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DIOCESAN BOYS' SCHOOL

G12 MOCK EXAMINATIONS

MATHEMATICS COMPULSORY PART PAPER 1

MARKING SCHEME

SECTION A(1) (35 marks)

	Solution	Remarks
1.	$\frac{m^{-5}}{(mn^3)^{-2}}=\frac{m^{-5}}{m^{-2}n^{-6}}$	1M for $(ab)^n=a^nb^n$
	$=m^{-3}n^6$	1M for $\frac{a^m}{a^n}=a^{m-n}$ or $a^{-n}=\frac{1}{a^n}$
	$=\frac{n^6}{m^3}$	1A
		----------(3)
2.	$b+1=\frac{4a+b}{a}$	
	$ab+a=4a+b$	1M
	$ab-b=3a$	1A
	$b(a-1)=3a$	
	$b=\frac{3a}{a-1}$	1A
		----------(3)
3. (a)	$x^2(x+3y)$	1A
(b)	$x^3+3x^2y-9x-27y$	
	$=x^2(x+3y)-9(x+3y)$	1M
	$=(x^2-9)(x+3y)$	
	$=(x+3)(x-3)(x+3y)$	1A
		----------(3)

	Solution	Marks
4.	Let \$$x$ be the marked price	
	$(1-10\%)x-\dfrac{x}{1+30\%}=340$	1M (1-10%) +1M $\left(\dfrac{1}{1+30\%}\right)$ +1A
	$x=2600$	
	Marked price is \$2600.	1A
		----------(4)

Alternate method	
Let \$$c$ be the cost	
$(1+30\%)(1-10\%)c-c=340$	1M (1-10%) +1M $(1+30\%)$ +1A
$c=2000$	
Marked price = $2000\times1.3=\$2600$	1A

	Solution	Marks
5. (a)	$\dfrac{x-1}{-4}\geq 3x-2$	
	$x-1\leq -12x+8$ and $11-x>5$	1M for putting x on one side
	$13x\leq 9$; $6>x$	
	$x\leq\dfrac{9}{13}$	1A for LHS or RHS
	$\therefore\ x\leq\dfrac{9}{13}$	1A
(b)	0	1A
		----------(4)
6.	Let x hours be the time used by car A	
	Then the time used by car B is $(x+0.05)$ hours	
	$\dfrac{60}{x}-\dfrac{60}{x+0.05}=2$	1M speed = distance/time +1A
	$60(x+0.05-x)=2x(x+0.05)$	
	$2x^2+0.1x-3=0$	1A
	$x=1.2$ or -1.25(rej.)	
	The time used by car A is 1.2 hours.	1A (or 1 hour 12 minutes)
		----------(4)

7. (a)	120°	1A
(b)	(i) $(13, 5^\circ)$	1A
	(ii) Length of each side = $2(13\sin 60^\circ)$	1M
	Perimeter of $\Delta ABC = 3\times 2(13\sin 60^\circ) = 39\sqrt{3}$	1A(accept 67.5)
		----------(4)
8. (a)	Reflex $\angle AOC = 2\times\angle ABC$	1M
	$= 264^\circ$	
	$\angle AOC = 360^\circ - 264^\circ$	
	$= 96^\circ$	1A
(b)	$\overset{\frown}{AB} : \overset{\frown}{BC} = 1:2$	
	$\angle BOC = 96^\circ \times \frac{2}{3}$	1M
	$= 64^\circ$	
	$\angle OCB = \frac{180^\circ - 64^\circ}{2} = 58^\circ$	1M
	$\angle AEC = 180^\circ - 58^\circ = 122^\circ$	1A
		----------(5)
9. (a)	Let $h(x) = r + sx$, where r and s are non-zero constants.	1M
	So we have $r - 4s = -76$ and $r + 9s = 80$	1M (any one)
	Solving, we have $r = -28$ and $s = 12$	
	Thus, we have $h(x) = 12x - 28$	1A
(b)	$h(x) = x^2 + x$	
	$12x - 28 = x^2 + x$	
	$x^2 - 11x + 28 = 0$	1M
	$x = 7$ or 4	1A
		----------(5)

