1. $\dfrac{(-3ab^{-1})^{-2}}{a^{-2}b}$

$= \dfrac{(-3)^{-2}a^{-2}b^{2}}{a^{-2}b}$

$= \dfrac{a^{-2}b^{2}}{9a^{-2}b}$

$= \dfrac{b}{9}$

2.(a) $ab + cd - ad - bc$

$= a(b-d) + c(d-b)$

$= (a-c)(b-d)$

2.(b) $144x^2 - 36$

$= 36(2x+1)(2x-1)$

3.(a)

$$\begin{aligned} 5(p-2q)-4 &= 7p-q \\ 5p-10q-4 &= 7p-q \\ -9q-4 &= 2p \\ p &= -\tfrac{9}{2}q-2 \end{aligned}$$

3.(b) The decrease in value of p

$= 18$

4.(a) The marked price

$= 1120 \div (1-30\%)$

$= \$1\,600$

4.(b) The profit percent

$= \dfrac{1\,600 - 340 - 1\,120}{1\,120} \times 100\%$

$= 12.5\%$

5.(a)

$$\begin{aligned} x-8 &< \tfrac{3-x}{2} \\ 2x-16 &< 3-x \\ x &< \tfrac{19}{3} \end{aligned}$$

or

$$\begin{aligned} 5x-2 &\le 4-x \\ x &\le 1 \end{aligned}$$

$\therefore \quad x < \frac{19}{3}$

5.(b) 6

6. Let a and b be the number of avocados owned by Amy and Ben respectively.

$$\begin{cases} a+5=3b \\ a-10=\frac{1}{2}(b+10) \end{cases}$$

$$3b-5-10=\frac{1}{2}(b+10)$$

$\therefore a=19$, $b=8$

$\therefore$ The required number is $19+8=27$.

7.(a) The least possible weight

$= 80-\dfrac{5}{2}$

$= 77.5$ g

7.(b) The least possible total weight

$= 77.5\times 150$

$= 11\ 625$ g

$= 11.625$ kg

> 11.55 kg

= the upper limit of 11.5 kg cor. to the nearest 0.1 kg

The claim is disagreed.

8.(a) In ΔADE & ΔBCE,

$\angle ADE = \angle BCE$ (given)

$\angle AED = \angle BEC$ (vert. opp. $\angle$s)

$\Delta ADE \sim \Delta BCE$ (AA)

8.(b) $\angle ADE = \angle BCE$ (given)

$= 46°$

$\angle EDB = 102° - 46°$

$= 56°$

In ΔEDB,

$\angle EBD = 180° - 85° - 56°$ ($\angle$ sum of Δ)

$= 39°$

In ΔBCE,

$\angle CBE = 85° - 46°$ (ext. $\angle$ of Δ)

$= 39°$

$\Delta ADE \sim \Delta BCE$ (proved)

$\angle BAD = \angle EBC$ (corr. $\angle$s, $\sim\Delta$s)

$= 39°$

$= \angle ABD$

$AD = BD$ (sides opp. equal $\angle$s)

ΔADB is an isos. Δ.

9.(a)

$$\frac{90-x}{360} \times 12\ 000 = 500$$
$$x = 75$$

9.(b)

$$12\ 000 \times \frac{75}{360} + n = 2 \times 12\ 000\left(1 - \frac{90+150+75}{360}\right)$$
$$n = 500$$

The required probability

$$= \frac{12\ 000 \times \frac{150}{360}}{12\ 000 + 500}$$
$$= \frac{2}{5}$$

10.

a	:	b	:	c	=	3	:	5		
		b	:	c	=			2	:	7
a	:	b	:	c	=	6	:	10	:	35

Let $a = 6k$, $b = 10k$ & $c = 35k$; where $k \neq 0$

$$\frac{a+2c}{3b+5c}$$
$$= \frac{6k + 2 \times 35k}{3 \times 10k + 5 \times 35k}$$
$$= \frac{76}{205}$$

11.

