Solution	Marks	Remarks
1. $\dfrac{(m^3n^{-2})^{-4}}{m^{-3}n^{10}}$		
$=\dfrac{m^{-12}n^8}{m^{-3}n^{10}}$	1M	for $(ab)^\ell = a^\ell b^\ell$ or $(a^h)^k = a^{hk}$
$= m^{-12-(-3)}n^{8-10}$	1M	for $\dfrac{c^p}{c^q} = c^{p-q}$ or $\dfrac{c^p}{c^q} = \dfrac{1}{c^{q-p}}$
$= m^{-9}n^{-2}$		
$=\dfrac{1}{m^9n^2}$	1A	
	(3)	
2. $x = \dfrac{y+2z}{3+z}$		
$x(3+z) = y+2z$		
$3x + xz = y+2z$	1M	
$xz - 2z = y - 3x$	1M	for putting z on one side
$(x-2)z = y-3x$		
$z = \dfrac{y-3x}{x-2}$	1A	or equivalent
	(3)	
3. **(a)** $a^2+3ab+2b^2$		
$=(a+b)(a+2b)$	1A	or equivalent
(b) $a^2+3ab+2b^2-4a^2b-8ab^2$		
$=(a+b)(a+2b)-4ab(a+2b)$	1M	for using the result of (a)
$=(a+2b)(a+b-4ab)$	1A	or equivalent
	(3)	
4. **(a)** Marked price of the wallet		
$=\$120\times(1+25\%)$	1M	
$=\$150$	1A	
(b) Selling price of the wallet		
$=\$150\times(1-10\%)$		
$=\$135$		
The required percentage profit		
$=\dfrac{\$(135-120)}{\$120}\times100\%$	1M	
$=12.5\%$	1A	
	(4)	

Solution	Marks	Remarks
5. **(a)** $-6x \geq 12$		
$x \leq -2$	1A	
$\frac{2x+5}{3} > 2(x+1)$		
$2x+5 > 6(x+1)$		
$2x+5 > 6x+6$		
$-4x > 1$		
$x < -\frac{1}{4}$	1A	$x < -0.25$
Therefore, we have $x \leq -2$ and $x < -\frac{1}{4}$.		
Thus, the solutions of (*) are $x \leq -2$.	1A	
(b) -2	1A	
	(4)	
6. Let x be the number of stamps Cathy has.		
Then, the number of stamps John has is $x - 6$.	1A	
$x+3 = 4[(x-6)-3]$	1A + 1M	1M for getting a linear equation
$x+3 = 4(x-9)$		
$x+3 = 4x-36$		
$39 = 3x$		
$x = 13$		
Thus, the total number of stamps Cathy and John have is 20.	1A	
Alternative Solution		
Let x and y be the numbers of stamps that Cathy and John have respectively.		
$\begin{cases} x-3 = y+3 \\ x+3 = 4(y-3) \end{cases}$	1A + 1A	
So, we have $x+3 = 4[(x-6)-3]$.	1M	for getting a linear equation in x or y only
Solving, we have $x = 13$ and $y = 7$.		
Thus, the total number of stamps Cathy and John have is 20.	1A	
	(4)	
7. **(a)** $\frac{1\times 43 + 2\times k + 3\times 20 + 4\times 62 + 5\times 20}{43+k+20+62+20} = 2.7$	1M	
$451 + 2k = 2.7(145+k)$		
$451 + 2k = 391.5 + 2.7k$		
$59.5 = 0.7k$		
$k = \underline{\underline{85}}$	1A	

Solution	Marks	Remarks
(b) Total number of customers $= 43 + 85 + 20 + 62 + 20$ $= 230$ The median of the ratings = 2 The required probability		
$= \dfrac{20 + 62 + 20}{230}$	1M	
$= \dfrac{51}{115}$	1A	r.t. 0.443
	(4)	
8. **(a)** Let $f(x) = \dfrac{h}{\sqrt{x}} + k$, where h and k are non-zero constants.	1A	
So, we have $\dfrac{h}{\sqrt{4}} + k = 15$ and $\dfrac{h}{\sqrt{25}} + k = 9$.	1M	for either substitution
i.e. $\dfrac{h}{2} + k = 15$ and $\dfrac{h}{5} + k = 9$		
Solving, we have $h = 20$ and $k = 5$.	1A	for both correct
Thus, we have $f(x) = \dfrac{20}{\sqrt{x}} + 5$.		
(b) $f(x) = 10$ $\dfrac{20}{\sqrt{x}} + 5 = 10$		
$\dfrac{20}{\sqrt{x}} = 5$	1M	
$\sqrt{x} = 4$		
$x = 16$	1A	
	(5)	
9. **(a)** The maximum absolute error $= 0.05$ m		
The lower limit of the actual length of the ribbon $= (2.6 - 0.05)$ m $= 2.55$ m	1A	for either one
The upper limit of the actual length of the ribbon $= (2.6 + 0.05)$ m $= 2.65$ m		
Thus, the required range is $2.55 \text{ m} \le \text{actual length of the ribbon} < 2.65 \text{ m}$.	1A	

