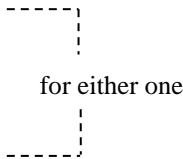


Solution	Marks	Remarks
1. $\frac{(x^{-3}y)^2 y^{-4}}{x^{-2}}$ $= \frac{x^{-6}y^2 y^{-4}}{x^{-2}}$ $= x^{-6-(-2)} y^{2+(-4)}$ $= x^{-4} y^{-2}$ $= \frac{1}{\underline{\underline{x^4 y^2}}}$	 1M 1M 1A	 for $(ab)^\ell = a^\ell b^\ell$ or $(a^h)^k = a^{hk}$ for $\frac{c^p}{c^q} = c^{p-q}$ or $\frac{c^p}{c^q} = \frac{1}{c^{q-p}}$
(3)		
2. $\frac{a}{a+1} = \frac{2}{b+1}$ $a(b+1) = 2(a+1)$ $ab + a = 2a + 2$ $ab - a = 2$ $a(b-1) = 2$ $a = \frac{2}{b-1}$	 1M 1M 1A	 for putting a on one side or equivalent
(3)		
3. (a) $2m^2 - 8n^2$ $= 2(m^2 - 4n^2)$ $= \underline{\underline{2(m+2n)(m-2n)}}$ (b) $3m - 2m^2 + 8n^2 - 6n$ $= 3m - 6n - 2m^2 + 8n^2$ $= 3m - 6n - (2m^2 - 8n^2)$ $= 3(m-2n) - 2(m+2n)(m-2n)$ $= (m-2n)[3 - 2(m+2n)]$ $= \underline{\underline{(m-2n)(3-2m-4n)}}$	 1A 1M 1A	 or equivalent for using the result of (a) or equivalent
(3)		
4. (a) $x - 2 \leq 0$ $x \leq 2$ $x - \frac{3x+2}{2} < 2$ $2x - (3x+2) < 4$ $2x - 3x - 2 < 4$ $-x < 6$ $x > -6$ <p>Therefore, we have $x \leq 2$ and $x > -6$.</p> <p>Thus, the required range is $-6 < x \leq 2$.</p>	 1A 1A 1A	

Solution	Marks	Remarks
(b) 4	1A (4)	
<p>5. (a) Let $y = \frac{k}{x^2}$, where k is a non-zero constant.</p> <p>So, we have $45 = \frac{k}{2^2}$.</p> <p>Solving, we have $k = 180$.</p> <p>Thus, we have $y = \frac{180}{x^2}$.</p> <p>(b) $20 = \frac{180}{x^2}$ $x^2 = 9$ $x = \underline{3}$ or $\underline{-3}$</p>	1A 1M 1A 1A (4)	for substitution
<p>6. Let $\\$C$ be the original cost price of the toy.</p> <p>Original selling price of the toy $= \\$C \times (1 + 20\%)$ $= \\$1.2C$</p> <p>New cost price of the toy $= \\$C \times (1 + 25\%)$ $= \\$1.25C$</p> <p>New selling price of the toy $= \\$(1.2C + 36)$</p> <p>$1.25C \times (1 + 20\%) = 1.2C + 36$ $1.5C = 1.2C + 36$ $0.3C = 36$ $C = 120$</p> <p>Thus, the original cost price of the toy is \$120.</p>	1A 1A 1M 1A (4)	
<p>7. The maximum absolute error $= \frac{1}{2} \times 1 \text{ cm}$ $= 0.5 \text{ cm}$</p> <p>The lower limit of the actual diameter of the semi-circle $= (8 - 0.5) \text{ cm}$ $= 7.5 \text{ cm}$</p> <p>The upper limit of the actual diameter of the semi-circle $= (8 + 0.5) \text{ cm}$ $= 8.5 \text{ cm}$</p>	1M	 <p>for either one</p>

