Diocesan Girls' School
Secondary 6 Mock Examinations (2017-2018)
Mathematics (Compulsory Part)
Paper 1

Feb 2018

Time Allowed: 2 hours 15 minutes | Total marks: 105

Name:____________________(   )   Class:____________   Set:__________

Instructions:

1. This paper consists of THREE sections, A(1), A(2) and B.
2. Attempt ALL questions. Write your answers in the spaces provided in this Question-Answer Book.
3. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class and class number on each sheet, and staple them INSIDE this book.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale.

Section A(1) (35 marks)

1. Simplify $\dfrac{(2^{-1}a^{-3}b)^3}{ab^2}$ and express your answer with positive indices.

(3 marks)

2. Factorize
   (a) $18m^2 - 60mn + 50n^2$,
   (b) $18m^2 - 60mn + 50n^2 + 10n - 6m$.

(3 marks)

3. (a) Find the range of values of x which satisfy the compound inequality
   $\frac{3x-10}{8} < x$ or $5(8-x) \geq 4(13-2x)$.
   (b) If x is a negative integer, write down the possible value(s) of x.

(4 marks)

4. In a sale of a department store, all items are sold at a discount of 20%. Amy wants to buy a bag marked at $320 and a pair of shoes marked at $650.
   (a) Find the amount that Amy needs to pay.
   (b) Now, a further 20% discount on the reduced prices will be offered for buying 3 items. So, Amy wants to buy one more item in addition. Find the marked price of the extra item so that the amount she needs to pay after the extra discount is equal to the amount found in (a).

(4 marks)

5. R is rotated anti-clockwise about the origin O through 90° to R'. R'' is the reflected image of R' with respect to the y-axis. It is given that the coordinates of R' are (8, 4).
   (a) Write down the coordinates of R and R''.
   (b) Is $\Delta RR'R''$ an isosceles triangle? Explain your answer.

(4 marks)

6. In a polar coordinate system, *O* is the pole. The polar coordinates of the points *A* and *B* are (9, 136°) and (4, 316°) respectively.
   (a) Describe the geometric relationship among *A*, *O* and *B*.
   (b) If the polar coordinates of point *C* are (k, 226°) and the area of ΔABC is 65 square units, find the value of k.
   (c) *D* is another point on the same coordinate plane such that ΔABD has the same area as ΔABC. Write down one possible polar coordinates of *D*.

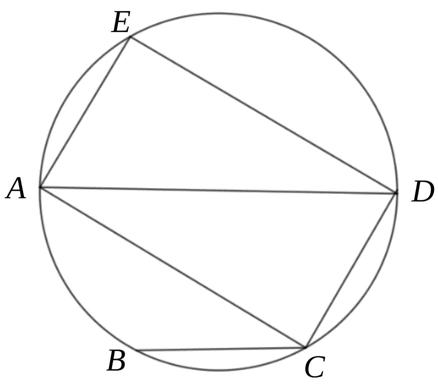
(4 marks)

7. There are 1 000 vitamin pills in a bottle. The label indicates that the net weight of the pills is 8 750g, correct to the nearest 20g.
   (a) Find the range of the actual weight of one pill.
   (b) The weekly intake of the vitamin pill must be less than 70g. How many pills should be taken at most in one week?

(4 marks)

8. In the figure, $ABCDE$ is a circle. $BC = CD$, $\angle CAD = 22.5°$ and AD is a diameter of the circle. Show that BE is the angle bisector of $\angle AED$.

(4 marks)



9. The frequency distribution table and the cumulative frequency table below show the distribution of passengers' waiting time at a station, where a, b and c are integers.

Waiting time (min)	Frequency
$0 \le x < 5$	8
$5 \le x < 10$	a
$10 \le x < 15$	15
$15 \le x < 20$	b

Waiting time less than (min)	Cumulative frequency
5	8
10	18
15	33
20	c

(a) If a passenger is selected randomly, the probability of his waiting time less than 5 min is $\frac{2}{9}$.