SECTION A(2) (35 marks)

	Solution	Remarks
10.(a)	$90° + \angle CBD + \angle BDC = 180°$	
	thus $\angle CBD + \angle BDC = 90°$	
	$\angle AED + \angle BDC = 90°$	
	$\angle EFD = 90°$	
	Thus $\angle DFE = \angle ADE$	
	$\angle DEF = \angle AED$ (common angle)	
	$\angle EDF = \angle EAD$ (remaining angle)	
	$\Delta DEF \sim \Delta AED$ (AAA)	
	Marking Scheme:	
	Case 1 Any correct proof with correct reasons	2
	Case 2 Any correct proof without reasons	1
(b)	$\angle DEF = \angle DBC$ (given)	
	$\angle FDE = \angle CDB$ (common angle)	
	$\angle DFE = \angle BCD$ (remaining angle)	
	$\Delta DEF \sim \Delta DBC$ (AAA)	
	Claim agreed.	1A f.t.
(c)	$BD = \sqrt{5^2 + 12^2} = 13$	1A
	$\Delta AED \sim \Delta DBC$	
	$\frac{DE}{AD} = \frac{BC}{DC}$	1M
	$\frac{DE}{5} = \frac{5}{12}$	
	$DE = \frac{25}{12}$	

$\Delta DEF \sim \Delta DBC$

$$\frac{DF}{DC} = \frac{DE}{DB}$$

$$\frac{DF}{12} = \frac{25/12}{13}$$

$$DF = \frac{25}{13} \text{ cm}$$

1A

----------(6)

11.(a)	$9+a+9+b+4=40$ $a+b=18$	
	The median = 1, we have	
	$9+b+4 \leq 19$ $b \leq 6$	1M
	Note that $4<b<9$	
	Thus, we have $\begin{cases} a=12 \\ b=6 \end{cases}$ or $\begin{cases} a=13 \\ b=5 \end{cases}$	1A for one pair +1A for all
(b)	At least one of the added data is 5	
	(i) Case 1: $a=12$ and $b=6$ five 1 and five 5 are added mean = 1.88 Case 2: $a=13$ and $b=5$ four 1 and six 5 are added mean = 1.92 Thus the greatest possible mean is 1.92	1A
	(ii) Case 1: $a=12$ and $b=6$ five 0 and five 5 are added IQR = 3 – 0 = 3 Case 2: $a=13$ and $b=5$ four 0 and six 5 are added IQR = 3 – 0 = 3 Thus the greatest possible IQR = 3 – 0 = 3	1A
	(iii) Case 1: $a=12$ and $b=6$ five 1, four 2 and one 5 are added variance = 1.5504 Case 2: $a=13$ and $b=5$ four 1, five 2 and one 5 are added variance = 1.5156 Thus the least possible variance = 1.5156	1A(r.t. 1.52) ----------(6)

12.(a)	Let A cm^2 be the final area of the wet surface	
	$\frac{A-1200\pi}{A}=\left(\frac{24}{36}\right)^2$	1M+1A
	$A-1200\pi=\frac{4}{9}A$	
	$A=2160\pi$	
	Thus, the final area is $2160\pi\text{ cm}^2$.	1A
(b)	Let r cm be the radius of water surface in the final volume.	
	$\pi r\sqrt{36^2+r^2}=2160\pi$	1M
	$r^2\left(36^2+r^2\right)=2160^2$	
	$r^4+36^2r^2-2160^2=0$	
	$r^2\approx 1607$ or -2903(rej.)	1M
	$\text{Volume}=\frac{1}{3}\pi r^2h$	
	$=\frac{1}{3}\pi(1607)(36)$	1M
	$\approx 60586>60000$	
	Claim disagreed.	1A f.t.
		----------(7)

13.(a)

$p(x)=(2x^2-3x-2)q(x)+4x-8$ 1M

$p(2)=0$

Thus, $x-2$ is a factor of $p(x)$. 1A f.t.

