CARMEL DIVINE GRACE FOUNDATION SECONDARY SCHOOL

SECOND TERM EXAMINATION 2020 − 2021

SECONDARY VI MATHEMATICS Compulsory Part

PAPER 1

Name : ________________________ ( )

Date : 23 − 2 − 2021

Time : $2\frac{1}{4}$ **hours**

Class : S. 6 _______

No. of pages : 24

INSTRUCTIONS

1. **Write your name, class and class number in the spaces provided on this cover.**
2. **This paper consists of THREE sections, A(1), A(2) and B.**
3. **Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.**
4. **Graph paper and supplementary answer sheets will be supplied on request. Write your name, class and class number, mark the question number box, and fasten them with string INSIDE this book.**
5. **Unless otherwise specified, all working must be clearly shown.**
6. **Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.**
7. **The diagrams in this paper are not necessarily drawn to scale.**
8. **No extra time will be given to students for filling in the question number boxes after the 'Time is up' announcement.**

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SECTION A(1) (35 marks)

1. Simplify $\dfrac{(x^{-2}y^{4})^{3}}{xy^{-3}}$ and express your answer with positive indices. (3 marks)

2. Factorize
   (a) $4p^2 - 4pq + q^2$,
   (b) $4p - 2q - 4p^2 + 4pq - q^2$.

(3 marks)

Answers written in the margins will not be marked.

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3. (a) Round up 190.98418 to the nearest ten.

(b) Round off 190.98418 to 3 significant figures.

(c) Round down 190.98418 to 4 decimal places.

(3 marks)

4. A man bought a number of watches for $6000. Three of them were broken. He then sold each of the remaining watches at 12.5% above its cost price and made a total profit of $480. How many watches did he buy? (4 marks)

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5. (a) Find the range of values of x which satisfy both $x - \frac{10 - x}{3} \geq -6$ and $21 - 3x > 0$.

(b) How many integers satisfy both inequalities in (a)?

(4 marks)

6. The following table shows the distribution of the number of mobile phones owned by a group of taxi drivers.

Number of mobile phones	1	2	3	4	5
Frequency	2	3	12	15	k

It is given that the mean of the distribution is 3.6.

(a) Find the value of k.

(b) Find the median and the standard deviation of the distribution.

(4 marks)

7. Consider the formula $3a = b(1 - 2y)$.

(a) Make y the subject of the above formula.

(b) If $a : b = 2 : 3$, find the value of y.

(4 marks)

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8. In Figure 1, $\angle ABC = \angle PQR$, $AB = 12$, $BC = 22.5$, $PQ = 8$, $QR = 15$ and $PR = 10$.

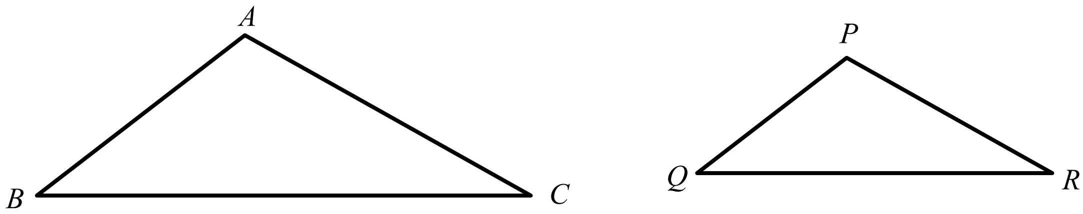

Figure 1

(a) Prove that $\Delta ABC \sim \Delta PQR$.

(b) Two paper cards in the shapes of the triangles in Figure 1 are made. Someone claims that a trapezium can be formed by the two cards. Is the claim correct? Explain your answer.

(5 marks)

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9. It is given that $g(x)$ is partly constant and partly varies as $(x+1)^2$. Suppose that $g(1) = 3$ and $g(-5) = -3$.

(a) Find $g(x)$.

(b) How many real roots does the equation $g(x) = 6$ have?

(5 marks)

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SECTION A(2)   (35 marks)

10. The stem-and-leaf diagram below shows the test scores of 30 students.

Stem (tens)	Leaf (units)
4	5 6
5	0 1 3
6	0 0 a a 3 3 6 6 8 9
7	1 1 2 3 8 8 8 b
8	3 6 7 8 9
9	2 3

It is given that the mode of the distribution is unique and the inter-quartile range of the distribution is greater than 16.

(a) Find a and b. (4 marks)

(b) The passing score of the test is 60. If a student is randomly chosen, find the probability that the student passes the test. (2 marks)

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11. The coordinates of the points A and B are $(2, 5)$ and $(-10, -1)$ respectively. P is a moving point in the rectangular coordinate plane such that $AP = BP$. Denote the locus of P by Γ.