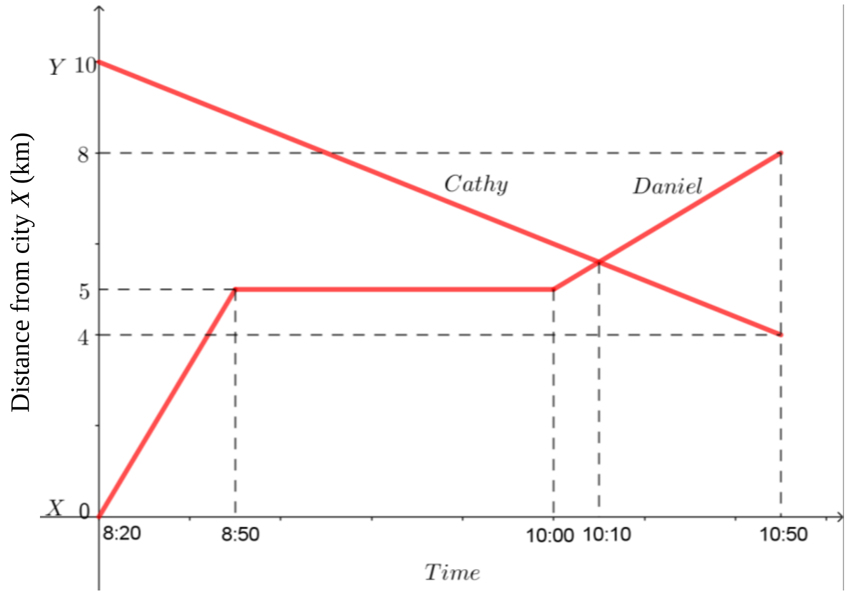
Find the values of a, b and c.

(b) Find the mean and the standard deviation of the above distribution.

(5 marks)

Section A(2) (35 marks)

10. The figure shows the graphs of Cathy and Daniel walking on the same straight road between city X and city Y during the period 8:20 to 10:50 in a morning. Cathy walks at a constant speed during that period. It is given that city X and city Y are 10 km apart.



(a) In which period of time (8:20-8:50, 8:50-10:00, or 10:00 -10:50) does Daniel walk the fastest? Explain your answer. (2 marks)

(b) How far from city X do Cathy and Daniel meet during the period? (3 marks)

(c) Daniel claims that his average walking speed is the same as that of Cathy's during the period 8:50 to 10:50 on that morning. Do you agree? Explain your answer. (2 marks)

11. It is given that $f(x)$ is the sum of two parts, one part varies directly as x and the other part varies inversely as x. Suppose that $f(3) = 7$ and $f(-1) = -5$.

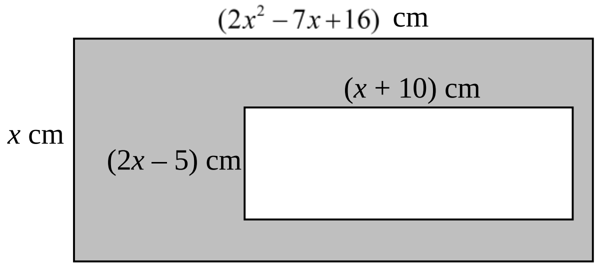
    (a) Find $f(x)$. (3 marks)

    (b) Solve $f(\log_2 x) = 7$. (4 marks)

12. Let $f(x) = ax^3 - 9x^2 + x + b$, where a and b are constants. When $f(x)$ is divided by x and $x + 1$, the remainders are 12 and 0 respectively.

(a) Find the values of a and b. (3 marks)

(b) The figure shows two rectangles. The length and width of the larger rectangle are $(2x^2 - 7x + 16)$ cm and x cm respectively, while the length and width of the smaller rectangle are $(x + 10)$ cm and $(2x - 5)$ cm respectively. Ada claims that there are more than one value of x such that the area of the shaded region is 38 cm^2. Do you agree? Explain your answer. (4 marks)



13. A closed inverted right circular conical vessel contains some water. The base radius and the height of the cone are 18 cm and 24 cm respectively. When the vessel is held vertically, the depth of water in the vessel is 12 cm.

(a) (i) Find the radius of the water surface.

(ii) Find the area of the wet curved surface of the vessel in terms of π.

(3 marks)

(b) The vessel is turned upside down. Find the new depth of water.

(3 marks)

14. In a rectangular coordinate plane, the coordinates of points A, B, C and D are (8, 0), (−4, 6), (−10, 0) and (2, 3) respectively. P is a moving point in the rectangular coordinate plane such that AP is perpendicular to BP. Denote the locus of P by Γ.

(a) (i) Find the equation of Γ.

(ii) Describe the geometric relation between Γ and D.

(3 marks)

(b) The equation of the straight line L is $x + 2y - 23 = 0$. It is found that Γ and L touch each other at Q.

(i) Find the coordinates of Q.

