = n$

$n = \underline{\underline{-21}}$

5. C

The maximum absolute error = $\frac{1}{2}(0.01) = 0.005$

The range of values of x is:

$1.90 - 0.005 \le x < 1.90 + 0.005$

$1.895 \le x < 1.905$

6. C

$-7(5 + 2x) \ge -5x + 1$ and $\frac{3x+4}{5} < -4$

$-35 - 14x \ge -5x + 1$ and $3x + 4 < -20$

$-9x \ge 36$ and $3x < -24$

$x \le -4$ and $x < -8$

$\therefore$ The solution is $x < -8$.

7. A

$y = (-x+1)^2 + 2$

$= (-x)^2 + 2(-x)(1) + 1^2 + 2$

$= x^2 - 2x + 3$

For I:

$\because$ coefficient of $x^2 > 0$

$\therefore$ The graph opens upwards.

$\therefore$ I is true.

For II:

$y = (-x+1)^2 + 2$

$= (-1)^2(x-1)^2 + 2$

$= (x-1)^2 + 2$

Compare with $y = a(x-h)^2 + k$

The vertex is (1, 2)

$\therefore$ II is false.

For III:

The y-intercept of the graph

$= (-0+1)^2 + 2$

$= 3$

$\therefore$ III is not true.

$\therefore$ The answer is A.

8. D

$\because$ $2x - 1$ is a factor of $g(x)$.

$\therefore$ $g\left(\frac{1}{2}\right) = 0$

$$k\left(\frac{1}{2}\right)^3 - 5\left(\frac{1}{2}\right)^2 - 2k\left(\frac{1}{2}\right) + 3 = 0$$

$$-\frac{7k}{8} + \frac{7}{4} = 0$$

$$k = 2$$

$\therefore$ $g(x) = 2x^3 - 5x^2 - 4x + 3$

$\therefore$ $g(-2) = 2(-2)^3 - 5(-2)^2 - 4(-2) + 3$

$= \underline{\underline{-25}}$

9. B

$z = \frac{k\sqrt{x}}{y}$, where k is a non-zero constant.

$$z^2 = \frac{k^2 x}{y^2}$$

$$\frac{x}{y^2 z^2} = \frac{1}{k^2}$$

$\therefore$ $\frac{x}{y^2 z^2}$ must be a constant.

10. B

Let x be Donald's weight.

Peter's weight $= (1 - 10\%)x = 0.9x$

John's weight

$= 0.9x \div (1 + 25\%)$

$= 0.72x$

$= (1 - 28\%)x$

$\therefore$ John's weight is 28% smaller than Donald's weight.

11. A

For I:

Slope of $L_1 = \frac{3}{a}$

$\because$ From the graph, slope of $L_1 < 0$.

$\therefore$ $\frac{3}{a} < 0$

$a < 0$

x-intercept of $L_1 = \frac{b}{3}$

$\because$ From the graph, x-intercept of $L_1 > 0$.

$\therefore$ $\frac{b}{3} > 0$

$b > 0$

Slope of $L_2 = \frac{c}{2}$

$\because$ From the graph, slope of $L_2 < 0$

$\therefore$ $\frac{c}{2} < 0$

$c < 0$

y-intercept of $L_2 = -\frac{d}{2}$

$\because$ From the graph, y-intercept of $L_2 > 0$

$\therefore$ $-\frac{d}{2} > 0$

$d < 0$

$\therefore$ $abcd < 0$

$\therefore$ I is true.

For II:

x-intercept of $L_1 = \frac{b}{3}$

x-intercept of $L_2 = \frac{d}{c}$

$\because$ From the graph, the x-intercepts of L_1 and L_2 are the same.

$\therefore$ $\frac{b}{3} = \frac{d}{c}$

$bc = 3d$

$\therefore$ II is true.

For III:

Slope of $L_1 = \frac{3}{a}$

Slope of $L_2 = \frac{c}{2}$

Note that $a < 0$ and $c < 0$.

$\because$ Slope of $L_1 <$ slope of L_2

$\therefore$ $\frac{3}{a} < \frac{c}{2}$

$6 > ac$

$ac < 6$

$\therefore$ III is not true.

