Mock Examination (2018-2019)

Form Six Mathematics Compulsory Part

Marking Scheme:

1. (a) $3h - 2 = \frac{3h+5k}{2}$

$6h - 4 = 3h + 5k$

$h = \frac{4+5k}{3}$

(b) The decrease in the value of h

$= 5$

2. $\frac{(x^2y^{-3})^5}{x^{-4}y^2}$

$= \frac{x^{10}y^{-15}}{x^{-4}y^2}$

$= \frac{x^{10+4}}{y^{15+2}}$

$= \frac{x^{14}}{y^{17}}$

3. $2a^3 - a^2b - 8a + 4b$

$= a^2(2a - b) - 4(2a - b)$

$= (a^2 - 4)(2a - b)$

$= (a + 2)(a - 2)(2a - b)$

4. Let x and y be the present ages of Peter and John respectively.

So, we have $x + y = 56$ and $x + 5 = 2(y + 5)$.

Therefore, we have $x + 5 = 2(56 - x + 5)$.

Solving, we have $x = 39$ and $y = 17$.

Thus, the present age of Peter is 39.

5. (a) $x-\frac{x-3}{5}\leq 4$

$5x-(x-3)\leq 20$

$4x\leq 17$

$x\leq\frac{17}{4}$

$\frac{1-x}{3}<1$

$x>-2$

Thus, the overall solution is $-2<x\leq\frac{17}{4}$.

(b) 6

6. (a) The required number is $6\times 100\times 5\%=30$.

(b) $\frac{1600-(970)(0.4)(x)-(970)(0.6)(1.2)}{1600}\times 100\%=12.7\%$

$x=1.8$

7. (a) $B(10,170°)$ and $C(10,210°)$

(b) $\angle BOC=40°$ and $\angle OCB=70°$

(c) $\angle POA=100°-30°=70°=\angle OCB$ by (b)

$\because$ A,O and C are collinear.

$\therefore$ BC is paralled to OP (corr. $\angle$s eq.)

Thus, the claim is agreed.

8. (a) $x=0.1$

(b) The possible area of the paper

$>\left(12-\frac{0.1}{2}\right)\left(4-\frac{0.1}{2}\right)+\left[\left(12-\frac{0.1}{2}\right)-\left(4+\frac{0.1}{2}\right)-\left(4+\frac{0.1}{2}\right)\right]\left(8-\frac{0.1}{2}\right)$

$=77.81$

>77

Thus, the claim is agreed.

9. (a) ΔANM

(b) $\frac{AB}{AN} = \frac{AC}{AM}$ (corr. sides, ~Δs)

$\frac{x+4}{x-2} = \frac{(x-2)+(x+4)}{4}$

$2x^2 - 6x - 20 = 0$

$x = 5$ or $x = -2$ (rej.)

(c) Note that $BC = 20$, $AB = (4+5) = 9$, $AC = [(5-2)+(5+4)] = 12$

$BC^2 = 20^2 = 400$

$AB^2 + AC^2$

$= 9^2 + 12^2$

$= 225 \neq 400$

Since $AB^2 + AC^2 \neq BC^2$, ΔABC is not a right-angled triangle.

10. (a) The range $= 6$

The mode $= 1$

The mean $= 2$

(b) (i) 1.5

(ii) 12

(iii) 1.44

11. (a) In ΔPST and ΔQRT,

(1) $PT = QT$ as $\angle TPQ = \angle TQP$ (sides opp. eq. ∠s)

(2) $\angle PTS = \angle QTS$ (vert. opp. ∠s)

(3) $\angle SPT = \angle RQT$ as $\angle SPR = \angle RQS$ (∠s in the same segment)

Thus, $\Delta PST \cong \Delta QRT$ (ASA)

Marking Scheme
Case 1 Any correct proof with correct reasons. 2
Case 2 Any correct proof without reasons. 1

(b) 4

12. (a) $P = hS + kS^2$

$\begin{cases} 255 = 1.5h + 2.25k \\ 780 = 3h + 9k \end{cases}$

1M for either one

$h = 80,\ k = 60$

$\therefore P = 80S + 60S^2$

The required profit

$= 80(4.5) + 60(4.5)^2$

$= \$\ 1\ 575$

(b) Profit of selling box C

$= 80(9 \times 1.5) + 60(9 \times 1.5)^2$

$= 12\ 015$

$> 9\ 180$

$= 36 \times 255$

Thus, Dora is correct.

13. (a) $f(x)$

$= -\frac{1}{3}x^2 - 2x + 9$

$= -\frac{1}{3}(x^2 + 6x + 3^2 - 3^2) + 9$

$= -\frac{1}{3}(x + 3)^2 + 12$

Thus, the coordinates of the vertex are $(-3, 12)$.

(b) The transformation is first reflected in the x-axis, then translated 5 units downwards.

The transformation is first translated 5 units upwards, then reflected in the x-axis.