Solution	Marks	Remarks
(b) The least possible total length of 180 smaller pieces of ribbon		
$= 180 \times 1.495$ cm	1M	
$= 269.1$ cm	1A	
$= 2.691$ m		
> 2.65 m		
Thus, Penny's claim is disagreed.	1A	f.t.
Alternative Solution		
Note that		
$\frac{2.65}{180}$ m	1M	
$\approx 0.014\ 722\ 222$ m		
$= 1.472\ 222\ 2$ cm	1A	
< 1.495 cm		
Thus, Penny's claim is disagreed.	1A	f.t.
Alternative Solution		
Note that		
$\frac{(2.65)(100)}{1.495}$	1M	
$\approx 177.257\ 525\ 1$	1A	
< 180		
Thus, Penny's claim is disagreed.	1A	f.t.
	(5)	
10. **(a)** The mid-point of $AB = (1, 4)$		
The slope of $AB = \frac{3-5}{-4-6}$	1M	
$= \frac{1}{5}$		
Thus, the slope of L is -5.		
The equation of L is		
$y - 4 = -5(x - 1)$	1M	
$5x + y - 9 = 0$	1A	or equivalent
(b) Note that perpendicular bisector L of chord AB passes through the centre of the circle.	1M	
The coordinates of the centre of the circle are $(a, -2a)$.		
By (a), we have $5a - 2a - 9 = 0$.	1M	
So, we have $a = 3$.	1A	
	(6)	

Solution	Marks	Remarks
11. (a) Note that $f(x)=(x-1)[x^2+(h-1)x-h]+k$.	1M	
Also, $f(-h)=5$ and $f(2)=0$.		
So, we have $(-h-1)[h^2-h(h-1)-h]+k=5$ and		
$(2-1)[2^2+2(h-1)-h]+k=0$.	1M	for either substitution
i.e. $k = 5$ and $2 + h + k = 0$		
Solving, we have $h = -7$ and $k = 5$.	1A + 1A	
(b) $f(x)=0$		
$(x-1)(x^2-8x+7)+5=0$ (By (a))		
$x^3-9x^2+15x-2=0$	1A	
$(x-2)(x^2-7x+1)=0$	1M	for $(x-2)(ax^2+bx+c)$
$x-2=0$ or $x^2-7x+1=0$		
$x=2$ or $x=\dfrac{7\pm 3\sqrt{5}}{2}$		
Note that the roots $\dfrac{7+3\sqrt{5}}{2}$ and $\dfrac{7-3\sqrt{5}}{2}$ are irrational numbers.		
Hence, the claim is disagreed.	1A	f.t.
	(7)	
12. (a) $\angle DAE=\angle BAE$ and $\angle CBE=\angle ABE$.		
$\angle DAB+\angle ABC=180°$ (int. $\angle$s, $AD \parallel BC$)		
$2\angle BAE+2\angle ABE=180°$		
$\angle BAE+\angle ABE=90°$		
$\angle BAE+\angle ABE+\angle AEB=180°$ ($\angle$ sum of $\triangle$)		
$90°+\angle AEB=180°$	1M	
$\angle AEB=90°$		
Thus, $\triangle ABE$ is a right-angled triangle.	1A	f.t.
(b) (i) $\triangle BPE$ is congruent to $\triangle BCE$.	1A	
In $\triangle BCE$ and $\triangle BPE$,		
$\because$ $\angle BCE=180°-\angle ADE$ (int. $\angle$s, $AD \parallel BC$)		
$\angle BPE=180°-\angle APE$ (adj. $\angle$s on st. line)		
$=180°-\angle ADE$ (corr. $\angle$s, $\cong\triangle$s)		
$\therefore$ $\angle BCE=\angle BPE$		
$\angle CBE=\angle PBE$		
$BE = BE$ (common side)		
$\triangle BCE\cong\triangle BPE$ (AAS)		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	