$\therefore$ The answer is A.

12. D

Let $T(n)$ be the number of dots in the nth pattern.

$T(1) = 4$

$T(2) = T(1 + 1) = 4 + 2(1) = 6$

$T(3) = T(2 + 1) = 6 + 2(2) = 10$

$T(4) = T(3 + 1) = 10 + 2(3) = 16$

$T(5) = T(4 + 1) = 16 + 2(4) = 24$

$T(6) = T(5 + 1) = 24 + 2(5) = 34$

$T(7) = T(6 + 1) = 34 + 2(6) = 46$

$T(8) = T(7 + 1) = 46 + 2(7) = 60$

$\therefore$ The 8th pattern consists of 60 dots.

13. B

The scale of map

$= \sqrt{\frac{26 \text{ cm}^2}{650 \text{ m}^2}}$

$= \sqrt{\frac{26 \text{ cm}^2}{650 \times 100^2 \text{m}^2}}$

$= \frac{1}{500}$

$= \underline{\underline{1:500}}$

14. A

$x : y = 3 : 2$

$\frac{x}{y} = \frac{3}{2}$

$4y = 5z$

$\frac{z}{y} = \frac{4}{5}$

$$\frac{x+z}{x+y} = \frac{\frac{x}{y} + \frac{z}{y}}{\frac{x}{y} + 1}$$

$$= \frac{\frac{3}{2} + \frac{4}{5}}{\frac{3}{2} + 1}$$

$$= \frac{\frac{23}{10}}{\frac{5}{2}}$$

$$= \underline{\underline{\frac{23}{25}}}$$

15. B

In $\triangle BCD$,

$BD = \sqrt{BC^2 + CD^2}$ (Pyth. theorem)

$= \sqrt{15^2 + 20^2}$ cm

$= 25$ cm

In $\triangle ABD$,

$AB^2 + BD^2 = AD^2$ (Pyth. theorem)

$AB = \sqrt{AD^2 - BD^2}$

$= \sqrt{65^2 - 25^2}$ cm

$= 60$ cm

Area of $ABCD$ = area of $\triangle BCD$ + area of $\triangle ABD$

$= \left(\frac{1}{2} \times 15 \times 20 + \frac{1}{2} \times 60 \times 25\right) \text{cm}^2$

$= (150 + 750)$ cm^2

$= \underline{\underline{900 \text{ cm}^2}}$

16. A

In $\triangle ABF$,

$$\frac{AB}{BF} = \sin\beta$$

$$AB = BF\sin\beta$$

In $\triangle CDE$,

$$\frac{DC}{CE} = \sin\alpha$$

$$DC = CE\sin\alpha$$

$$AB = DC \quad \text{(property of rectangle)}$$

$$BF\sin\beta = CE\sin\alpha$$

$$\frac{BF}{CE} = \underline{\underline{\frac{\sin\alpha}{\sin\beta}}}$$

17. D

18. C

Let k be the area of $\triangle FEC$.

$\because$ $DE = AB$ (opp. sides of//gram)

$\therefore$ $DE : EC = AB : EC = 2 : 1$

$\triangle ADC \sim \triangle FEC$ (AAA)

$$\therefore \quad \frac{\text{Area of } \triangle FEC}{\text{Area of } \triangle ADC} = \left(\frac{EC}{DC}\right)^2$$

$$\frac{k}{\text{Area of } \triangle ADC} = \left(\frac{1}{1+2}\right)^2$$

$$\text{Area of } \triangle ADC = 9k$$

$\because$ Area of $\triangle ADE$:area of $\triangle ADC = DE : DC = 2 : 3$

$\therefore$ Area of $\triangle ADE = 9k \times \frac{2}{3} = 6k$

$\therefore$ Area of $\triangle FEC$: area of $\triangle ADE$

$= k : 6k$

$= \underline{\underline{1 : 6}}$

19. A

Let r_1 and r_2 be the base radii and h_1 and h_2 be the heights of the right circular cylinder and the right circular cone respectively.

