2021-2022
MOCK EXAM
MATH CP

PAPER 1

DIOCESAN BOYS' SCHOOL

MOCK EXAMINATION

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

January 2022

Time allowed: 2¼ hours

This paper must be answered in English

Name	
Class	
Class Number	

Group (Teacher's initials)
Please tick your group

DSM1 (HYL)		PMM1 (ZY)		JT1 (CHN)	
DSM2 (KWN)		PMC1 (FYL)		JT2 (HLC)	
DSC (KSL)		PMC2 (LTC)		JT3 (CYW)	

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Name, Class, Class Number and Group in the spaces provided on Page 1.
2. This paper consists of THREE sections, A(1), A(2) and B.
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name, Class, Class Number, Group and the question number on each sheet, and fasten them with string INSIDE this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.
8. No extra time will be given to candidates for writing any information after the 'Time is up' announcement.

Section A(1)	
Section A(2)	
Section B	
Paper Total	

SECTION A(1) (35 marks)

1. Simplify $\dfrac{m^{-5}}{\left(mn^3\right)^{-2}}$ and express your answer with positive indices. (3 marks)

2. Make b the subject of the formula $b+1=\dfrac{4a+b}{a}$. (3 marks)

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3. Factorize

(a) x^3+3x^2y,

(b) $x^3+3x^2y-9x-27y$.

(3 marks)

4. The marked price of a dress is 30% above the cost. If the dress is sold at a discount of 10% on the marked price, then the profit is \$340. Find the marked price of the dress. (4 marks)

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5. Consider the compound inequality

$$\frac{x-1}{-4} \geq 3x-2 \quad \text{and} \quad 11-x>5 \qquad \ldots\ldots\ldots\ldots(*)$$

(a) Solve (*).

(b) Write down the greatest integer satisfying (*).

(4 marks)

6. Two cities, P and Q, are 60 km apart. Car A and car B both travel from city P to city Q. They start at the same time but car B arrives at city Q later than car A by 3 minutes. If the difference between the average speed of car A and the average speed of car B is 2 km/h, find the travelling time of car A. (4 marks)

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7. In a polar coordinate system, O is the pole. The polar coordinates of the points A and B are $(13,125°)$ and $(13,245°)$ respectively.

(a) Find $\angle AOB$.

(b) Let C be a point such that the number of folds of rotational symmetry of ΔABC about O is 3.

(i) Write down the polar coordinates of C.

(ii) Find the perimeter of ΔABC.

(4 marks)

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8. In Figure 1, O is the centre of the circle $ABCD$. It is given that $\angle ABC = 132°$, $AD \parallel BC$ and $\overset{\frown}{BC} = 2\overset{\frown}{AB}$.

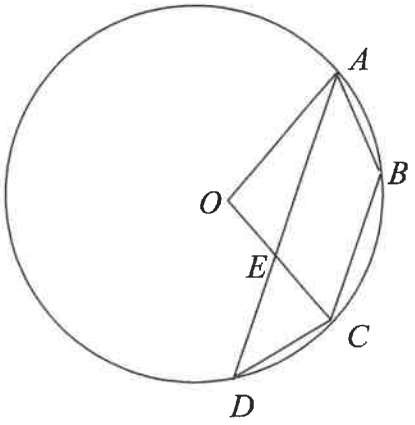

Figure 1

Find

(a) $\angle AOC$,

(b) $\angle AEC$.

(5 marks)

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9. It is given that $h(x)$ is partly constant and partly varies as x. Suppose that $h(-4)=-76$ and $h(9)=80$.

(a) Find $h(x)$.

(b) Solve the equation $h(x)=x^2+x$.

(5 marks)

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SECTION A(2) (35 marks)

10. In Figure 2, $ABCD$ is a rectangle. E is a point on CD such that $\angle AED = \angle CBD$. AE and BD intersect at F.

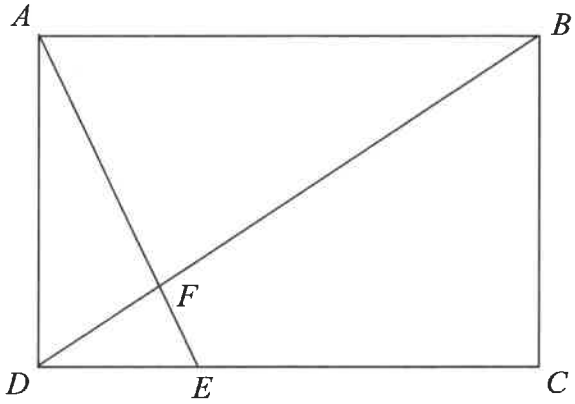

Figure 2

(a) Prove that $\Delta DEF \sim \Delta AED$. (2 marks)

(b) A student claims that $\Delta DEF \sim \Delta DBC$. Is the claim correct? Explain your answer. (1 mark)

(c) It is given that $BC = 5$ cm and $CD = 12$ cm. Find the exact length of DF. (3 marks)