(b)

$4x^3+bx+c=(2x^2-3x-2)(mx+n)+4x-8$ 1M

$4x^3+bx+c=2mx^3+(2n-3m)x^2+(4-2m-3n)x-2n-8$

Comparing like terms on both sides,

$$\begin{cases} 4=2m \\ 0=2n-3m \\ b=4-2m-3n \\ c=-2n-8 \end{cases} \Rightarrow \begin{cases} m=2 \\ n=3 \\ b=-9 \\ c=-14 \end{cases}$$ } 1M

$$\begin{aligned} p(x) &= 4x^3-9x-14 \\ &= (x-2)(4x^2+8x+7) \end{aligned}$$ 1M

Consider $4x^2+8x+7=0$

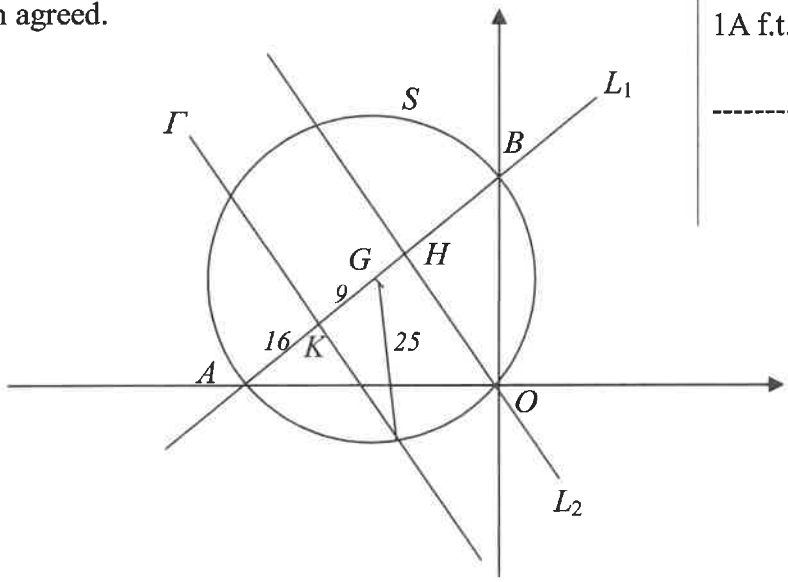
$$\begin{aligned} \Delta &= 8^2-4\times4\times7 \\ &= -48<0 \end{aligned}$$ 1M

Thus $4x^2+8x+7=0$ has no real roots

Claim disagreed. 1A f.t.

----------(7)

14.(a)	Centre $G = (-20,15)$ and radius $= 25$	1M+1A
	Equation of *C*: $(x+20)^2 + (y-15)^2 = 625$	1A
14.(b)(i)	Γ is the perpendicular bisector of *AH*.	1A
14.(b)(ii)	Coordinates of *H* are $(-14.4,19.2)$	1A
14.(b)(iii)	$GH = \sqrt{(20-14.4)^2 + (19.2-15)^2} = 7$	1M(dist. formula)
	Γ cuts L_1 at $K(-27.2,9.6)$	
	$GK = 9$	
	$\cos\angle KGT = \dfrac{9}{25}$ $\angle KGT \approx 68.89980398°$	1M
	Area of the segment containing *G*	
	$= \dfrac{360° - 2\angle KGT}{360°} \times \pi(25)^2 + \dfrac{1}{2}(25)^2 \sin 2\angle KGT$ $\approx 1421.829 > 450\pi$	1M
	Claim agreed.	1A f.t.
		----------(9)



SECTION B (35 marks)

	Solution	Remarks
15.(a)	Number of different queues = $6!P_3^7 = 151200$	1M(P_3^7)+1A
(b)	The required probability	
	$=\dfrac{5!P_3^6 2!+5!C_1^2 P_2^6 2!}{151200}$	1M(for either case in numerator)
	$=\dfrac{43200}{151200}$	
	$=\dfrac{2}{7}$	1A
		----------(4)