Then, $r_1 = 3r_2$.

$\because$ Volume of the right circular cone

= volume of the right circular cylinder $\times 2$

$$\therefore \quad \frac{1}{3}\pi r_2^2 h_2 = \pi r_1^2 h_1 \times 2$$

$$\frac{1}{3} r_2^2 h_2 = (3r_2)^2 h_1 \times 2$$

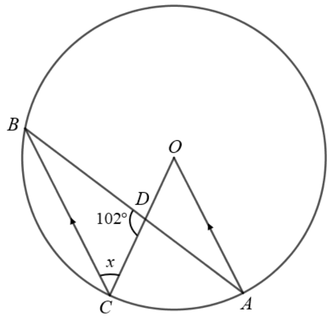
$$\frac{1}{3} r_2^2 h_2 = 9 r_2 h_1 \times 2$$

$$h_2 = 54 h_1$$

$$\frac{h_1}{h_2} = \frac{1}{54}$$

$\therefore$ The required ratio is 1 : 54.

20. D



Let $\angle BCO = x$.

$\angle AOC = \angle BCO = x$ (alt. $\angle$s, OA // BC)

$$\angle ABC = \frac{1}{2}\angle AOC = \frac{x}{2} \quad (\angle \text{ at centre twice } \angle \text{at } \odot^{ce})$$

In $\triangle BCD$,

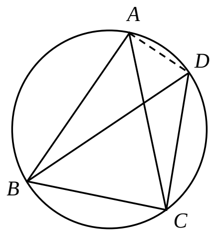
$$x + \frac{x}{2} + 102° = 180° \quad (\angle \text{ sum of } \triangle)$$

$$\frac{3x}{2} = 78°$$

$$x = 52°$$

$\therefore$ $\angle BCO = \underline{\underline{52°}}$

21. C



Join AD.

$\because$ $AC = AB$

$\therefore$ $\angle ACB = \angle ABC$ (base $\angle$s, isos. $\triangle$)

$= 70°$

$\angle BCD = \angle ACB + \angle ACD$

$= 70° + 20°$

$= 90°$

$\therefore$ BD is a diameter of the circle. (converse of $\angle$ in semi-circle)

$\angle BAD = 90°$ ($\angle$ in semi-circle)

$\angle ABD = \angle ACD$ ($\angle$s in same segment)

$= 20°$

In $\triangle ABD$,

$$\cos 20° = \frac{12 \text{ cm}}{BD}$$

$$BD \approx 12.770\,133\,27 \text{ cm}$$

$\angle DBC = \angle ABC - \angle ABD$

$= 70° - 20°$

$= 50°$

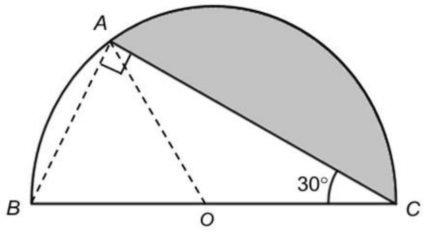
In $\triangle BCD$,

$$\sin 50° \approx \frac{CD}{12.770\,133\,27 \text{ cm}}$$

$CD = \underline{\underline{10 \text{ cm}}}$ (corr. to the nearest cm)

22. D

Let O be the centre of the semi-circle.
Join AB.



In $\triangle OAC$,

$\because$ $OA = OC$ (radii)

$\therefore$ $\angle OAC = \angle OCA$ (base $\angle$s, isos. $\triangle$)

$= 30°$

$\angle AOC + \angle OAC + \angle OCA = 180°$ ($\angle$ sum of $\triangle$)