(a) Find the equation of Γ. (2 marks)

(b) Γ cuts the x-axis at C and the y-axis at D. E is a point on the rectangular coordinate plane such that $ADCE$ is a parallelogram. Find the equation of the line which passes through E and cuts $ADCE$ into two congruent triangles. (4 marks)

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12. Figure 2(a) shows a traditional Japanese skill toy, kendama. Mr Chan wants to print the ball of the kendama by a 3D printer. Figure 2(b) shows the longitudinal section of the ball. V, A and B are points on the circle. The hollow part is in the form of a frustum which is made by cutting off the upper part of the right circular cone VAB. EF is the height of the frustum, which lies on the diameter of the circle through V. The radius of the ball is 5 cm. $EF = 6$ cm and $AB = 2$ cm.

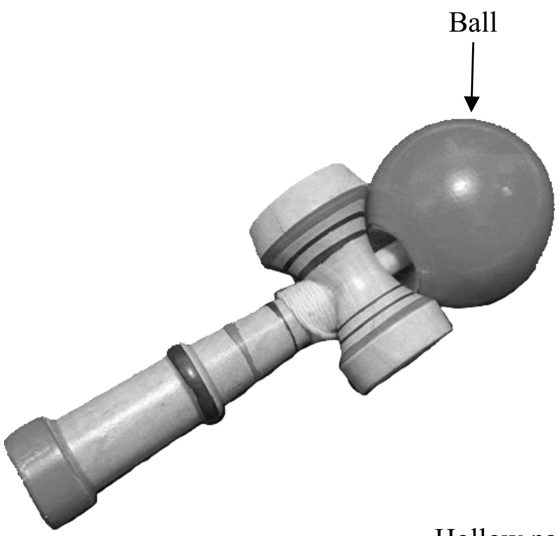

Figure 2(a)

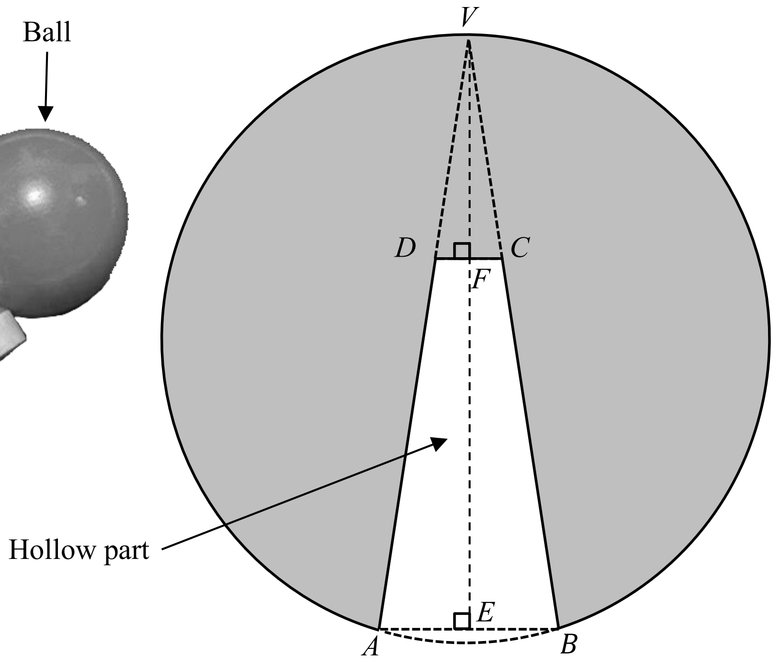

Figure 2(b)

(a) Find VE. (2 marks)

(b) Mr Chan claims that at least 520 cm^3 of printing material is required to print the ball. Do you agree? Explain your answer. (5 marks)

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13. In Figure 3, O is the centre of the circle. AEC is a diameter of the circle and BED is a straight line. It is given that $\overset{\frown}{AB} : \overset{\frown}{CD} = 3 : 5$ and $\angle AEB = 112°$.

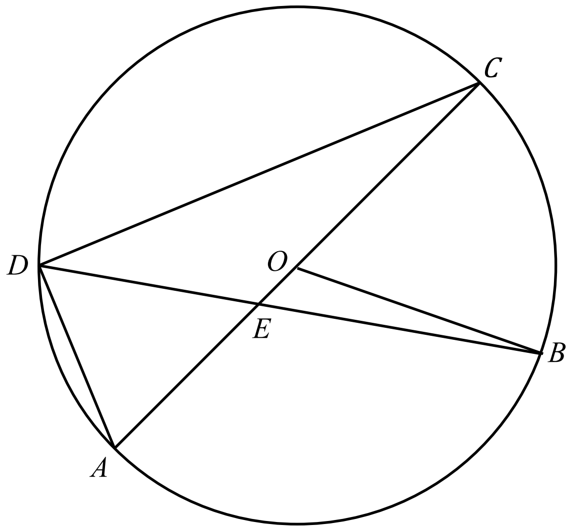

Figure 3

Find $\angle BOC$ and $\angle OBE$. (7 marks)

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14. Let $f(x) = hx^3 + 3x^2 - 5x + k$ and $g(x) = x^3 - (k+24)x^2 - (h+20)x - 3$, where h and k are constants. It is given that $x+1$ is a factor of $f(x)$.