Solution	Marks	Remarks
(ii) $AB^2 = AE^2 + BE^2$ (Pyth. theorem) $AB = \sqrt{12^2 + 5^2}$ cm $= 13$ cm $AD = AP$ (corr. sides, $\cong \triangle$s) $BC = BP$ (corr. sides, $\cong \triangle$s) $AD + BC = AP + BP$ $= AB$ $= \underline{\underline{13 \text{ cm}}}$	1M 1A	
	(7)	
13. (a) Note that the range of the distribution is 35 cm. Thus, the length of the shortest trial pipe = (159 − 35) cm = 124 cm $\therefore \quad a = \underline{\underline{4}}$ Note that the lower quartile and the inter-quartile range of the distribution are 128 cm and 13 cm respectively. Thus, the upper quartile = (128 + 13) cm = 141 cm $\therefore \quad b = \underline{\underline{1}}$	 1A 1M 1A	
(b) Let x cm and y cm be the lengths of the two added trial pipes, where $x \le y$. Note that the original mean is 137 cm. Thus, we have $\dfrac{x + y + 137(20)}{22} = 137 - 1$. Solving, we have $x + y = 252$. Since the range is increased by 1 cm, the new range is 36 cm. There are two cases. Case 1: $x = 123$ Since $x + y = 252$, we have $y = 129$. Case 2: $y = 160$ Since $x + y = 252$, we have $x = 92$. In this case, the new range is 68 cm. It is impossible. Thus, the lengths of the two added trial pipes are 123 cm and 129 cm.	 1M 1M 1A + 1A	 either one (Case 1 / Case 2)
	(7)	

Solution	Marks	Remarks
14. **(a)** Let h cm be the height of a smaller square pyramid. The height of the largest square pyramid cut off is equal to the side of the square base. Since the 27 smaller square pyramids are similar to the larger square pyramids, the side of the square base of a smaller square pyramid is also h cm.	1M	
$27\times\frac{1}{3}\times(h\times h)\times h = 4\times\left[12^3-\frac{1}{3}\times(12\times12)\times12\right]$	1M	
$9h^3 = 4608$ $h = 8$		
Thus, the height of a smaller square pyramid is 8 cm.	1A	
(b) $\frac{\text{Total surface area of a smaller square pyramid}}{\text{Total surface area of a larger square pyramid}}$		
$=\left(\frac{8}{12}\right)^2$	1M	
$=\frac{4}{9}$		
Thus, the required ratio is 4 : 9.	1A	
(c) Let a cm and H cm be the length of the sides of the metal cubes used at the beginning and the new height of a smaller square pyramid respectively. Thus, the new height of the largest square pyramid cut off is a cm.		
$27\times\frac{1}{3}\times(H\times H)\times H = 4\times\left[a^3-\frac{1}{3}\times(a\times a)\times a\right]$	1M	
$9H^3 = \frac{8a^3}{3}$ $\left(\frac{H}{a}\right)^3 = \frac{8}{27}$ $\frac{H}{a} = \frac{2}{3}$		
Therefore, we have $\left(\frac{H}{a}\right)^2 = \frac{4}{9}$.	1M	
i.e. The ratio in (b) remains unchanged. Thus, Kelvin's claim is disagreed.	1A	f.t.
	(8)	

Solution	Marks	Remarks
15. $\dfrac{\log_{16} y - 1}{x - 0} = \dfrac{0 - 1}{-4 - 0}$	1M	
$\dfrac{\log_{16} y - 1}{x} = \dfrac{1}{4}$		
$\log_{16} y = \dfrac{1}{4}x + 1$		
$y = 16^{\frac{1}{4}x+1}$	1M	
$= 2^{x+4}$	1A	
Alternative Solution		
$\dfrac{\log_{16} y - 1}{x - 0} = \dfrac{0 - 1}{-4 - 0}$	1M	
$\dfrac{\log_{16} y - 1}{x} = \dfrac{1}{4}$		
$\log_{16} y = \dfrac{1}{4}x + 1$		
$\log_{16} y = \left(\dfrac{1}{4}x + 1\right)\log_{16} 16$		
$\log_{16} y = \log_{16} 16^{\frac{1}{4}x+1}$	1M	
$\log_{16} y = \log_{16} 2^{x+4}$		
$y = 2^{x+4}$	1A	
	(3)	
16. **(a)** The required probability		
$= \dfrac{9!4!}{(4+8)!}$	1M	
$= \dfrac{1}{55}$	1A	r.t. 0.0182
(b) The required probability		
$= \dfrac{C_4^9 4!8!}{(4+8)!}$	1M	
$= \dfrac{14}{55}$	1A	r.t. 0.255
	(4)	
17. **(a)** Let m and σ be the mean and the standard deviation of the scores respectively.		
Note that the standard scores of Ray and Cindy are 0 and –1.5 respectively.		
Thus, we have $\dfrac{68 - m}{\sigma} = 0$ and $\dfrac{50 - m}{\sigma} = -1.5$.	1M	for either one
Solving, we have m = 68 and σ = 12.	1A + 1A	
Thus, the mean and the standard deviation of the scores are 68 marks and 12 marks respectively.		