$\angle AOC + 30° + 30° = 180°$

$\angle AOC = 120°$

Area of the shaded region

$$= \pi(OC)^2\left(\frac{120°}{360°}\right) - \frac{1}{2}(OA)(OC)\sin 120°$$

$$= \left[\pi(6)^2\left(\frac{120°}{360°}\right) - \frac{1}{2}(6)(6)\sin 120°\right]\text{cm}^2$$

$$= \left[12\pi - \frac{1}{2}(6)(6)\frac{\sqrt{3}}{2}\right]\text{cm}^2$$

$$= \underline{\underline{(12\pi - 9\sqrt{3})\text{cm}^2}}$$

23. C

Rectangular coordinates of $B = (-2,\ -(-5)) = (-2,\ 5)$

Rectangular coordinates of $A = (-2,\ 5-3) = (-2,\ 2)$

Let (r, θ) be the polar coordinates of A,
where $90° < \theta < 180°$.

$$\begin{cases} r\cos\theta = -2 & \text{......(1)} \\ r\sin\theta = 2 & \text{......(2)} \end{cases}$$

(2) ÷ (1): $\tan\theta = -1$

$\theta = 135°$ or $\theta = 315°$ (rejected)

By substituting $\theta = 135°$ into (2), we have

$r\sin 135° = 2$

$\frac{1}{\sqrt{2}}r = 2$

$r = 2\sqrt{2}$

$\therefore$ Polar coordinates of $A = \underline{\underline{(2\sqrt{2},\ 135°)}}$

24. C

$\because$ Slope of $L_1 = -\frac{1}{3}$ and slope of $L_2 = \frac{-3}{-1} = 3$

$\therefore$ L_1 is not parallel to L_2.

$\therefore$ The locus of P is the angle bisectors of the angles formed by L_1 and L_2, which are a pair of perpendicular lines.

25. C

$\because$ $AP \perp AB$

$\therefore$ The equation of the locus of P is

$$\frac{y-(-6)}{x-3} \times \frac{2-(-6)}{-5-3} = -1$$

$$\frac{y+6}{x-3} = 1$$

$$x - y - 9 = 0$$

26. B

Centre of the circle $= \left(-\frac{-10}{2}, -\frac{8}{2}\right) = (5, -4)$

$\because$ (4, –8) is the mid-point of PQ.

$\therefore$ The line connecting (4, –8) and the centre of the circle is perpendicular to PQ, i.e. perpendicular to L.
(line joining centre to mid-point of chord $\perp$ chord)

Let k be y-intercept of L.

$$\frac{k-(-8)}{0-4} \times \frac{-4-(-8)}{5-4} = -1$$

$$k = -7$$

$\therefore$ The y-intercept of L is –7.

27. A

The equation of the circle C is:

$$2x^2 + 2y^2 + 8x + 4y - 5 = 0$$

$$x^2 + y^2 + 4x + 2y - \frac{5}{2} = 0$$

Centre of $C = \left(-\frac{4}{2}, -\frac{2}{2}\right) = (-2, -1)$

Radius of C

$$= \sqrt{\left(\frac{4}{2}\right)^2 + \left(\frac{2}{2}\right)^2 - \left(-\frac{5}{2}\right)} = \sqrt{\frac{15}{2}}$$

For option A:

Circumference of C

$$= \pi\left(2\sqrt{\frac{15}{2}}\right) \approx 17.2072 < 20$$

$\therefore$ Option A is true.

For option B:

$\because$ The x-coordinate of the centre of C is not -4.

$\therefore$ Option B is not true.

For option C:

$$\begin{cases} 2x^2 + 2y^2 + 8x + 4y - 5 = 0 & \text{......(1)} \\ y = 0 & \text{......(2)} \end{cases}$$

By substituting (2) into (1), we have

$$2x^2 + 8x - 3 = 0$$

Hence, we have $\Delta = 8^2 - 4(2)(-5) = 104 > 0$

$\therefore$ C intersects the x-axis at 2 points.

$\therefore$ Option C is not true.

For option D:

Distance between the origin (0, 0) and the centre of C

$$= \sqrt{(-2-0)^2 + (-1-0)^2}$$

$$= \sqrt{(-2)^2 + (-1)^2}$$

$$= \sqrt{5}$$

$$< \sqrt{\frac{15}{2}} = \text{the radius of } C$$

i.e. The origin lies inside C.

$\therefore$ Option D is not true.

$\therefore$ The answer is B.

28. D

The table below shows all the possible outcomes.