(a) Show that $x+1$ is also a factor of $g(x)$. (3 marks)

(b) If the graph of $y = f(x) + g(x)$ has only one x-intercept, find the range of possible values of h. (6 marks)

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SECTION B (35 marks)

15. Five married couples are standing in a room. If 4 people are chosen at random, find the probability that no married couples are chosen. (3 marks)

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16. Let α and β be the roots of the equation $x^2 - 6x + 16 = 0$.

(a) Find the value of $\alpha^2 + \beta^2$. (2 marks)

(b) The 1st term and the 2nd term of a geometric sequence are $\alpha + \beta$ and $\alpha^2 + \beta^2$ respectively. Let $S(n)$ and $S(\infty)$ be the sum of the first n terms and the sum to infinity of the sequence respectively. Find the least value of n such that $S(\infty) - S(n) < 10^{-12}$.

(4 marks)

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17. Let $f(x) = -x^2 - 1$. $f(x)$ is transformed to $g(x) = f(x+4)$.

(a) Describe the geometric meaning of the above transformation. (1 mark)

(b) The graph of $y = h(x)$ is then obtained by reflecting the graph of $y = g(x)$ about the x-axis.

(i) Find $h(x)$.

(ii) P is a point on the graph of $y = f(x)$. Q is the image of P when $f(x)$ is transformed to $g(x)$. R is the image of Q when $g(x)$ is transformed to $h(x)$. If the x-coordinate of P is a, find the exact values of a such that Q, the in-centre of ΔPQR and the origin are collinear.

(5 marks)

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18. In Figure 4(a), $ABCD$ is a quadrilateral paper card. It is given that $\angle ABC = 60°$, $AB = CB = 40$ cm, $AD = CD = 22$ cm and $BD < AB$.

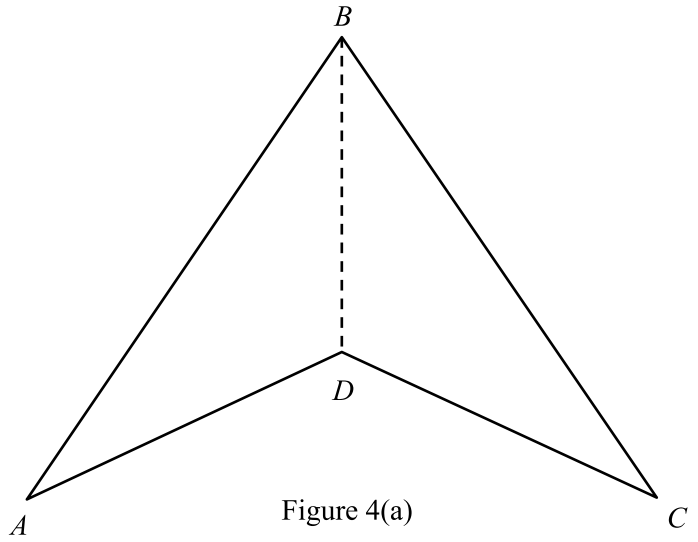

Figure 4(a)

(a) Find BD. (2 marks)

(b) The paper card in Figure 4(a) is folded along BD such that $\angle ADC = 90°$ and the plane ACD lies on a horizontal ground (see Figure 4(b)).

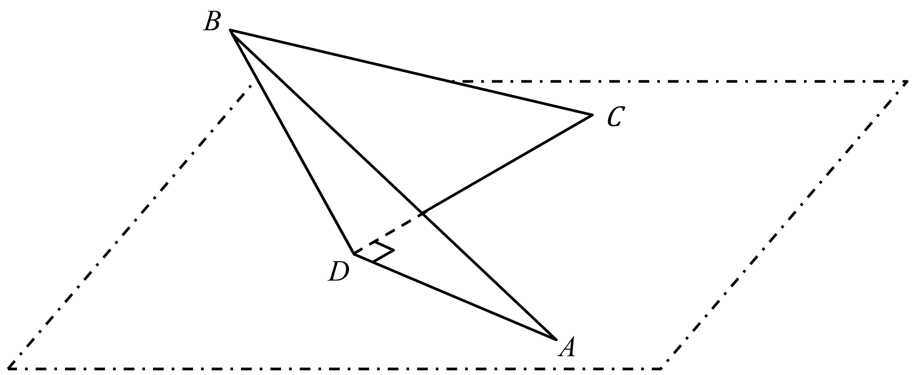

Figure 4(b)

(i) Find the angle between BD and the horizontal ground.

(ii) Find the shortest distance from D to the plane ABC.

(6 marks)

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19. (a) In ΔABE, H and K are points on AE and BE respectively. Prove that, if $ABKH$ is a cyclic quadrilateral, then $AE \cdot HE = BE \cdot KE$. (3 marks)

(b) In a rectangular coordinate plane, C is a circle passing through $P(-1, 10)$, Q, R and S. PS is a diameter of C. PS produced intersects QR produced at $T(35, -26)$. It is given that $QR = 12$, $RT = 36$.