Solution	Marks	Remarks
<p>The lower limit of the actual area of the semi-circle</p> $= \frac{1}{2} \times \pi \times \left(\frac{7.5}{2}\right)^2 \text{ cm}^2$ $= 7.031\,25\pi \text{ cm}^2$ $\approx 22.089\,323\,34 \text{ cm}^2$ <p>The upper limit of the actual area of the semi-circle</p> $= \frac{1}{2} \times \pi \times \left(\frac{8.5}{2}\right)^2 \text{ cm}^2$ $= 9.031\,25\pi \text{ cm}^2$ $\approx 28.372\,508\,65 \text{ cm}^2$ <p>$\therefore 22.089\,323\,34 \text{ cm}^2 \leq \text{actual area of the semi-circle}$ $< 28.372\,508\,65 \text{ cm}^2$</p> <p>Note that 30 cm^2 is not within the range of the actual area of the semi-circle.</p> <p>Thus, the claim is disagreed.</p>	<p>1A</p> <p>1A</p> <p>1A</p> <p>(4)</p>	<p>f.t.</p>
<p>8. (a) $f(x) = (x^2 - 4)(x^{2018} + 2x^{2017}) + 3x + k$, where k is a constant.</p> <p>$\therefore f(x)$ is divisible by $x - 2$.</p> <p>$\therefore f(2) = 0$</p> $\therefore (2^2 - 4)(2^{2018} + 2 \times 2^{2017}) + 3(2) + k = 0$ $k = \underline{\underline{-6}}$ <p>(b) Remainder</p> $= f(-1)$ $= [(-1)^2 - 4][(-1)^{2018} + 2(-1)^{2017}] + 3(-1) - 6$ $= (-3)(-1) - 3 - 6$ $= \underline{\underline{-6}}$	<p>1M</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>(5)</p>	
<p>9. Let x and y be the numbers of red balls and black balls respectively.</p> $\begin{cases} \frac{x+1}{x+y+1} = \frac{1}{2} \\ \frac{y+1}{x+y+1} = \frac{3}{5} \end{cases}$ <p>So, we have $\begin{cases} x = y - 1 \\ 3x - 2y - 2 = 0 \end{cases}$.</p>	<p>} 1A + 1A</p>	

Solution	Marks	Remarks						
<p>Therefore, we have $3(y-1) - 2y - 2 = 0$.</p> <p>Solving, we have $x = 4$ and $y = 5$.</p> <p>The required probability = $\frac{4+1}{(4+1)+(5+1)}$ $= \frac{5}{11}$</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>(5)</p>	<p>for getting a linear equation in x or y only</p> <p>for both correct</p>						
<p>10. (a) Note that</p> <p>$\angle ADC + \angle BAD = 180^\circ$ (int. \angles, $AB \parallel DC$).</p> <p>So, we have $\angle ADC = 180^\circ - \theta$.</p> <p>Also, note that</p> <p>$\angle CBA + \angle ADC = 180^\circ$ (opp. \angles, cyclic quad.).</p> <p>So, we have $\angle CBA + (180^\circ - \theta) = 180^\circ$.</p> <p>Therefore, we have $\angle CBA = \theta$.</p> <p>(b) $\therefore \angle BAD = \angle CBA$ (proved in (a))</p> <p>$\therefore \frac{\widehat{BCD}}{\widehat{ADC}} = \frac{\angle BAD}{\angle CBA} = \frac{1}{1}$ (arcs prop. to \angles at \odot^{ce})</p> <p>$\therefore \widehat{BCD} = \widehat{ADC}$</p> <p>$\widehat{BC} + \widehat{CD} = \widehat{AD} + \widehat{CD}$</p> <p>$\therefore \widehat{BC} = \widehat{AD}$</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td colspan="2">Marking Scheme:</td> </tr> <tr> <td>Case 1 Any correct proof with correct reasons.</td> <td style="text-align: center;">2</td> </tr> <tr> <td>Case 2 Any correct proof without reasons.</td> <td style="text-align: center;">1</td> </tr> </table> <p>(c) $\therefore \widehat{AD} = \widehat{BC}$ (proved in (b))</p> <p>$\therefore \angle AOD = \angle BOC$ (equal arcs, equal \angles)</p> <p>$= 2\angle BAC$ (\angle at centre twice \angle at \odot^{ce})</p> <p>Thus, the claim is agreed.</p>	Marking Scheme:		Case 1 Any correct proof with correct reasons.	2	Case 2 Any correct proof without reasons.	1	<p>1A</p> <p>1A</p> <p>1A</p> <p>(6)</p>	<p>f.t.</p>
Marking Scheme:								
Case 1 Any correct proof with correct reasons.	2							
Case 2 Any correct proof without reasons.	1							