$$2\pi(x+3)(25-3x) + 2\pi(x+3)^2 = 64\pi$$
$$(x+3)(25-3x) + (x+3)^2 = 32$$
$$-2x^2 + 22x + 52 = 0$$

$x = -2$ or 13 (rejected)

$x = -2$

12.(a)

Let $f(x) = (3x+k)(2x^2+x-1) + mx + n$

$$f\left(\frac{1}{2}\right) = 1$$
$$\frac{m}{2} + n = 1 \ldots (1)$$
$$f(-1) = 7$$
$$-m + n = 7 \ldots (2)$$

$m = -4$, $n = 3$

$\therefore g(x) = -4x + 3$

12.(b)

$$(3x+k)(2x^2+x-1) - 4x + 3 = 1$$
$$(3x+k)(2x-1)(x+1) - 2(2x-1) = 0$$
$$(2x-1)[(3x+k)(x+1) - 2] = 0$$
$$(2x-1)[3x^2 + (3+k)x + k - 2] = 0$$
$$x = \frac{1}{2}$$

The discriminant of $3x^2 + (3+k)x + k - 2 = 0$

$$\Delta = (3+k)^2 - 4(3)(k-2)$$
$$= k^2 - 6k + 33$$
$$= (k-3)^2 + 24$$
$$> 0 \text{ for all values of } k$$

The roots of the equation $3x^2 + (3+k)x + k - 2 = 0$ are real. $\therefore$ The claim is agreed.

13.(a) $2\pi r = 2\pi(120) \times \frac{216°}{360°}$

$r = 72$

$h = \sqrt{120^2 - r^2}$

$= \sqrt{120^2 - 72^2}$

$= 96$

(b)(i) $\frac{\text{volume of water in the container}}{\text{Capacity of the container}} = (25\%)^{\frac{3}{2}} = \frac{1}{8}$

The volume of the water in the container

$= \frac{1}{8} \times \frac{1}{3}\pi(72)^2(96)$

$= 20\ 736\pi$ cm^3

$\frac{\text{Radius of water surface}}{\text{Base radius of container}} = \frac{\text{Depth of water}}{\text{Height of container}}$ $= \sqrt{25\%}$ $= \frac{1}{2}$ Radius of water surface $= 0.5 \times 72$ $= 36$ Depth of water $= 0.5 \times 96$ $= 48$ Required volume $= \frac{1}{3}\pi(36)^2(48)$ $= 20\ 736\pi$ cm^3

(b)(ii) The area of the inner curved surface of the container which is not wet

$= (1 - 25\%) \times \pi(120)^2 \times \frac{216°}{360°}$

≈ 20357.5204 cm^2

≈ 2.03575204 m^2

> 2 m^2

$\therefore$ The claim is agreed.

14.(a) $a = 0$

$(30 + d) - (10 + 0) = 24$

$d = 4$

$(20 + c) - (10 + b) = 15$

$c = b + 5$

As $3 \leq b \leq 6$ & $6 \leq c \leq 9$,

$\begin{cases} b = 3 \\ c = 8 \end{cases}$ or $\begin{cases} b = 4 \\ c = 9 \end{cases}$.

14.(b) When $b = 3$ & $c = 8$,
the standard deviation $\approx 7.744\ 245\ 175$
When $b = 4$ & $c = 9$,
the standard deviation $\approx 7.732\ 758\ 599$
The least possible standard deviation is 7.73 .

15.