Solution	Remarks
Alternate method (1)	
The required probability	
$=\dfrac{5\times 2\times 5!P_2^6+2\times 1\times 5!P_2^6}{151200}$	1M(for either case in numerator)
$=\dfrac{43200}{151200}$	
$=\dfrac{2}{7}$	1A

Solution	Remarks
Alternate method (2)	
The required probability	
$=\dfrac{5}{7}\times\dfrac{2}{6}+\dfrac{2}{7}\times\dfrac{1}{6}$	1M(for either case)
$=\dfrac{2}{7}$	1A

16.(a)	
$f(x)=-\frac{1}{4}(x^2-24x)-45$	
$=-\frac{1}{4}((x-12)^2-144)-45$	1M
$=-\frac{1}{4}(x-12)^2-9$	
Vertex $=(12,-9)$	1A
(b)	
Vertex of $g(x)=(0,-9)$	
Thus, $g(x)==-\frac{1}{4}x^2-9$	1A
(c)	
$f(-2x)=-\frac{1}{4}(-2x)^2+6(-2x)-45$	1M
$=-x^2-12x-45$	
The new graph is obtained by reducing the graph of $y=f(x)$ along the x-axis to the half of its original and	1A(~~)
then reflected along the y-axis.	1A
	----------(6)

Alternate method of (c)	
$f(2x+24)=-\frac{1}{4}(2x+24-12)^2-9$	1M
$=-\frac{1}{4}(2x+12)^2-9$	
$=-x^2-12x-45$	
The new graph is obtained by translating the graph of $y=f(x)$ to the left by 24 units and then reducing the	1A(~~)
graph along the x-axis to the half of its original.	1A

17.(a)	Let a be the first term, r be the common ratio	
	$\begin{cases} ar = 24 \\ ar^6 = 768 \end{cases}$	1M(any one)
	Thus, $r = 2$ and $a = 12$	1A
(b)	$12\left(\dfrac{2^{2n}-1}{2-1}\right)-12\left(\dfrac{2^n-1}{2-1}\right)>10^{20}$	1M
	Let $x = 2^n$	
	$12(x^2-1)-12(x-1)>10^{20}$ $12x^2-12x-10^{20}>0$ $x<-2886751345$ or $x>2886751346$	1M
	$2^n > 2886751346$ $n > \log_2 2886751346$ $n > 31.4267997$	1M(taking log)
	Thus, the least value of n is 32.	1A
		----------(6)

18.(a)

$$\frac{AD}{\sin \angle ABD} = \frac{AB}{\sin \angle ADB}$$ 1M

$$\frac{AB}{\sin 65^\circ} = \frac{85}{\sin 35^\circ}$$

$$AB \approx 134 \text{ cm} \quad (134.3084496)$$ 1A

18.(b)(i)

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos 30^\circ$$ 1M

$$= 134^2 + 85^2 - 2 \times 134 \times 85 \times \cos 30^\circ$$

$$AC \approx 74.09648881$$

$$CD = AB - 2 \times 85\cos 80^\circ \approx 104.7882594$$

$$\cos \angle ADC = \frac{AD^2 + CD^2 - AC^2}{2 \times AD \times CD}$$

$$= \frac{85^2 + 104.8^2 - 74.1^2}{2 \times 85 \times 104.8}$$

$$\angle ADC \approx 44.5^\circ \quad (44.4566502^\circ)$$ 1A(r.t. 44.5°)

18.(b)(ii)

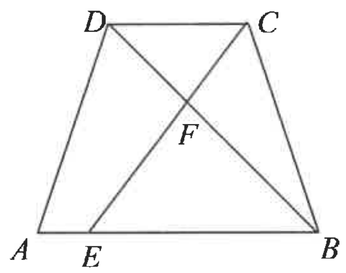
Let E on AB such that CE cuts BD perpendicularly at F.