Solution	Marks	Remarks
<p>11. (a) In $\triangle CBQ$ and $\triangle CDP$,</p> <p>$CB = CD$ (property of square)</p> <p>$\angle CBQ = 180^\circ - \angle ABC$ (adj. \angles on st. line)</p> <p>$= 180^\circ - 90^\circ$ (property of square)</p> <p>$= 90^\circ$</p> <p>$= \angle CDP$ (property of square)</p> <p>$BQ = DP$ (given)</p> <p>$\therefore \triangle CBQ \cong \triangle CDP$ (SAS)</p>		
<p>Marking Scheme:</p> <p>Case 1 Any correct proof with correct reasons. 2</p> <p>Case 2 Any correct proof without reasons. 1</p>		
<p>(b) Note that $\angle BCD = 90^\circ$ (property of square).</p> <p>So, we have $\angle BCP = 90^\circ - \angle DCP$.</p> <p>Also, note that $\angle BCQ = \angle DCP$ (corr. \angles, $\cong \triangle$s).</p> <p>So, we have $\angle PCQ = (90^\circ - \angle DCP) + \angle BCQ$.</p> <p>i.e. $\angle PCQ = 90^\circ$</p> <p>Thus, $\triangle CPQ$ is a right-angled triangle.</p>	<p>1M</p> <p>1A</p>	<p>f.t.</p>
<p>(c) $CP = CQ = 8$ cm (corr. sides, $\cong \triangle$s)</p> <p>Area of right-angled triangle CPQ</p> $= \frac{8 \times 8}{2} \text{ cm}^2$ $= 32 \text{ cm}^2$ <p>$PQ^2 = CP^2 + CQ^2$ (Pyth. theorem)</p> $PQ = \sqrt{8^2 + 8^2} \text{ cm}$ $= \sqrt{128} \text{ cm}$ <p>Let d cm be the shortest distance from C to PQ.</p> $\frac{\sqrt{128} \times d}{2} = 32$ $d = \frac{64}{\sqrt{128}}$ <p>Note that $\frac{64}{\sqrt{128}} \text{ cm} > 5 \text{ cm}$.</p> <p>Thus, there is no point F on PQ such that the distance between F and C is 5 cm.</p>	<p>1M</p> <p>1A</p> <p>(6)</p>	<p>f.t.</p>

Solution	Marks	Remarks
<p>12. (a) Let $\{a, b, c, d, e\}$ be the time taken by the girls, arranged in ascending order.</p> <p>The median of the time taken by the boys = 7 min The median of the time taken by the girls = 8 min So, we have $c = 8$.</p> <p>The mode of the time taken by the boys = 8 min The mode of the time taken by the girls = 9 min So, we have $d = e = 9$.</p> <p>The mean of the time taken by the boys = 6 min The mean of the time taken by the girls = 7 min</p> <p>Thus, we have $\frac{a+b+8+9+9}{5} = 7$.</p> <p>Solving, we have $a + b = 9$.</p> <p>The inter-quartile range of the time taken by the girls $= \left(\frac{9+9}{2} - \frac{a+b}{2} \right)$ min $= \left(9 - \frac{9}{2} \right)$ min $= 4.5$ min</p> <p>The inter-quartile range of the time taken by the boys $= (8 - 4)$ min = 4 min</p> <p>Thus, the inter-quartile range of the time taken by the girls is not 1 minute more than that of the boys.</p> <p>(b) The range of the time taken by the boys $= (8 - 2)$ min = 6 min</p> <p>Thus, the range of the time taken by the girls is 7 min. So, we have $9 - a = 7$.</p> <p>Therefore, we have $a = 2$.</p> <p>Since $a + b = 9$, we have $b = 7$.</p> <p>The standard deviation of the time taken by the girls $\approx \underline{\underline{2.61}}$ min</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p>f.t.</p> <p>----- for a, b both correct -----</p>
	(7)	