1st card \ 2nd card	1	2	4	5	7	8
1		3	5	6	8	9
2	3		6	7	9	10
4	5	6		9	11	12
5	6	7	9		12	13
7	8	9	11	12		15
8	9	10	12	13	15	

$\therefore \quad P(\text{sum is less than } 10) = \frac{18}{30} = \underline{\underline{\frac{3}{5}}}$

29. C

$$\text{Mean number of children} = \frac{0\times8+1\times n+2\times7+3\times9+4\times5}{8+n+7+9+5}$$

$$2 = \frac{61+n}{29+n}$$

$$58+2n = 61+n$$

$$n = 3$$

Standard deviation of the distribution

$$= \sqrt{\frac{(0-2)^2\times8+(1-2)^2\times3+(2-2)^2\times7+(3-2)^2\times9+(4-2)^2\times5}{8+3+7+9+5}}$$

$$= \sqrt{2}$$

$$= \underline{\underline{1.41}} \text{ (cor. to 2 d.p.)}$$

30. B

For III:

$\because$ Inter-quartile range> 24 marks

$\therefore \quad (30+n)-(10+m) > 24$

$$20+n-m > 24$$

$$n-m > 4$$

$\therefore$ III is true.

For I and II:

From the diagram, we have

$0 \le m \le 9$ and $0 \le n \le 8$.

Take $m = 4$,

$$n-4 > 4$$

$n > 8$, which is impossible

Similarly, it is impossible for $m = 5, 6, 7, 8, 9$.

$\therefore \quad 0 \le m < 4$

$\therefore$ I is true.

Take $n = 4$,

$$4-m > 4$$

$m < 0$, which is impossible

Similarly, it is impossible for $n = 0, 1, 2, 3$.

$\therefore \quad 4 < n \le 8$

$\therefore$ II is not true.

$\therefore$ The answer is B.

31. D

$$4^{16}+8^{16} = (2^2)^{16}+(2^3)^{16}$$

$$= 2^{32}+2^{48}$$

$$= (2^4)^8+(2^4)^{12}$$

$$= 1\times16^8+1\times16^{12}$$

$$= \underline{\underline{1000100000000_{16}}}$$

32. A

The graph of $y = f(-x)$ can be obtained by reflecting the graph of $y = f(x)$ about the y-axis.

Note that $f[-(x-1)] = f(-x+1)$.

So, the graph of $y = f(-x+1)$ can be obtained by translating the graph of $y = f(-x)$ rightwards by 1 unit.

$\therefore$ The answer is A.

33. A

$$(\log_9 x)^2 + \log_9 x^2 + k = \log_9 x$$

$$(\log_9 x)^2 + 2\log_9 x + k = \log_9 x$$

$$(\log_9 x)^2 + \log_9 x + k = 0$$

Then, we have

$$\log_9 \alpha + \log_9 \beta = -1$$

$$\log_9 \alpha\beta = -1$$

$$\alpha\beta = 9^{-1}$$

$\log_3 \alpha\beta$

$= \log_3 9^{-1}$

$= \underline{\underline{-2}}$

34. B

$$\frac{y^3-12}{\log_5 x - 0} = \frac{12-0}{0-3}$$

$$y^3 - 12 = -4\log_5 x$$

$$3 - \frac{y^3}{4} = \log_5 x$$

$$x = 5^{3-\frac{y^3}{4}}$$

When $y = 2$, $x = 5^{3-\frac{2^3}{4}} = \underline{\underline{5}}$

35. A

For I:

$$T(2)-T(1) = 2-3m-(1-2m)$$

$$= -m+1$$

$$T(3)-T(2) = 3-4m-(2-3m)$$

$$= -m+1$$

$$T(4)-T(3) = 4-5m-(3-4m)$$

$$= -m+1$$

$\because \quad T(2)-T(1) = T(3)-T(2) = T(4)-T(3)$

$\therefore$ It is an arithmetic sequence.

$\therefore$ I is true.