Solution	Marks	Remarks
(b) Note that the score of Ray is equal to the mean of the scores. So, the mean of the scores remains unchanged and the distribution of the scores is more dispersed. Therefore, the standard deviation of the scores is larger. Hence, the standard score of Cindy will be less negative. i.e. The standard score of Cindy will increase. Thus, Cindy's claim is agreed.	 1M 1A	 f.t.
	(5)	
18. (a) $b_1 + b_2 + b_3 + \ldots = 12\ 500$ $\frac{3125}{m} + \frac{3125}{m^2} + \frac{3125}{m^3} + \ldots = 12\ 500$ $\frac{\frac{3125}{m}}{1 - \frac{1}{m}} = 12\ 500$ $\frac{3125}{m} = 12\ 500\left(1 - \frac{1}{m}\right)$ $\frac{15\ 625}{m} = 12\ 500$ $m = \frac{5}{4}$	 1M 1A	 1.25
(b) Note that z_k is purely imaginary. We have $a_1 + a_2 + \ldots + a_k = 0$ $-38 + (-19) + \ldots + (19k - 57) = 0$ $\frac{k}{2}[-38 + (19k - 57)] = 0$ $\frac{k}{2}(19k - 95) = 0$ $k = 5$ Thus, $z_5 = (b_1 + b_2 + b_3 + b_4 + b_5)i$ $= \left[\frac{3125}{\frac{5}{4}} + \frac{3125}{\left(\frac{5}{4}\right)^2} + \frac{3125}{\left(\frac{5}{4}\right)^3} + \frac{3125}{\left(\frac{5}{4}\right)^4} + \frac{3125}{\left(\frac{5}{4}\right)^5}\right]i$ $= 8404i$	 1M 1M 1A	
	(5)	

Solution	Marks	Remarks
19. **(a)** Note that *VA* = *AB* and *VM* = *MB*. So, we have $AM \perp VB$. (prop. of isos. △) $\angle ABM = \frac{180°}{3} = 60°$ (prop. of equil. △) Consider right-angled triangle *ABM*. $\sin \angle ABM = \frac{AM}{AB}$ $\sin 60° = \frac{AM}{a \text{ cm}}$ $AM = \frac{\sqrt{3}a}{2}$ cm	1M	
Note that *VM* = *MB* and *VN* = *NC*. So, we have $MN = \frac{BC}{2} = \frac{a}{2}$ cm. (mid-pt. theorem)	1M	
Let *P* be the point on *MN* such that $AP \perp MN$. Since *AM* = *AN* and $AP \perp MN$, *MP* = *NP* and $\angle MAP = \angle NAP$. (prop. of isos. △) $MP = \frac{MN}{2} = \frac{a}{4}$ cm Consider right-angled triangle *AMP*. $\sin \angle MAP = \frac{MP}{AM}$	1M	
$= \frac{\frac{a}{4}}{\frac{\sqrt{3}a}{2}}$ $\angle MAP \approx 16.778\ 654\ 88°$ $\angle MAN = 2\angle MAP$ $\approx 2 \times 16.778\ 654\ 88°$ $= 33.557\ 309\ 76°$ $\approx \underline{\underline{33.6°}}$	1A	r.t. 33.6°

Solution	Marks	Remarks
Alternative Solution Note that $VA = AB$ and $VM = MB$. So, we have $AM \perp VB$. (prop. of isos. $\triangle$) $\angle ABM = \frac{180°}{3} = 60°$ (prop. of equil. $\triangle$) Consider right-angled triangle ABM. $\sin \angle ABM = \frac{AM}{AB}$ $\sin 60° = \frac{AM}{a \text{ cm}}$ $AM = \frac{\sqrt{3}a}{2}$ cm	1M	
Similarly, $AN = \frac{\sqrt{3}a}{2}$ cm . Note that $VM = MB$ and $VN = NC$. So, we have $MN = \frac{BC}{2} = \frac{a}{2}$ cm . (mid-pt. theorem)	1M	
By cosine formula, we have $\cos \angle MAN = \frac{AM^2 + AN^2 - MN^2}{2(AM)(AN)}$	1M	
$= \frac{2AM^2 - MN^2}{2AM^2}$ $= 1 - \frac{MN^2}{2AM^2}$ $= \frac{5}{6}$ $\angle MAN \approx 33.6°$	1A	r.t. 33.6°