Solution	Marks	Remarks
<p>13. (a) Let $x^2 + y^2 + Dx + Ey + F = 0$ be the equation of C.</p> <p>Putting $x = 0, y = 2$ in C, we have</p> $0^2 + 2^2 + D(0) + E(2) + F = 0$ <p>Putting $x = 4, y = 0$ in C, we have</p> $4^2 + 0^2 + D(4) + E(0) + F = 0$ <p>Putting $x = 9, y = 5$ in C, we have</p> $9^2 + 5^2 + D(9) + E(5) + F = 0$ <p>So, we have $\begin{cases} 2E + F + 4 = 0 \\ 4D + F + 16 = 0 \\ 9D + 5E + F + 106 = 0 \end{cases} .$</p> <p>Solving, we have $D = -8, E = -10$ and $F = 16$.</p> <p>Thus, the equation of C is $x^2 + y^2 - 8x - 10y + 16 = 0$.</p>	<p>1M</p> <p>1A</p> <p>1A</p>	<p>for substituting P, Q and R</p>
<p>(b) Centre of $C = \left(-\frac{(-8)}{2}, -\frac{(-10)}{2} \right) = (4, 5)$</p> <p>Radius of $C = \sqrt{\left(\frac{-8}{2}\right)^2 + \left(\frac{-10}{2}\right)^2} - 16 = 5$</p> <p>Distance between M and the centre of C</p> $= \sqrt{(5-4)^2 + (8-5)^2}$ $= \sqrt{10}$ < 5 <p>Thus, M lies inside C.</p>	<p>1M</p> <p>1</p>	
<p>(c) (i) G, M and N are collinear.</p> <p>(ii) The slope of the straight line which passes through M and N</p> <p>= the slope of the straight line which passes through M and G</p> $= \frac{8-5}{5-4}$ $= 3$ <p>The required equation is</p> $y - 5 = 3(x - 4)$ $y - 5 = 3x - 12$ $3x - y - 7 = 0$	<p>1M</p> <p>1A</p> <p>(8)</p>	<p>or equivalent</p>

Solution	Marks	Remarks
<p>14. (a) $f(12) = 12^3 - 96(12) - 576$ $= 0$ $\therefore x - 12$ is a factor of $f(x)$. $\therefore f(x) = \underline{\underline{(x - 12)(x^2 + 12x + 48)}}$</p>	<p>1M 1A</p>	
<p>(b) (i) Let h_A cm and h_B cm be the depths of water of containers A and B after pumping water for 44 seconds respectively. Note that the corresponding radius of the water surface of container $A = \frac{9}{18} h_A$ cm $= \frac{h_A}{2}$ cm. So, we have $\frac{1}{3} \times \pi \times \left(\frac{h_A}{2}\right)^2 \times h_A = 2\pi \times 35$ $\frac{h_A^3 \pi}{12} = 70\pi$ $h_A^3 = 840$ $h_A \approx 9.435\ 387\ 960$ Also, we have $\pi \times 4^2 \times h_B = \pi \times 44$ $h_B = 2.75$ $\therefore h_A > h_B$ \therefore The water level in container A is higher after pumping water for 44 seconds.</p>	<p>1A 1A 1A</p>	<p>f.t.</p>
<p>(ii) Let h cm be the depth of water when the water reaches the same level in the two containers. Note that the corresponding radius of the water surface of container $A = \frac{9}{18} h$ cm $= \frac{h}{2}$ cm. $\frac{\frac{1}{3} \times \pi \times \left(\frac{h}{2}\right)^2 \times h - 2\pi \times 35}{\pi} = \frac{\pi \times 4^2 \times h - 44\pi}{2\pi}$ $\frac{1}{12} h^3 - 70 = 8h - 22$ $h^3 - 96h - 576 = 0$ $(h - 12)(h^2 + 12h + 48) = 0$</p>	<p>1M 1M</p>	<p>for using the result of (a)</p>