AO	$= BO$	(radii)
$\angle OAB$	$= \angle OBA$	(base $\angle$s, isos. Δ)
	$= \theta$	
$\angle CDB$	$= \angle CAB$	($\angle$s in the same segment)
	$= \theta$	
$\angle ADC$	$= 90°$	($\angle$ in semi-circle)
$x + \theta$	$= 90°$	
x	$= 90° - \theta$	

In ΔADC,

CD	$= DA$	(given)
$\angle ADC$	$= 90°$	($\angle$ in semi-circle)
$\angle DCA$	$= \angle DAC$	(base $\angle$s, isos. Δ)
	$= (180° - 90°) \div 2$	($\angle$ sum of Δ)
	$= 45°$	

In ΔADE,

y	$= x + 45°$	(ext. $\angle$ of Δ)
	$= 90° - \theta + 45°$	
	$= 135° - \theta$	

16.(a) $A'(6, 345°)$

16.(b) In ΔOAC,

$\frac{OC}{\sin 120°} = \frac{6}{\sin 30°}$

$OC = 6 \div \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= 6\sqrt{3}$

$C(6\sqrt{3}, 15°)$

16.(c) Area of $OACA'$

$= 2 \times \frac{1}{2} \times 6^2 \sin 120°$

$= 18\sqrt{3}$

17.

$$\begin{aligned} 1 &= \log_a 2 - \log_a 2 + b \\ b &= 1 \\ 3 &= \log_a 98 - \log_a 2 + 1 \\ 3 &= \log_a \tfrac{98}{2} + 1 \\ 2 &= \log_a 49 \\ a &= 7 \end{aligned}$$

$$\begin{aligned} y &= \log_7 x - \log_7 2 + 1 \\ y - 1 &= \log_7 \tfrac{x}{2} \\ x &= 2 \times 7^{y-1} \text{ or } \tfrac{2}{7^{1-y}} \end{aligned}$$

18.(a) Let μ marks and σ marks be the mean and the standard deviation of the test scores respectively.

$$\begin{cases} \dfrac{50-\mu}{\sigma} = -1.5 \\ \dfrac{80-\mu}{\sigma} = 2.25 \end{cases}$$

$\mu = 62$ and $\sigma = 8$

The mean score is 62 marks and the standard deviation is 8 marks.

18.(b) When the score 62 marks is deleted, the mean remains unchanged, but the standard deviation σ increases to σ_1.

Alan's new standard score

$= \dfrac{50-62}{\sigma_1}$

$= -\dfrac{12}{\sigma_1}$

$> -\dfrac{12}{\sigma}$

= Alan's original standard score

Hence, the standard score of Alan increases.

19. The required probability

$= \dfrac{2! \times 7!}{8!}$

$= \dfrac{1}{4}$

20.(a) Let a and r be the first term and the common ratio of the geometric sequence respectively.

$\begin{cases} ar = 600 \\ ar^4 = 2\ 025 \end{cases}$

$a = 400$ and $r = 1.5$

The first term is 400.

20.(b)

$$ar^n + ar^{n+1} + ar^{n+2} + \cdots + ar^{2n-1} > 1.5 \times 10^{18}$$

$$ar^n\left(\frac{r^n-1}{r-1}\right) > 1.5 \times 10^{18}$$

$$ar^{2n} - ar^n - (r-1) \times 1.5 \times 10^{18} > 0$$

$$r^{2n} - r^n - \frac{r-1}{a} \times 1.5 \times 10^{18} > 0$$

$$1.5^{2n} - 1.5^n - 1.875 \times 10^{15} > 0$$

$1.5^n > 43\ 301\ 270.69$ or $1.5^n < -43\ 301\ 269.69$ (rej.)

$$n > \frac{\log 43\ 301\ 270.69}{\log 1.5}$$

$n > 43.366\ 721\ 79$

The least value of n is 44.

21.(a) Let r be the radius.

$$\sqrt{12(12-7)(12-8)(12-9)} = \frac{1}{2}(7)r + \frac{1}{2}(8)r + \frac{1}{2}(9)r$$

$$\sqrt{720} = 12r$$

$$r = \sqrt{5}$$

The radius is $\sqrt{5}$.

(b)(i)

$$\tan 60° = \frac{VO}{r}$$

$$VO = \sqrt{5} \times \sqrt{3} = \sqrt{15}$$

The required volume

$= \frac{1}{3}(\text{Area of } \Delta ABC)(VO)$

$= \frac{1}{3} \times \sqrt{720} \times \sqrt{15}$

$= 20\sqrt{3}$

(b)(ii) Let D be the point of intersection of the inscribed circle of ΔABC and AC .