(i) Find the equation of C.

(ii) Someone claims that the area of $PQRS$ exceeds 90 square units. Do you agree? Explain your answer.

(9 marks)

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END OF PAPER

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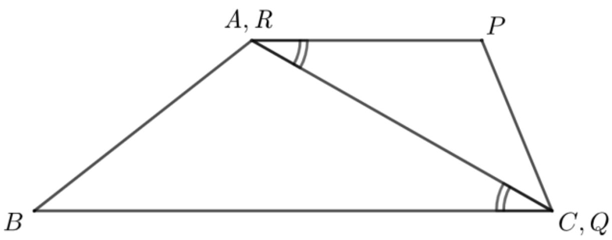
SECOND TERM EXAMINATION 2020 – 2021
SECONDARY VI MATHEMATICS Compulsory Part
PAPER 1
Marking scheme

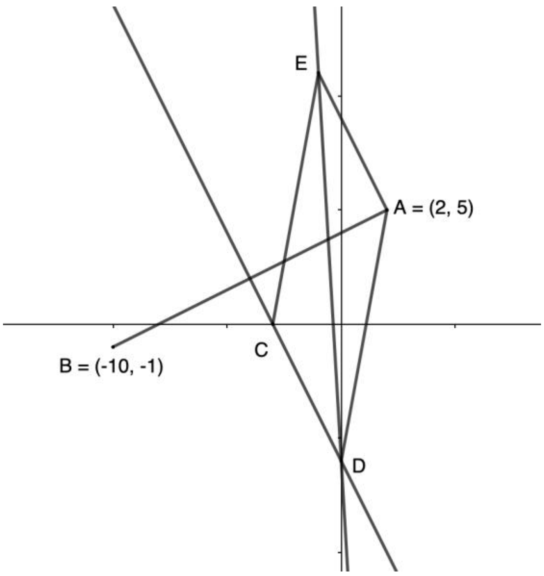
1.	$\frac{(x^{-2}y^4)^3}{xy^{-3}} = \frac{x^{-6}y^{12}}{xy^{-3}}$ $= \frac{y^{12-(-3)}}{x^{1-(-6)}}$ $= \frac{y^{15}}{x^7}$	1M 1M 1A	for $(a^h)^k = a^{hk}$ or $(ab)^l = a^l b^l$ for $\frac{c^p}{c^q} = c^{p-q}$ or $d^{-r} = \frac{1}{d^r}$
2.(a)	$4p^2 - 4pq + q^2$ $= (2p-q)^2$	1A	or equivalent
2.(b)	$4p - 2q - 4p^2 + 4pq - q^2$ $= 2(2p-q) - (4p^2 - 4pq + q^2)$ $= 2(2p-q) - (2p-q)^2$ $= (2p-q)(2-2p+q)$	1M 1A	for using the results of (a)
3.(a)	200	1A	
3.(b)	191	1A	
3.(c)	190.9841	1A	
4.	Let the number of watches he bought be x. Then the cost price of each watch $= \$\left(\frac{6000}{x}\right)$ The selling price of each watch $= \$\left(\frac{6000}{x}\right)(1+12.5\%)$ $= \$\left(\frac{6750}{x}\right)$ $\frac{6750}{x}(x-3) = 6000 + 480$ $6750x - 20250 = 6480x$ $270x = 20250$ $x = 75$ $\therefore$ He bought 75 watches.	1M 1M+1A 1A	for $p(1+12.5\%)$ 1M for total selling price = unit selling price $\times (x-3)$

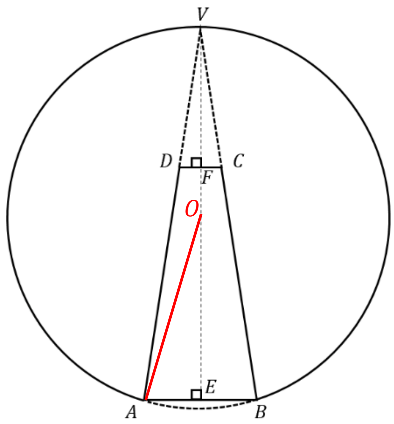
5.(a)	$x-\frac{10-x}{3}\geq -6$		
	$3x-10+x\geq -18$		
	$4x\geq -8$		
	$x\geq -2$ (1)	1A	
	$21-3x>0$		
	$3x<21$		
	$x<7$ (2)	1A	
	Combining (1) and (2), we have		
	$-2\leq x<7$	1M	
5.(b)	9	1A	
6.(a)	$\frac{1\times 2+2\times 3+3\times 12+4\times 15+5k}{2+3+12+15+k}=3.6$	1M	
	$104+5k=3.6(32+k)$		
	$=115.2+3.6k$		
	$1.4k=11.2$		
	$k=8$	1A	
6.(b)	Median $=4$	1A	
	Standard deviation $=1.04$	1A	
7.(a)	$3a=b(1-2y)$		
	$3a=b-2by$	1M	for expanding
	$2by=b-3a$		
	$y=\frac{b-3a}{2b}$	1A	or equivalent
7.(b)	$a:b=2:3$		
	$\frac{a}{b}=\frac{2}{3}$		
	$y=\frac{b-3a}{2b}$		
	$=\frac{1}{2}-\frac{3}{2}\left(\frac{a}{b}\right)$		
	$=\frac{1}{2}-\frac{3}{2}\left(\frac{2}{3}\right)$	1M	for substitution
	$=-\frac{1}{2}$	1A	