$$CF = 85\sin 45^\circ \approx 60.1$$

$$BF = 85\cos 45^\circ \approx 60.1$$

$$FE = 85\cos 45^\circ \tan 35^\circ \approx 42.1$$

$$BE = \frac{85\cos 45^\circ}{\cos 35^\circ} \approx 73.4$$



After folding, distance between C and E

$$= \sqrt{85^2 + BE^2 - 2(85)(BE)\cos 30^\circ}$$ 1M

$$\approx 42.5 \quad (42.50066993)$$

Angle between two planes = $\angle CFE$ 1M

$$\angle CFE = \cos^{-1}\left(\frac{60.1^2 + 42.1^2 - 42.5^2}{2 \times 60.1 \times 42.1}\right)$$

$$\approx 45.0^\circ \quad (44.99815791^\circ)$$ 1A(r.t. 45.0°)

----------(7)

19.(a)	$x^2+(5-2k)^2-22x-10(5-2k)-6k(5-2k)$ $-160k^2+30k+146=0$	1M
	$x^2+25-20k+4k^2-22x-50+20k-30k+12k^2$ $-160k^2+30k+146=0$	
	$x^2-22x-144k^2+121=0$	
	$(x-11-12k)(x-11+12k)=0$	
	$x=11+12k$ or $11-12k$	
	Thus, $P=(11-12k,5-2k)$ and $Q=(11+12k,5-2k)$	1A
19.(b)	Coordinates of *G* is $(11,5+3k)$	
	Slope of *PG* = $\frac{5}{12}$	
	Equation of *PG*:	
	$\frac{y-5-3k}{x-11}=\frac{5}{12}$	1M
	$y=\frac{5}{12}x-\frac{55}{12}+5+3k$	
	$y=\frac{5}{12}x+\frac{5}{12}+3k$	1A
19.(c)(i)	Let *r* be radius of *S*. Note that *PG* = *QG*.	
	So the coordinates of the centre of *S* are $(11,5-2k+r)$.	1M
	Hence, the equation of *S* is	
	$(x-11)^2+(y+2k-5-r)^2=r^2$	1M
	Putting $y=\frac{5x+5}{12}+3k$ in equation of *S*	
	$(x-11)^2+\left(\frac{5x+5}{12}+5k-5-r\right)^2=r^2$	
	$x^2-22x+121+\left(\frac{5x+5}{12}\right)^2$ $+2\left(\frac{5x+5}{12}\right)(5k-5-r)+(5k-5-r)^2-r^2=0$	

$$169x^2+(-120r+600k-3718)x$$
$$+3600k^2-1440kr-6600k+1320r+20449=0$$

Since PG is tangent to S, we have

$$(-120r+600k-3718)^2$$
$$-4(169)(3600k^2-1440kr-6600k+1320r+20449)=0$$ 1M(Δ=0)

After simplify,

$$5r^2+288kr-720k^2=0$$
$$r=2.4k \text{ or } -60k(\text{rej.})$$

Thus, we have $r=2.4k$ 1A

19.(c)(ii) Suppose it is possible. Let W be centre of S.

Since PV bisects $\angle GPQ$, slope of PW = slope of PV. 1M

$$\frac{8-5+2k}{6-11+12k}=\frac{1}{5}$$
$$k=10$$ 1A

Thus, coordinates of P, T and U are $(-109,-15)$, $\left(\frac{23}{13},\frac{405}{13}\right)$ and $(11,-15)$ respectively

Let M be the mid-point of TU.
"$PT=PU$" + "PV bisects $\angle GPQ$"
⇓
M is the point of intersection 1M

Coordinates of M = $\left(\frac{83}{13},\frac{105}{13}\right)$

x-coordinate of M > x-coordinate of V and
y-coordinate of M > y-coordinate of V

Thus M does not lie on PV. Thus not possible. 1A f.t.

----------(12)