For II:

$$T(2)-T(1) = \log m^2 - \log m$$

$$= \log\left(\frac{m^2}{m}\right)$$

$$= \log m$$

$$T(3)-T(2) = \log m^3 - \log m^2$$

$$= \log\left(\frac{m^3}{m^2}\right)$$

$$= \log m$$

$$T(4)-T(3) = \log m^4 - \log m^3$$

$$= \log\left(\frac{m^4}{m^3}\right)$$

$$= \log m$$

$\because \quad T(2)-T(1) = T(3)-T(2) = T(4)-T(3)$

$\therefore$ It is an arithmetic sequence.

$\therefore$ II is true.

For III:

$T(2)-T(1)=2^{m+2}-2^{m+1}$

$=2^{m+1}(2-1)$

$=2^{m+1}$

$T(3)-T(2)=2^{m+3}-2^{m+2}$

$=2^{m+2}(2-1)$

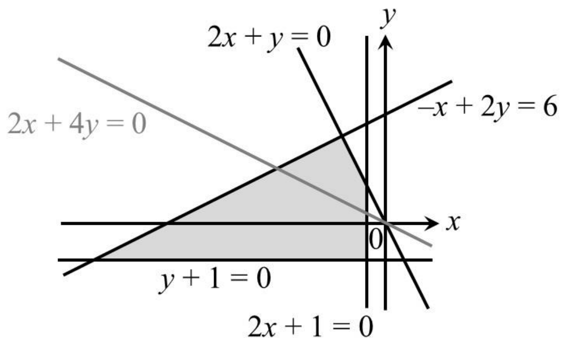
$=2^{m+2}$

36. C

$$\frac{i^{2022}}{k+i^{2021}}=\frac{i^{4\times505+2}}{k+i^{4\times505+1}}$$
$$=\frac{(i^4)^{505}i^2}{k+(i^4)^{505}i}$$
$$=\frac{i^2}{k+i}$$
$$=\frac{-1}{k+i}\times\frac{k-i}{k-i}$$
$$=\frac{-k+i}{k^2-i^2}$$
$$=\frac{-k+i}{k^2-(-1)}$$
$$=\frac{-k}{k^2+1}+\frac{1}{k^2+1}i$$

$\therefore$ The imaginary part of $\frac{i^{2022}}{k+i^{2021}}$ is $\frac{1}{k^2+1}$.

37. B



From the graph, $2x+4y+5=0$ attains its maximum at the point of intersection of $2x+y=0$ and $-x+2y=6$.

$$\begin{cases}2x+y=0 & ......(1)\\ -x+2y=6 & ......(2)\end{cases}$$

(1) + 2×(2):

$5y=12$

$y=2.4$

By substituting $y=2.4$ into (1), we have

$2x+2.4=0$

$x=-1.2$

i.e. $2x+y=0$ intersects $-x+2y=6$ at $(-1.2, 2.4)$.

At$(-1.2, 2.4)$, $2x+4y+5=2(-1.2)+4(2.4)+5=12.2$

$\therefore$ The greatest value of $2x+4y+5=\underline{\underline{12.2}}$

38. D

$$\begin{cases}2x+y+k=0 & ......(1)\\ x^2+y^2+12x-8y+32=0 & ......(2)\end{cases}$$

From (1), we have

$y=-2x-k$ ......(3)

By substituting (3) into (2), we have

$$x^2+(-2x-k)^2+12x-8(-2x-k)+32=0$$
$$x^2+4x^2+4kx+k^2+12x+16x+8k+32=0$$
$$5x^2+4(k+7)x+k^2+8k+32=0 \quad ......(*)$$

$\because$ The circle and the straight line do not intersect.

$\therefore$

$$\Delta \text{ of } (*)<0$$
$$[4(k+7)]^2-4(5)(k^2+8k+32)<0$$
$$4(k+7)^2-5(k^2+8k+32)<0$$
$$4k^2+56k+196-5k^2-40k-160<0$$
$$-k^2+16k+36<0$$
$$k^2-16k-36>0$$
$$(k+2)(k-18)>0$$

Hence, we have $k<-2$ or $k>18$.