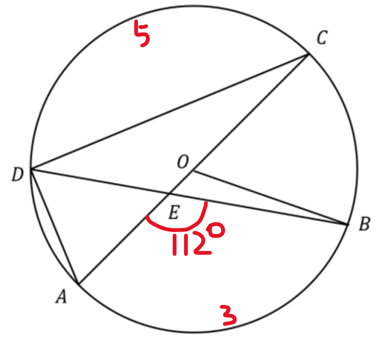
8.(a)	$\angle ABC = \angle PQR$ (given)		
	$\frac{AB}{PQ} = \frac{12}{8} = \frac{3}{2}$		
	$\frac{BC}{QR} = \frac{22.5}{15} = \frac{3}{2}$		
	$\therefore \Delta ABC \sim \Delta PQR$ (ratio of 2 sides, inc. $\angle$)		
	Marking Scheme:		
	Case 1 Any correct proof with correct reasons.	2	
	Case 2 Any correct proof without reasons.	1	
8.(b)	$\because \Delta ABC \sim \Delta PQR$		
	$\angle PRQ = \angle ACB$		
	$\frac{AC}{PR} = \frac{3}{2}$		
	$AC = \frac{3}{2}(10)$		
	$= 15$	1A	
	$= QR$		
	If AC and RQ are joined together, $AP//BC$ since	1M	
	$\angle PRQ = \angle ACB$.		
	$\therefore APCB$ is a trapezium and the claim is agreed.	1A	f.t.



9.(a)	Let $g(x) = a + b(x+1)^2$, where a and b are non-zero constants.	1A	
	$g(1) = 3$		
	$a + b(1+1)^2 = 3$		
	$a + 4b = 3 \quad (1)$	1M	for either substitution
	$g(-5) = -3$		
	$a + b(-5+1)^2 = -3$		
	$a + 16b = -3 \quad (2)$		
	$(2) - (1)$		
	$12b = -6$		
	$b = -\frac{1}{2} \quad (3)$		
	Subs. (3) into (1)		
	$a + 4\left(-\frac{1}{2}\right) = 3$		
	$a = 5$		
	$\therefore g(x) = -\frac{1}{2}(x+1)^2 + 5$	1A	
9.(b)	Since the maximum value of $y = g(x)$ is $5 < 6$, the equation	1M	
	$g(x) = 6$ has no real roots.	1A	
	<u>Alternative solution:</u>		
	$g(x) = 6$		
	$-\frac{1}{2}(x+1)^2 + 5 = 6$		
	$(x+1)^2 = -2$		
	$x^2 + 2x + 3 = 0$		
	$\Delta = 2^2 - 4(1)(3)$	1M	
	$= -8 < 0$		
	$\therefore$ The equation $g(x) = 6$ has no real roots.	1A	

10.(a)	Inter-quartile range $= 70 + b - 60 - a > 16$ $b > a + 6$ Since the mode is unique, the possible values of (a, b) are $(0, 9), (3, 9), (1, 8), (1, 9), (2, 8), (2, 9)$. Thus we have $\begin{cases} a = 0 \\ b = 9 \end{cases}, \begin{cases} a = 1 \\ b = 8 \end{cases}, \begin{cases} a = 1 \\ b = 9 \end{cases}$ and $\begin{cases} a = 2 \\ b = 9 \end{cases}$.	 1M 1A 1M 1A	 for all answers
10.(b)	The required probability $= \frac{25}{30}$ $= \frac{5}{6}$	 1M 1A	 for numerator
11.(a)	Let (x, y) be the coordinates of P. $\sqrt{(x-2)^2 + (y-5)^2} = \sqrt{[x-(-10)]^2 + [y-(-1)]^2}$ $x^2 - 4x + 4 + y^2 - 10y + 25 = x^2 + 20x + 100 + y^2 + 2y + 1$ $24x + 12y + 72 = 0$ $2x + y + 6 = 0$	 1M 1A	
11.(b)	$C = (-3, 0)$ $D = (0, -6)$ $E = (-3 + (2 - 0), 0 + [5 - (-6)])$ $= (-1, 11)$ The required straight line is the line passing through D and E. $\therefore$ The equation of the required straight line is $y = \frac{11 - (-6)}{-1 - 0} x - 6$ $y = -17x - 6$ $17x + y + 6 = 0$  	1M 1M 1M 1A	for both C and D