Solution	Marks	Remarks
<p>Hence, we have $h - 12 = 0$ or $h^2 + 12h + 48 = 0$.</p> <p>Note that $h^2 + 12h + 48 = 0$ has no real solutions.</p> <p>So, we have $h = 12$.</p> <p>Thus, the depth of water when the water reaches the same level in the two containers is 12 cm.</p>	1A (8)	f.t.
<p>15. (a) The required probability</p> $= \frac{C_5^6}{C_5^{10}}$ $= \frac{1}{42}$	1M 1A	for numerator r.t. 0.0238
<p><u>Alternative Solution</u></p> <p>The required probability</p> $= \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times \frac{2}{6}$ $= \frac{1}{42}$	1M 1A	r.t. 0.0238
<p>(b) The required probability</p> $= \frac{1}{42} + \frac{C_4^6 C_1^4}{C_5^{10}} + \frac{C_3^6 C_2^4}{C_5^{10}}$ $= \frac{1}{42} + \frac{5}{21} + \frac{10}{21}$ $= \frac{31}{42}$	1M 1A	for (a) + $p_1 + p_2$ r.t. 0.738
<p><u>Alternative Solution</u></p> <p>The required probability</p> $= 1 - \frac{C_2^6 C_3^4}{C_5^{10}} - \frac{C_1^6 C_4^4}{C_5^{10}}$ $= 1 - \frac{5}{21} - \frac{1}{42}$ $= \frac{31}{42}$	1M 1A	for $1 - p_3 - p_4$ r.t. 0.738

Solution	Marks	Remarks
<p style="text-align: center;"><u>Alternative Solution</u></p> <p>The required probability</p> $= \frac{1}{42} + C_1^5 \times \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times \frac{4}{6} + C_2^5 \times \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} \times \frac{3}{6}$ $= \frac{31}{42}$	<p style="text-align: center;">1M</p> <p style="text-align: center;">1A</p>	<p style="text-align: center;">for (a) + $p_5 + p_6$</p> <p style="text-align: center;">r.t. 0.738</p>
<p>16. Let m and σ be the mean and the standard deviation of the salaries respectively and x_1, x_2 be the salaries of Chris and John respectively.</p> <p>Note that the standard scores of Chris and John are 1 and -2 respectively.</p> <p>So, we have $\frac{x_1 - m}{\sigma} = 1$ and $\frac{x_2 - m}{\sigma} = -2$.</p> <p>Solving, we have $\frac{x_1 - x_2}{\sigma} = 3$.</p> <p>Note that the difference in their salaries is \$3000.</p> <p>i.e. $x_1 - x_2 = 3000$</p> <p>So, we have $\frac{3000}{\sigma} = 3$.</p> <p>Thus, we have $\sigma = 1000$.</p> <p>Variance of the distribution of the salaries</p> $= \sigma^2$ $= \$(1000^2)$ $= \underline{\underline{\$1\,000\,000}}$	<p style="text-align: center;">(4)</p> <p style="text-align: center;">1M</p> <p style="text-align: center;">1A</p> <p style="text-align: center;">1A</p> <p style="text-align: center;">(4)</p>	<p style="text-align: center;">for either one</p>

Solution	Marks	Remarks
<p>17. (a) $f(x) = 24x - 2x^2$ $= -2(x^2 - 12x)$ $= -2(x^2 - 12x + 6^2) + 2(6^2)$ $= -2(x - 6)^2 + 72$</p> <p>Thus, the coordinates of the vertex are (6, 72).</p>	1M 1A	
<p>(b) (i) Note that $\triangle AEF \sim \triangle ACB$ (AAA).</p> $\frac{AF}{AB} = \frac{EF}{CB} \quad (\text{corr. sides, } \sim \triangle\text{s})$ $\frac{AF}{18 \text{ cm}} = \frac{x \text{ cm}}{12 \text{ cm}}$ $AF = \frac{3}{2}x \text{ cm}$ <p>Hence, $ED = \left(18 - \frac{3}{2}x\right) \text{ cm}.$</p> <p>Area of rectangle $BDEF$</p> $= x \left(18 - \frac{3}{2}x\right) \text{ cm}^2$ $= \underline{\underline{\left(18x - \frac{3}{2}x^2\right) \text{ cm}^2}}$	1M 1A	or equivalent
<p>(ii) Area of rectangle $BDEF$</p> $= \left(18x - \frac{3}{2}x^2\right) \text{ cm}^2$ $= \frac{3}{4}(24x - 2x^2) \text{ cm}^2$ $= \frac{3}{4}[-2(x - 6)^2 + 72] \text{ cm}^2$ $= \left[-\frac{3}{2}(x - 6)^2 + 54\right] \text{ cm}^2$ <p>The greatest value of the area of rectangle $BDEF$ is $54 \text{ cm}^2 < 60 \text{ cm}^2$.</p> <p>Thus, Peter's claim is disagreed.</p>	1M 1A	for using the result of (a) f.t.