39. B

Let $P(a, 0)$ be the coordinates of the circumcentre,
$A=(-2, 4)$ and $B=(8, 6)$.
P lies on the perpendicular bisector of AB.
So, $PA=PB$

$$\sqrt{[a-(-2)]^2+(0-4)^2}=\sqrt{(a-8)^2+(0-6)^2}$$
$$a^2+4a+4+16=a^2-16a+64+36$$
$$20a=80$$
$$a=\underline{\underline{4}}$$

40. D

$\angle BAF=\angle ACB=25°$ ($\angle$ in alt. segment)

$\angle DCB=90°$($\angle$ in semi-circle)

$\angle DCE=\angle DCB-\angle ACB$

$=90°-25°$

$=65°$

$\angle DBA=\angle DCA=65°$ ($\angle$s in the same segment)

In $\triangle BAF$,

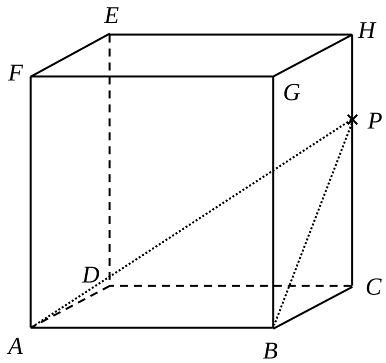
$\angle DBA=\angle BAF+\angle BFA$ (ext. $\angle$ of $\triangle$)

$65°=25°+\angle BFA$

$\angle BFA=\underline{\underline{40°}}$

41. B

Join AP and BP.



The angle between AP and the plane $BCHG$ is $\angle APB$, i.e. $\angle APB = \theta$.

In $\triangle PCB$,

$$BP^2 = BC^2 + CP^2 \quad \text{(Pyth. theorem)}$$

$$BP = \sqrt{12^2 + 9^2} \text{ cm}$$

$$= 15 \text{ cm}$$

$\because$ $ABCDEFGH$ is a cuboid.

$\therefore$ $AB \perp BP$

In $\triangle ABP$,

$$AP^2 = AB^2 + BP^2 \quad \text{(Pyth. theorem)}$$

$$AP = \sqrt{20^2 + 15^2} \text{ cm}$$

$$= 25 \text{ cm}$$

$$\therefore \quad \sin\theta = \frac{AB}{AP} = \frac{20}{25} = \underline{\underline{\frac{4}{5}}}$$

42. B

Number of different queues formed

$= P_2^8 \times 8!$

$= \underline{\underline{2\ 257\ 920}}$

43. D

The required probability

$$= 1 - \frac{C_3^6 C_1^{10}}{C_4^{16}} - \frac{C_4^6}{C_4^{16}}$$

$$= \underline{\underline{\frac{321}{364}}}$$

44. B

Let $\bar{x}$ and σ be the mean and the standard deviation of the test scores respectively.

The standard score of Lily is −1.5, we have

$$\frac{45 - \bar{x}}{\sigma} = -1.5$$

$$45 - \bar{x} = -1.5\sigma \quad \text{……(1)}$$

The standard score of Henry is 1, we have

$$\frac{70 - \bar{x}}{\sigma} = 1$$

$$70 - \bar{x} = \sigma \quad \text{……(2)}$$

(2) – (1): $25 = 2.5\sigma$

$\sigma = 10$

$\therefore$ The standard deviation of the test scores is 10 marks.

45. C

Add 5 to each datum of {a – 5, b – 5, c – 5, d – 5, e – 5, f – 5}, we get another data set {a, b, c, d, e, f} and its median, range and variance are m_1 + 5, r_1 and v_1 respectively.

Multiply each datum of {a, b, c, d, e, f} by 2, we get {2a, 2b, 2c, 2d, 2e, 2f } and its median, range and variance are 2(m_1 + 5), 2r_1 and 2^2v_1 respectively.

$\therefore$ $m_2 = 2(m_1 + 5) = 2m_1 + 10$

which is NOT $2m_1 + 5$

$r_2 = 2r_1$

$v_2 = 2^2v_1 = 4v_1$

$\therefore$ only III and III are true.

$\therefore$ The answer is C.