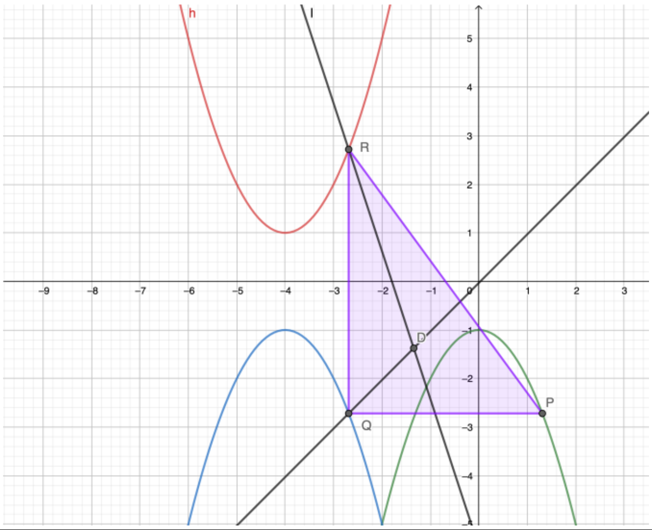
12.(a)	Let O be the centre of the ball.  $OA = OV = 5$ cm $AE = \frac{2}{2} = 1$ cm $OE = \sqrt{OA^2 - AE^2}$ $= \sqrt{5^2 - 1^2}$ $= \sqrt{24}$ cm $VE = 5 + \sqrt{24}$ $= 9.90$ cm	 1M 1A	 r.t. 9.90 cm
12.(b)	Volume of cone VAB $= \frac{1}{3}\pi(1^2)(5 + \sqrt{24})$ $= \frac{(5 + \sqrt{24})\pi}{3}$ cm^3 Volume of the frustum $= \frac{(5 + \sqrt{24})\pi}{3} \cdot \frac{(5 + \sqrt{24})^3 - (5 + \sqrt{24} - 6)^3}{(5 + \sqrt{24})^3}$ ≈ 9.732754172 cm^3 $\therefore$ The volume of material required $< \frac{4}{3}\pi(5^3) - 9.732754172$ ≈ 513.8660214 < 520 cm^3 $\therefore$ The claim is disagreed.	 1M 1M + 1A 1M 1A	 for $\frac{1}{3}\pi r^2 h$ 1M for $\frac{V_2}{V_1} = \left(\frac{h_1}{h_2}\right)^3$ f.t.

13.			
	Let $\angle ADB = 3x$.		
	Then $\angle CAD = 3x\left(\frac{5}{3}\right)$	1M	
	$= 5x$		
	$\angle ADB + \angle CAD = 112°$		
	$3x + 5x = 112°$	1M	
	$8x = 112°$		
	$x = 14°$		
	$\angle ADC = 90°$	1A	
	$\angle BDC = \angle ADC - \angle ADB$		
	$= 90° - 3(14°)$		
	$= 48°$		
	$\angle BOC = 2\angle BDC$	1M	
	$= 2(48°)$		
	$= 96°$	1A	
	$\angle BOE + \angle BOC = 180°$		
	$\angle BOE = 180° - 96°$		
	$= 84°$	1M	
	$\angle BOE + \angle OBE = \angle AEB$		
	$84° + \angle OBE = 112°$		
	$\angle OBE = 28°$	1A	

14.(a)	By Factor Theorem,		
	$f(-1) = 0$	1M	
	$h(-1)^3 + 3(-1)^2 - 5(-1) + k = 0$		
	$-h + 3 + 5 + k = 0$		
	$k = h - 8$		
	$g(x) = x^3 - (k + 24)x^2 - (h + 20)x - 3$		
	$= x^3 - (h - 8 + 24)x^2 - (h + 20)x - 3$		
	$= x^3 - (h + 16)x^2 - (h + 20)x - 3$		
	$g(-1) = (-1)^3 - (h + 16)(-1)^2 - (h + 20)(-1) - 3$	1M	
	$= -1 - h - 16 + h + 20 - 3$		
	$= 0$		
	$\therefore x + 1$ is also a factor of $g(x)$.	1A	f.t.
14.(b)	Subs. $y = 0$ into $y = f(x) + g(x)$		
	$f(x) + g(x) = 0$		
	$hx^3 + 3x^2 - 5x + h - 8 + x^3 - (h + 16)x^2 - (h + 20)x - 3 = 0$		
	$(h + 1)x^3 - (h + 13)x^2 - (h + 25)x + (h - 11) = 0 \quad (*)$	1M	for considering cases
	If $h = -1$, (*) becomes		
	$-12x^2 - 24x - 12 = 0$		
	$x^2 + 2x + 1 = 0$		
	$x = -1$ (repeated)		
	$\therefore$ The graph of $y = f(x) + g(x)$ has only 1 x-intercept		
	when $h = -1$.	1A	
	If $h \neq -1$,		
	$(x + 1)[(h + 1)x^2 - 2(h + 7)x + (h - 11)] = 0$	1M	for $(x + 1)(px^2 + qx + r)$
	$x = -1$ or $(h + 1)x^2 - 2(h + 7)x + (h - 11) = 0 \quad (**)$		
	Since the graph of $y = f(x) + g(x)$ has only 1 x-intercept,		
	(*) has exactly 1 real root,		
	Δ of $(**) = [-2(h + 7)]^2 - 4(h + 1)(h - 11) < 0$	1M +1A	
	$h^2 + 14h + 49 - h^2 + 10h + 11 < 0$		
	$24h + 60 < 0$		
	$h < -\frac{5}{2}$	1A	
	$\therefore h < -\frac{5}{2}$ or $h = -1$		