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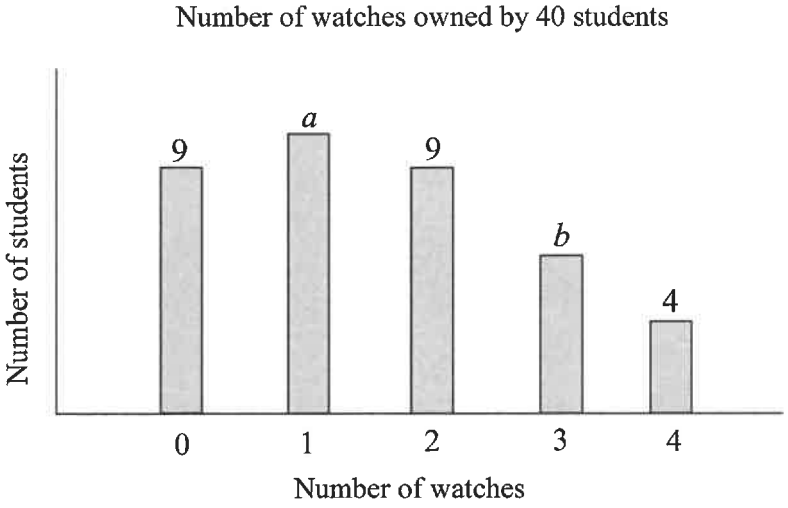
11. The following bar chart shows the distribution of the numbers of watches owned by 40 students. It is given that $a > 9$ and $4 < b < 9$. The median of the numbers of watches owned by the students is 1.



(a) Find a and b. (3 marks)

(b) Ten more students are interviewed and their numbers of watches are added to the distribution. It is found that the median of the distribution remains unchanged and the range of the distribution is increased by 1. Write down

(i) the greatest possible mean of the distribution,

(ii) the greatest possible inter-quartile range of the distribution,

(iii) the least possible variance of the distribution.

(3 marks)

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12. An inverted right circular conical vessel contains some water. The vessel is held vertically. The initial depth of water in the vessel is 24 cm. George then pours some water into the vessel without overflowing until the depth of water reaches 36 cm. He finds that the area of the wet curved surface of the vessel is increased by $1\,200\pi$ cm^2.

(a) Express the final area of the wet curved surface of the vessel in terms of π. (3 marks)

(b) George claims that the final volume of water in the vessel is less than 60 000 cm^3. Do you agree? Explain your answer. (4 marks)

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13. Let $p(x)$ be a polynomial. When $p(x)$ is divided by $2x^2-3x-2$, the remainder is $4x-8$.

(a) Prove that $x-2$ is a factor of $p(x)$. (2 marks)

(b) It is given that $p(x)=4x^3+bx+c$, where b and c are real constants. Someone claims that $p(x)=0$ has 3 distinct real roots. Do you agree? Explain your answer. (5 marks)

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14. The coordinates of the points O, A and B are $(0,0)$, $(-40,0)$ and $(0,30)$ respectively. The circle C passes through O, A and B. Denote the centre of C by G.

(a) Find the equation of C. (3 marks)

(b) The straight line L_1 passes through B and G. L_2 is another straight line which passes through O and cuts L_1 perpendicularly at H. Let P be a moving point in the rectangular coordinate plane such that $AP = PH$. Denote the locus of P by Γ.

(i) Describe the geometric relationship between Γ and the line segment AH.

(ii) Write down the coordinates of H.

(iii) Denote the region bounded by Γ and C that contains G by S. Someone claims that the area of S exceeds 450π. Is the claim correct? Explain your answer.

(6 marks)

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SECTION B (35 marks)

15. A queue is randomly formed by 6 boys and 3 girls such that no 2 girls are next to each other.

(a) How many different queues can be formed? (2 marks)

(b) David and Emily are in the queue. Find the probability that they are next to each other. (2 marks)

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16. Let $f(x) = -\frac{1}{4}x^2 + 6x - 45$.

(a) Using the method of completing the square, find the coordinates of the vertex of the graph of $y = f(x)$. (2 marks)

(b) The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ horizontally such that the vertex of the graph of $y = g(x)$ lies on the y-axis. Write down $g(x)$. (1 mark)

(c) Under two transformations, $f(x)$ is changed to $-x^2 - 12x - 45$. Describe the geometric meaning of the transformations. (3 marks)

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17. The 2nd term and the 7th term of a geometric sequence are 24 and 768 respectively.

(a) Find the 1st term of the sequence. (2 marks)

(b) Find the least value of n such that the sum of the first $2n$ terms exceeds the sum of the first n terms by more than 10^{20}. (4 marks)

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18. A paper card $ABCD$ is in the shape of a trapezium, where AB // DC. It is given that $AD = BC = 85$ cm, $\angle BAD = 80°$ and $\angle ADB = 65°$.

(a) Find the length of AB. (2 marks)

(b) The paper card is folded along BD such that $\angle ABC = 30°$, as shown in Figure 3.

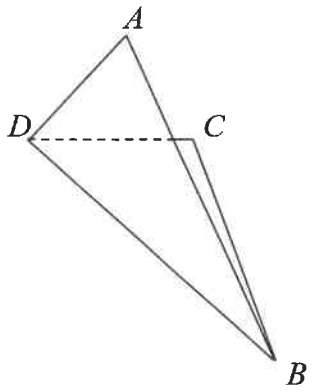

Figure 3

(i) Find $\angle ADC$.

(ii) Find the angle between the plane ABD and the plane BCD.

(5 marks)

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19. The equation of the circle C is $x^2+y^2-22x-10y-6ky-160k^2+30k+146=0$, where k is a positive constant. The straight line $L: y=5-2k$ cuts C at P and Q, where P is on the left of Q. Denote the centre of C by G.

(a) Express, in terms of k, the coordinates of P and Q. (2 marks)

(b) Express, in terms of k, the equation of the straight line which passes through P and G. (2 marks)

(c) Denote the inscribed circle of ΔGPQ by S and the radius of S by r.

(i) Using the result of (b), express r in terms of k.

(ii) Suppose that GP is the tangent to S at the point T and PQ is the tangent to S at the point U. It is given that the coordinates of the point V are $(6,8)$. Is it possible that the line segment PV bisects $\angle GPQ$ and intersects the line segment TU? Explain your answer. (8 marks)

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END OF PAPER

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