(c) $-\frac{1}{3}x^2 - 2x + 9 = 0$

$x = -9$ or $x = 3$

The required area

$= \frac{(3-(-9))(12-6)}{2}$

$= 36$

14. (a) The required volume

$= \frac{(12^2)(16\pi)}{3} \times \left(\frac{8\pi}{16\pi}\right)^3$

$= 96\pi \text{ cm}^3$

(b) The curved surface area of the vessel

$= 9\pi\sqrt{9^2 + 12^2}$

$= 135\pi \text{ cm}^2$

Let A cm^2 be the required area

$$\left(\frac{96\pi}{\frac{(9^2\pi)(12)}{3}}\right)^2 = \left(\frac{A}{135\pi}\right)^3$$

$$\left(\frac{8}{27}\right)^2 = \left(\frac{A}{135\pi}\right)^3$$

$A = 60\pi$

$\therefore$ The required area is $60\pi \text{ cm}^2$.

Alternative solution:

Let h cm be the depth of water in the vessel.

$$96\pi = \frac{1}{3}\pi h\left(\frac{9}{12}h\right)^2$$

$h = 8$

The required area

$= \pi\left(\frac{9}{12}\cdot 8\right)\sqrt{8^2 + \left(\frac{9}{12}\cdot 8\right)^2}$

$= 60\pi \text{ cm}^2$

15. (a) $f(x) = (x + 1)(x^2 + hx + k) + 36$

$f(x) = x^3 + (h + 1)x^2 + (h + k)x + k + 36$

$f(x) = x^3 + (k + 2)x^2 + (3h + 4)x + k + 36$

$$\begin{cases} h + 1 = k + 2 \\ h + k = 3h + 4 \end{cases}$$

Solving, we have $h = -5$ and $k = -6$.

(b) $f(x) = 0$

$x^3 - 4x^2 - 11x + 30 = 0$

$(x - 2)(x^2 - 2x - 15) = 0$

$(x - 2)(x + 3)(x - 5) = 0$

$x = 2$ or $x = -3$ or $x = 5$

Note that -3 is not a natural number.

Thus, the claim is disagreed.

16. (a) By sine formula,

$$\frac{AC}{\sin\angle ABC}=\frac{BC}{\sin\angle BAC}$$

$$\frac{AC}{\sin 60^\circ}=\frac{8}{\sin 20^\circ}$$

$AC\approx 20.25671109$

$AC\approx 20.3$ cm

$\angle ACE=80^\circ$

$AE^2=AC^2+CE^2-2(AC)(CE)\cos\angle ACE$

$AE^2=20.30^2+8^2-2(20.3)(8)\cos 80^\circ$

$AE\approx 20.44636126$

$AE\approx 20.4$ cm

(b) In $\Delta AB'C$,

$s=\frac{20.3+20.4+8}{2}=24.4$

By Heron's Formula,

The area of $\Delta AB'C$ $=\sqrt{24.4(24.4-20.3)(24.4-20.4)(24.4-8)}$

$=79.8$

The height of $\Delta AB'C$ with base AB'

$=\frac{79.8\times 2}{20.4}$

$=7.81$

In $\Delta B'CD$,

$\sin\angle CB'D=\frac{CD}{B'C}$

$CD=8\sin 60^\circ$

Let θ be the required angle.

$\sin\theta=\frac{8\sin 60^\circ}{7.81}$

$\theta=62.6^\circ$

The angle between the plane $AB'C$ and the plane $AB'D$ is 62.6° .

Alternative solution:

In $\Delta CB'D$,

$\cos\angle CB'D = \frac{B'D}{B'C}$

1M either one

$B'D = 8\cos 60°$

$B'D = 4$ cm

In ΔCAD ,

$\cos\angle CAD = \frac{AD}{AC}$

$AD = (20.3)(\cos 20°)$

$AD \approx 19.03508193$

$AD \approx 19.0$ cm

In ΔABD ,

$AD^2 = AB^2 + BD^2 - 2(AB)(BD)\cos\angle AB'D$

$(19.0)^2 = (20.4)^2 + 4^2 - 2(20.4)(4)\cos\angle AB'D$

$\angle AB'D \approx 63.99431541°$

$\angle AB'D \approx 64.0°$

Let Q be the foot of the perpendicular from D to AB' .

In $\Delta B'DQ$,

$\sin\angle AB'D = \frac{DQ}{B'D}$

$DQ = 4\sin 64.0°$

$DQ \approx 3.595002196$

$DQ \approx 3.60$cm

In $\Delta B'CD$,

$\sin\angle CB'D = \frac{CD}{B'C}$

$CD = 8\sin 60°$

In ΔCDQ ,

$\tan\angle CQD = \frac{CD}{DQ}$

1M

$\tan\angle CQD = \frac{8\sin 60°}{3.60}$

$\angle CQD \approx 62.575478°$

$\angle CQD \approx 62.6°$

Thus, the angle between the plane $AB'C$ and the plane $AB'D$ is $62.6°$.