Solution	Marks	Remarks
<p><u>Alternative Solution</u></p> <p>Assume that the area of rectangle $BDEF > 60 \text{ cm}^2$.</p> $18x - \frac{3}{2}x^2 > 60$ $\frac{3}{2}(12x - x^2) > 60$ $12x - x^2 > 40$ $x^2 - 12x + 40 < 0$ $\Delta = (-12)^2 - 4(1)(40) = -16 < 0$ <p>Therefore, we have $x^2 - 12x + 40 > 0$.</p> <p>This is impossible.</p> <p>Thus, Peter's claim is disagreed.</p>	<p>1M</p> <p>1A</p>	<p>f.t.</p>
	(7)	
<p>18. (a) Consider $\triangle ABC$.</p> <p>By cosine formula, we have</p> $\cos \angle ABC = \frac{AB^2 + BC^2 - AC^2}{2(AB)(BC)}$ $\cos \angle ABC = \frac{29^2 + 92^2 - 75^2}{2(29)(92)}$ $\cos \angle ABC = \frac{20}{29}$ <p>Consider right-angled triangle ABD.</p> $\cos \angle ABD = \frac{BD}{AB}$ $\frac{20}{29} = \frac{BD}{29 \text{ cm}}$ $BD = \underline{\underline{20 \text{ cm}}}$ $AD = \sqrt{AB^2 - BD^2} \quad (\text{Pyth. theorem})$ $= \sqrt{29^2 - 20^2} \text{ cm}$ $= \underline{\underline{21 \text{ cm}}}$	<p>1M</p> <p>1A</p> <p>1A</p>	

Solution	Marks	Remarks
<p><u>Alternative Solution</u></p> <p>Consider $\triangle ABC$.</p> <p>By cosine formula, we have</p> $\cos \angle ACB = \frac{AC^2 + BC^2 - AB^2}{2(AC)(BC)}$ $\cos \angle ACB = \frac{75^2 + 92^2 - 29^2}{2(75)(92)}$ $\cos \angle ACB = \frac{24}{25}$ <p>Consider right-angled triangle ACD.</p> $\cos \angle ACB = \frac{CD}{AC}$ $\frac{24}{25} = \frac{CD}{75 \text{ cm}}$ $CD = 72 \text{ cm}$ $AD = \sqrt{AC^2 - CD^2} \quad (\text{Pyth. theorem})$ $= \sqrt{75^2 - 72^2} \text{ cm}$ $= \underline{\underline{21 \text{ cm}}}$ $BD = BC - CD = (92 - 72) \text{ cm} = \underline{\underline{20 \text{ cm}}}$	<p>1M</p> <p>1A</p> <p>1A</p>	
<p>(b) (i) Consider $\triangle ABC$.</p> <p>By cosine formula, we have</p> $BC^2 = AC^2 + AB^2 - 2(AC)(AB)\cos \angle CAB$ $BC = \sqrt{75^2 + 29^2 - 2(75)(29)\cos 60^\circ} \text{ cm}$ $BC = \sqrt{4291} \text{ cm}$ <p>Note that $CD = (92 - 20) \text{ cm} = 72 \text{ cm}$.</p> <p>Consider $\triangle BCD$.</p> <p>By cosine formula, we have</p> $\cos \angle CDB = \frac{CD^2 + BD^2 - BC^2}{2(CD)(BD)}$ $\cos \angle CDB = \frac{72^2 + 20^2 - (\sqrt{4291})^2}{2(72)(20)}$ $\angle CDB \approx 63.323\ 128\ 68^\circ$ $\approx 63.3^\circ$ <p>Note that $AD \perp BD$ and $AD \perp CD$.</p> <p>Hence, the required angle is 63.3°.</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>r.t. 63.3°</p> <p>f.t.</p>