$VD = \sqrt{VO^2 + r^2}$

$= \sqrt{15 + 5}$

$= \sqrt{20}$

Area of ΔVAC

$= \frac{1}{2}(AC)(DV)$

$= \frac{1}{2} \times 9 \times \sqrt{20}$

$= 9\sqrt{5}$

(b)(iii) Volume of $VABC = \frac{1}{3}(\text{area of } \Delta VAC)(BM)$

$$20\sqrt{3} = \frac{1}{3}(9\sqrt{5})(BM)$$
$$BM = \sqrt{\frac{80}{3}}$$
$$BM \approx 5.16$$

$$AM^2 = AB^2 - BM^2$$
$$= 7^2 - \frac{80}{3}$$
$$AM = \sqrt{\frac{67}{3}}$$

$$CM^2 = BC^2 - BM^2$$
$$= 64 - \frac{80}{3}$$
$$CM = \sqrt{\frac{112}{3}}$$
$$\neq \sqrt{\frac{67}{3}} = AM$$

∴ The claim is disagreed.

$\angle BMA = \angle BMC = \angle BMV = 90°$
If M is the circumcenter,
then $MV = MA = MC$.
$$\begin{cases} AB^2 = BM^2 + MA^2 \\ BC^2 = BM^2 + MC^2 \end{cases}$$
If $MA = MC$, $AB = BC$.
However, $AB = 7$
$\neq 8$
$= BC$
The claim is disagreed.

22.(a) Let $(h, 0)$ be the coordinates of A.

$$\begin{cases}(x-h)^2+y^2=h^2 \text{ ... (1)} \\ 4x-3y+90=0 \text{ ... (2)}\end{cases}$$

From (2)

$y=\frac{4}{3}x+30$...(3)

Put (3) into (1)

$$(x-h)^2+\left(\frac{4}{3}x+30\right)^2=h^2$$

$$\frac{25}{9}x^2+2(40-h)x+900 = h^2$$

$$2^2(40-h)^2-4\times\frac{25}{9}\times 900 = 0$$

$$h^2-80h-900 = 0$$

$h=-10$ or $h=90$ (rej.)

The equation of C is

$(x+10)^2+y^2=100$ or $x^2+y^2+20x=0$

(b)(i) Γ is a circle with centre A and radius AP.

(b)(ii) $(x+10)^2+y^2=1\,000$ or

$x^2+y^2+20x-900=0$

(c)(i)

$$\begin{cases}x^2+y^2+20x-900=0 \text{ ... (4)} \\ y=\dfrac{4}{3}x+30 \text{ ... (3)}\end{cases}$$

Put (3) into (4),

$$x^2+\left(\frac{4}{3}x+30\right)^2+20x-900=0$$

$$x^2+\frac{16}{9}x^2+80x+900+20x-900 = 0$$

$$\frac{25}{9}x^2+100x = 0$$

$x=0$ (rej.) or $x=-36$

$R(-36,-18)$

Let $(0, s)$ be the coordinates of S.

$$PS = RS$$

$$(30-s)^2 = 36^2+(s+18)^2$$

$$900-60s+s^2 = 1296+s^2+36s+324$$

$$s = -7.5$$

$S(0,-7.5)$

(c)(ii) Area of ΔPRS

$=36\times(7.5+30)\div 2$

$=675$

1.	C	11.	C	21.	C	31.	A	41.	B
2.	A	12.	B	22.	A	32.	A	42.	A
3.	D	13.	D	23.	D	33.	C	43.	D
4.	B	14.	B	24.	A	34.	B	44.	C
5.	C	15.	A	25.	B	35.	C	45.	C

6.	A	16.	C	26.	D	36.	A
7.	C	17.	D	27.	A	37.	C
8.	B	18.	B	28.	C	38.	C
9.	D	19.	C	29.	D	39.	B
10.	D	20.	D	30.	C	40.	A