15.	The required probability $= \frac{C_4^5 (C_1^2)^4}{C_4^{10}}$ $= \frac{8}{21}$ <u>Alternative method:</u> The required probability $= \frac{C_4^{10} - C_2^5 - C_1^5 (C_2^8 - 4)}{C_4^{10}}$ $= \frac{8}{21}$	1M+1M 1A 1M+1M 1A	1M for numerator 1M for denominator 1M for numerator 1M for denominator
16.(a)	$\alpha + \beta = 6$ $\alpha\beta = 16$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= 6^2 - 2(16)$ $= 4$	1M 1A	Both correct
16.(b)	Common ratio $= \frac{4}{6}$ $= \frac{2}{3}$ $S(\infty) = \frac{6}{1 - \frac{2}{3}}$ $= 18$ $S(n) = \frac{6\left[1 - \left(\frac{2}{3}\right)^n\right]}{1 - \frac{2}{3}}$ $= 18\left[1 - \left(\frac{2}{3}\right)^n\right]$ $S(\infty) - S(n) < 10^{-12}$ $18 - 18\left[1 - \left(\frac{2}{3}\right)^n\right] < 10^{-12}$ $\left(\frac{2}{3}\right)^n < \frac{10^{-12}}{18}$ $n \log \frac{2}{3} < \log \frac{10^{-12}}{18}$ $n > \frac{\log \frac{10^{-12}}{18}}{\log \frac{2}{3}}$ $n > 75.27501692$ $\therefore$ The least value of n is 76.	1M (either one: $S(\infty)$ or $S(n)$) 1M 1M 1A	either one

<table>
<tr><td>17.(a)</td><td>$f(x)$ is translated to the left by 4 units.</td><td>1A</td><td></td></tr>
<tr><td>17.(b)(i)</td><td>$h(x) = -g(x)$
$= -f(x+4)$
$= -[-(x+4)^2 - 1]$
$= x^2 + 8x + 16 + 1$
$= x^2 + 8x + 17$</td><td>1M

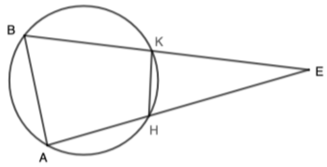
1A</td><td></td></tr>
<tr><td>17.(b)(ii)</td><td>$P = (a, -a^2 - 1)$
$Q = (a - 4, -a^2 - 1)$
Note that Q is below the x-axis. Since P is at the right side of Q and R is vertically above, the angle of inclination the angle bisector of $\angle PQR$ is 45°. If Q, the in-centre of ΔPQR and the origin are collinear, the angle bisector passes through the origin. Thus, the equation of the angle bisector is $y = x$.
Set $a - 4 = -a^2 - 1$
$a^2 + a - 3 = 0$
$a = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-3)}}{2(1)}$
$= \dfrac{-1 \pm \sqrt{13}}{2}$