Solution	Marks	Remarks
<p>(ii) Let P be the point on BD produced such that $CP \perp BD$.</p> <p>\therefore The plane BCD is perpendicular to the horizontal ground.</p> <p>\therefore CP is the height of the tetrahedron $ABCD$ with the base $\triangle ABD$.</p> <p>Consider right-angled triangle CDP.</p> $\sin \angle CDP = \frac{CP}{CD}$ $CP \approx 72 \sin 63.323\ 128\ 68^\circ \text{ cm}$ <p>Volume of the tetrahedron $ABCD$</p> $= \frac{1}{3} \times \text{area of } \triangle ABD \times CP$ $\approx \frac{1}{3} \times \frac{20 \times 21}{2} \times 72 \sin 63.323\ 128\ 68^\circ \text{ cm}^3$ $\approx 4503.505\ 572 \text{ cm}^3$ $\approx \underline{\underline{4500 \text{ cm}^3}}$	<p>1M</p> <p>1A</p> <p>(8)</p>	<p>r.t. 4500 cm^3</p>
<p>19. (a) (i) Amount of donation on 31st December 2018</p> $= 50 \times 1.04 \times 0.1 \text{ million dollars}$ $= \underline{\underline{5.2 \text{ million dollars}}}$ <p>(ii) Amount of donation on 31st December 2021</p> $= 50 \times 1.04^4 \times 0.9^3 \times 0.1 \text{ million dollars}$ $= 4.264\ 134\ 451 \text{ million dollars}$ $\approx \underline{\underline{4.26 \text{ million dollars}}}$ <p>(b) Total amount of donation</p> $= \frac{50 \times 1.04 \times 0.1}{1 - 1.04 \times 0.9} \text{ million dollars}$ $= 81.25 \text{ million dollars}$ $< 85 \text{ million dollars}$ <p>Hence, the total amount of donation cannot exceed 85 million dollars.</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p>	<p>for $S(\infty) = \frac{a}{1-r}$</p> <p>f.t.</p>

Solution	Marks	Remarks
<p>(c) (i) Amount of donation on 31st December 2021</p> $= \left\{ \begin{array}{l} [(50 \times 1.04 \times 0.9 + 60) \times \\ 1.04 \times 0.9 + 60] \times \\ 1.04 \times 0.9 + 60 \\ 1.04 \times 0.1 \end{array} \right\} \text{ million dollars}$ $= \left(\begin{array}{l} 50 \times 1.04^4 \times 0.9^3 + \\ 60 \times 1.04^3 \times 0.9^2 + \\ 60 \times 1.04^2 \times 0.9 + \\ 60 \times 1.04 \end{array} \right) \times 0.1 \text{ million dollars}$ <p>$\approx 21.811\ 613\ 49$ million dollars $\approx \underline{\underline{21.8}} \text{ million dollars}$</p>	<p>1M</p> <p>1A</p>	<p>r.t. 21.8 million dollars</p>
<p>(ii) Let n be the required number of years.</p> $\left[\begin{array}{l} 50 \times 1.04^n \times 0.9^{n-1} + \\ 60 \times 1.04 \times \frac{1 - (1.04 \times 0.9)^{n-1}}{1 - 1.04 \times 0.9} \end{array} \right] \times 0.1 > 85$ $\left(\begin{array}{l} 0.32 \times 1.04^n \times 0.9^{n-1} + 6 \times 1.04 - \\ 6 \times 1.04^n \times 0.9^{n-1} \end{array} \right) > 5.44$ $(1.04 \times 0.9)^n < \frac{9}{71}$ $n \log 0.936 < \log \frac{9}{71}$ $n > \frac{\log \frac{9}{71}}{\log 0.936}$ <p>Solving, we have $n > 31.228\ 628\ 17$.</p> <p>\therefore The amount of donation will first exceed 85 million dollars on 31st December 2049 (i.e. on 31st December of the 32th year).</p>	<p>1M</p> <p>1M</p> <p>1A</p> <p>1A</p>	<p>f.t.</p>
	<p>(12)</p>	