Solution	Marks	Remarks
(b) Note that $VX < \frac{1}{2}VB$ and $VY < \frac{1}{2}VC$.		
So, we have $VX < VM$ and $VY < VN$.	1M	
Since $AM \perp VB$, AM is the shortest distance from A to VB. So, we have $AX > AM$.		
Since $VX = VY$, $AX = AY$.		
Let U be the point on XY such that $AU \perp XY$.		
Since $AX = AY$ and $AU \perp XY$,		
$XU = YU$ and $\angle XAU = \angle YAU$. (prop. of isos. $\triangle$)		
Since $XY < MN$, $XU < MP$.		
Consider right-angled triangle AXU.		
$\sin \angle XAU = \frac{XU}{AX}$		
$< \frac{MP}{AM}$	1M	
$= \sin \angle MAP$		
So, we have $\angle XAU < \angle MAP$.	1M	
i.e. $\angle XAY < \angle MAN$		
Thus, Tommy's claim is agreed.	1A	f.t.
Alternative Solution		
Note that $VX < \frac{1}{2}VB$ and $VY < \frac{1}{2}VC$.		
So, we have $VX < VM$ and $VY < VN$.	1M	
Also, $VX = VY$.		
Thus, $AX = AY$ and $XY < MN$.		
Since $AM \perp VB$, AM is the shortest distance from A to VB. So, we have $AX > AM$.		
By cosine formula, we have		
$\cos \angle XAY = \frac{AX^2 + AY^2 - XY^2}{2(AX)(AY)}$		
$= \frac{2AX^2 - XY^2}{2AX^2}$		
$= 1 - \frac{XY^2}{2AX^2}$		
$> 1 - \frac{MN^2}{2AM^2}$	1M	
$= \cos \angle MAN$		
So, we have $\angle XAY < \angle MAN$.	1M	
Thus, Tommy's claim is agreed.	1A	f.t.
	(8)	

Solution	Marks	Remarks
20. **(a)** Since $\angle ABC + \angle ADC = 180°$,		
$ABCD$ is a cyclic quadrilateral. (opp. $\angle$s supp.)	1A	
A, B, C and D lie on the same circle, any triangle formed by three of the four points share the same circumcentre.		
So, the circumecentre of $\triangle ABC$ is also the circumcentre of $\triangle ABD$.	1A	
(b) **(i)** The coordinates of points Y and Z are $(-1, 7)$ and $(-1, 7 - h)$ respectively.	1A	for correct coordinates of Z
Note that OX and OZ are perpendicular to each other.		
Thus, we have		
$\frac{7-0}{7-0} \times \frac{(7-h)-0}{-1-0} = -1$	1M	
$7 - h = 1$		
$h = \underline{\underline{6}}$	1A	
(ii) Since $\angle XYZ + \angle XOZ = 90° + 90° = 180°$,		
the circumcentre of $\triangle XYZ$ is also the circumcentre of $\triangle OXY$. (By (a))	1M	
Thus, Isaac's claim is disagreed.	1A	f.t.
(iii) Since the circumcentre of $\triangle XYP$ is also the circumcentre of $\triangle XYZ$, X, Y, Z and P lie on the same circle.		
i.e. P lies on the circumcircle of $\triangle XYZ$.		
So, the locus of P is the circumcircle of $\triangle XYZ$.	1A	
Let $x^2 + y^2 + Dx + Ey + F = 0$ be the equation of Γ.		
So, we have $\begin{cases} 7^2 + 7^2 + 7D + 7E + F = 0 \\ (-1)^2 + 7^2 - D + 7E + F = 0 \\ (-1)^2 + 1^2 - D + E + F = 0 \end{cases}$.	1M	or equivalent
Solving, we have $D = -6$, $E = -8$ and $F = 0$.		
Thus, the equation of Γ is $x^2 + y^2 - 6x - 8y = 0$.	1A	$(x-3)^2 + (y-4)^2 = 25$

Solution	Marks	Remarks
Alternative Solution		
P lies on the circumcircle of $\triangle XYZ$.		
So, the locus of P is the circumcircle of $\triangle XYZ$.	1A	
Since $\angle XYZ = 90°$, XZ is a diameter of the circumcircle of $\triangle XYZ$. (converse of $\angle$ in semi-circle)	1M	
Thus, the coordinates of the circumcentre are (3, 4).		
Radius of the circumcircle of $\triangle XYZ$ $= \sqrt{(7-3)^2 + (7-4)^2}$ $= 5$		
Thus, the equation of Γ is $(x-3)^2 + (y-4)^2 = 25$.	1A	$x^2 + y^2 - 6x - 8y = 0$
	(10)	