</td><td>
1M

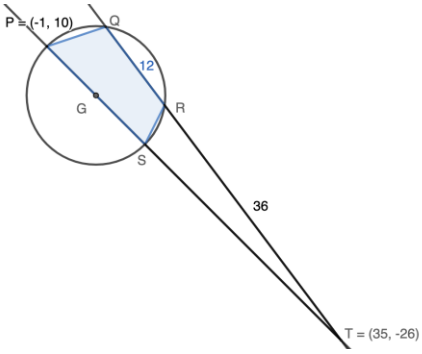
1M

1A</td><td></td></tr>
</table>

18.(a)	$\angle ABD = \frac{60°}{2}$ $= 30°$ $AD^2 = AB^2 + BD^2 - 2(AB)(BD)\cos\angle ABD$ $22^2 = 40^2 + BD^2 - 2(40)(BD)\cos 30°$ $BD^2 - (80\cos 30°)BD + 1116 = 0$ $BD \approx 25.47586476$ or 43.80616754 (rejected) $\therefore BD \approx 25.5$ cm	 1M 1A	 r.t. 25.5 cm
18.(b)(i)	Let M be the mid-point of AC. $AC = \sqrt{AD^2 + CD^2}$ $= \sqrt{22^2 + 22^2}$ $= 22\sqrt{2}$ cm $AM = \frac{1}{2}AC$ $= 11\sqrt{2}$ cm $BM = \sqrt{AB^2 - AM^2}$ $= \sqrt{40^2 - (11\sqrt{2})^2}$ $= \sqrt{1358}$ cm $\because \Delta ADM$ is an isosceles right-angled triangle. $DM = AM$ $= 11\sqrt{2}$ cm $\cos\angle BDM = \frac{BD^2 + DM^2 - BM^2}{2(BD)(DM)}$ $\approx \frac{(25.47586476)^2 + (11\sqrt{2})^2 - 1358}{2(25.47586476)(11\sqrt{2})}$ $\angle BDM \approx 126.0972975°$ $\therefore$ The angle between BD and the horizontal ground $\approx 180° - 126.09792975°$ $\approx 53.9°$	 1A 1M 1A	

18.(b)(ii)	Let the shortest distance from D to plane ABC be h cm. $\frac{1}{3}(\text{Area of }\Delta ACD)(BD\sin\angle BDM)=\frac{1}{3}(\text{Area of }\Delta ABC)h$ $\frac{1}{2}(AD)(CD)(BD)\sin\angle BDM=\frac{1}{2}(AC)(BM)h$ $h=\frac{(AD)(CD)(BD)\sin\angle BDM}{(AC)(BM)}$ $\approx\frac{(22)(22)(25.47586476)\sin 126.0972975^\circ}{\left(22\sqrt{2}\right)\left(\sqrt{1358}\right)}$ ≈ 8.68975622 ≈ 8.69 cm $\therefore$ The shortest distance from D to the plane ABC is 8.69 cm.	1M +1M 1A	1M for $V=\frac{1}{3}b_1h_1$ 1M for $A=\frac{1}{2}b_2h_2$ r.t. 8.69 cm

19.(a)			
	$\angle AEB = \angle KEH$ (common $\angle$)		
	$\angle ABE = \angle KHE$ (ext. $\angle$, cyclic quad.)		
	$\therefore \Delta ABE \sim \Delta KHE$ (AA)		
	$\frac{AE}{KE} = \frac{BE}{HE}$ (corr. sides, $\sim\Delta$s)		
	$\therefore AE \cdot HE = BE \cdot KE$		
	Marking Scheme:		
	Case 1 Any correct proof with correct reasons.	3	
	Case 2 Any correct proof without reasons.	2	
	Case 3 Incomplete proof with any one correct step and one correct reason.	1	
19.(b)(i)	$PT = \sqrt{[35-(-1)]^2 + (-26-10)^2}$		
	$= 36\sqrt{2}$		
	From (a),		
	$ST \cdot PT = RT \cdot QT$		
	$ST(36\sqrt{2}) = 36(36+12)$	1M	for using (a)
	$ST = 24\sqrt{2}$		
	Radius of $C = \frac{1}{2}(36\sqrt{2} - 24\sqrt{2})$		
	$= 6\sqrt{2}$	1A	
	Let G be the centre of C.		
	$PG : GT = 6\sqrt{2} : (36\sqrt{2} - 6\sqrt{2})$		
	$= 1 : 5$		
	$G = \left(\frac{5(-1)+35}{5+1}, \frac{5(10)+(-26)}{5+1}\right)$	1M	
	$= (5, 4)$		
	$\therefore$ The equation of C is		
	$(x-5)^2 + (y-4)^2 = (6\sqrt{2})^2$	1M	
	$(x-5)^2 + (y-4)^2 = 72$	1A	
	$x^2 + y^2 - 10x - 8y - 31 = 0$		

	 		
19.(b)(ii)	Let M be the mid-point of QR. $QM = \frac{1}{2}(12)$ $= 6$ $GM \perp QR$ $GM^2 = GQ^2 - QM^2$ $GM = \sqrt{(6\sqrt{2})^2 - 6^2}$ $= 6$	1A	
	Area of $\Delta QGR = \frac{1}{2}(12)(6)$ $= 36$ sq. units $\frac{\text{Area of } \Delta QGT}{\text{Area of } \Delta QGR} = \frac{12+36}{12}$ Area of $\Delta QGT = 4(36)$	1M	
	$= 144$ sq. units $\frac{\text{Area of } \Delta QPT}{\text{Area of } \Delta QGT} = \frac{5+1}{5}$ Area of $\Delta QPT = \frac{6}{5}(144)$ $= 172.8$ sq. units $\because \Delta QPT \sim \Delta SRT$ $\frac{\text{Area of } PQRS}{\text{Area of } \Delta QPT} = 1 - \left(\frac{RT}{PT}\right)^2$	1M	
	Area of $PQRS = \left[1 - \left(\frac{36}{36\sqrt{2}}\right)^2\right](172.8)$ $= 86.4$ sq. units < 90 sq. units Thus, the claim is disagreed.	1A	f.t.