17. (a) The required probability $= \frac{C_6^8 + C_6^8}{C_6^{16}}$

$= \frac{1}{143}$

(b) The required probability $= \frac{C_4^{14} \times 2! \times 5!}{P_6^{16}}$

$= \frac{1}{24}$

18. (a) The required amount

$= 1\,000(1+10\%)^n + 1\,000(1+10\%)^{n-1}(1+21\%) + 1\,000(1+10\%)^{n-2}(1+21\%)^2$

$... + 1\,000(1+10\%)(1+21\%)^{n-1}$

$= 1000(1.1)^n + 1000(1.1)^{n-1}(1.21) + 1000(1.1)^{n-2}(1.21)^2 + \cdots + 1000(1.1)(1.21)^{n-1}$

$= 1000(1.1)^n \left[\frac{1-\left(\frac{1.21}{1.1}\right)^n}{1-\frac{1.21}{1.1}}\right]$

$= 10000(1.1)^n(1.1^n - 1)$

(b) $10000(1.1)^k(1.1^k - 1) \geq 1\,000\,000$

$(1.1^k)^2 - (1.1)^k - 1\,00 \geq 0$

$1.1^k \geq 10.51249 \ \ or \ 1.1^k \leq -9.51249$

$\log 1.1^k \geq \log 10.51249$

$k \log 1.1 \geq \log 10.51249$

$k \geq 24.683$

$\therefore k = 25$

The least value of k is 25.

19. (a) The equation of L_2 is $\frac{x}{200}+\frac{y}{80}=1$.

$y=-0.4x+80$ or $2x+5y-40=0$

Solving $2x+3y=300$ and $\frac{x}{200}+\frac{y}{80}=1$.

The required coordinates are $(75, 50)$.

(b) Let the profit be $\$P$, where $P=1200x+2100y$.

The constraints are $\begin{cases} 400x+600y\leq 60000 \\ 1000x+2500y\leq 200000 \\ x, y \text{ are nonnegative integers} \end{cases}$

i.e. $\begin{cases} 2x+3y\leq 300 \\ 2x+5y\leq 400 \\ x, y \text{ are nonnegative integers} \end{cases}$.

By (a),

At $(0,0)$, $P=1200(0)+2100(0)=0$.

At $(0, 80)$, $P=1200(0)+2100(80)=168000$.

At $(75,50)$, $P=1200(75)+2100(50)=195000$.

At $(150,0)$, $P=1200(150)+2100(0)=180000$.

Thus, the maximum profit is $195 000 , which is less than $200 000.

Hence, the claim is disagreed.

20. (a) (i) Let (x, y) be the coordinates of M

$\sqrt{[(x-(-1)]^2+(y-3)^2} = \sqrt{(x-2)^2+(y-4)^2}$

The equation of S is $y = -3x + 5$ or $3x + y - 5 = 0$

(ii) S is the perpendicular bisector of the line segment AB.

(b) (i) The x-coordinate of $U = \frac{5-k}{3}$.

(ii) The equation of C is

$\left(x - \frac{5-k}{3}\right)^2 + (y-k)^2 = \left(2 - \frac{5-k}{3}\right)^2 + (4-k)^2$

$x^2 - \frac{2x(5-k)}{3} + \left(\frac{5-k}{3}\right)^2 + y^2 - 2ky + k^2 = 2^2 - \frac{4(5-k)}{3} + \left(\frac{5-k}{3}\right)^2 + 4^2 - 2k(4) + k^2$

$x^2 + y^2 - \frac{2x(5-k)}{3} - 2ky = 20 - \frac{4(5-k)}{3} - 8k$

$3x^2 + 3y^2 - 2(5-k)x - 6ky = 60 - 4(5-k) - 24k$

$3x^2 + 3y^2 + 2(k-5)x - 6ky - 60 + 20 - 4k + 24k = 0$

$3x^2 + 3y^2 + 2(k-5)x - 6ky + 20k - 40 = 0$

(iii) Put $y = 5$ into C, we have $3x^2 + 2x(k-5) + (35-10k) = 0$

By considering the difference of two roots, we have

$\left(\frac{-2(k-5)}{3}\right)^2 - 4\left(\frac{35-10k}{3}\right) = 1^2$

$\frac{4(k^2+20k-80)}{9} = 1$

$k^2 + 20k - 82.25 = 0$

$k = 3.5$ or $k = -23.5$

Mock Examination (2018-2019)

Form Six Mathematics Compulsory Part Paper 2 Answer Keys

Questions No.	Key	Questions No.	Key
1.	A	26.	A
2.	C	27.	C
3.	A	28.	D
4.	A	29.	C
5.	D	30.	D
6.	B	31.	C
7.	A	32.	A
8.	A	33.	D
9.	B	34.	A
10.	D	35.	B
11.	D	36.	C
12.	A	37.	C
13.	D	38.	C
14.	C	39.	D
15.	A	40.	B
16.	B	41.	C
17.	B	42.	B
18.	D	43.	A
19.	B	44.	D
20.	C	45.	B
21.	A		
22.	B		
23.	C		
24.	D		
25.	B		