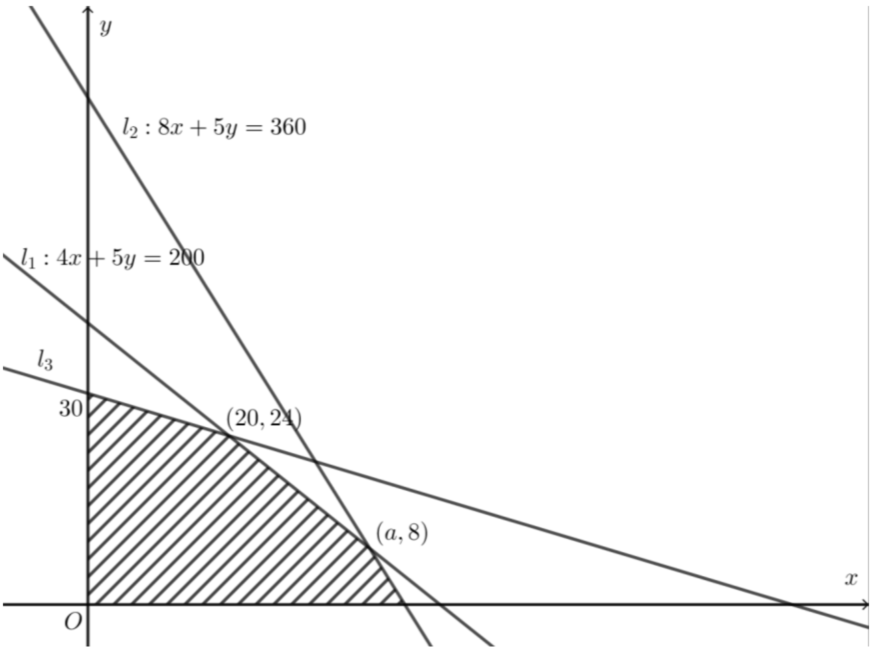
(ii) Jason claims that the ratio of the area of ΔACD to the area of ΔACQ is 1 : 9. Do you agree? Explain your answer.

(5 marks)

Section B (35 marks)

15. (a) In the figure, l_1: $4x + 5y = 200$ and l_2: $8x + 5y = 360$ intersect at $(a, 8)$. l_1 and l_3 intersect at $(20, 24)$. The y-intercept of l_3 is 30. The shaded region (including the boundaries) represents the solution of a system of inequalities. Find the system of inequalities.

(3 marks)



(b) Belle's Bakery produces mousse and cookies from three ingredients: butter, sugar and flour. The ratio of profits when selling a dozen of mousse and a dozen of cookies is 2 : 3. To produce a dozen of each dessert, the amount of the ingredients (in cups) are shown below:

Ingredients (cups) / Desserts (dozen)	Butter	Sugar	Flour
Mousse	4	8	3
cookie	5	5	10

The bakery has only 200 cups of butter, 360 cups of sugar and 300 cups of flour in their supply room. If x dozens of mousse and y dozens of cookies are produced and sold, find the values of x and y when the total profit of selling them attains its maximum.

(4 marks)

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16. A circle with radius 3 is tangential to the x-axis at A. $C(4, a)$ is the centre of the circle, where $a > 0$. Straight line L passes through the origin O and intersects with the circle at B and D, where D is on the right of B.

   (a) Find the equation of the circle. (2 marks)

   (b) If the slope of L is k, find the coordinates of the mid-point of BD, and express the answer in terms of k. (4 marks)

   (c) Let $\angle COB = \alpha$, $\angle CDO = \beta$ and $k > \frac{3}{4}$. Show that $\sin\alpha = \frac{3}{5}\sin\beta$. (2 marks)

17. There are 10 questions in a question bank.
    - James knows the answers of 4 questions.
    - Ellen knows the answers of those 4 questions that James can do. She also knows the answers of 2 more questions.

    Now 3 questions are randomly selected from the question bank to form a test. To pass the test, one must answer at least 2 out of 3 questions correctly.

    (a) Find the probability that James passes the test. (2 marks)

    (b) It is given that James fails the test. Find the probability that Ellen passes the test. (2 marks)

18. Every year during the spawning season from January to March, the fishermen in a certain area are not permitted to fish and at the end of the season the number of salmon increases by 20% with respect to the number at the beginning of the season. After the spawning season, the fishermen are permitted to fish and the maximum number of salmon they can catch is X thousand per year. This year at the beginning of the spawning season, there are approximately 100 thousand salmon (K_0) in that area.

    (a) Find the minimum number of salmon in thousand at the end of the first year in terms of X.

    (1 mark)

    (b) Find the maximum value of X (correct to 3 decimal places) such that the amount of salmon will be at least doubled after 50 years.

    (5 marks)

19. In Figure I, $ABCD$ is a rectangle. $AB = 8$ cm and $BC = 6$ cm. ΔABD is folded along BD as shown in Figure II. Denote the new position of A as A_1. It is given that the projection of A_1 on plane BCD is E which lies on CD.

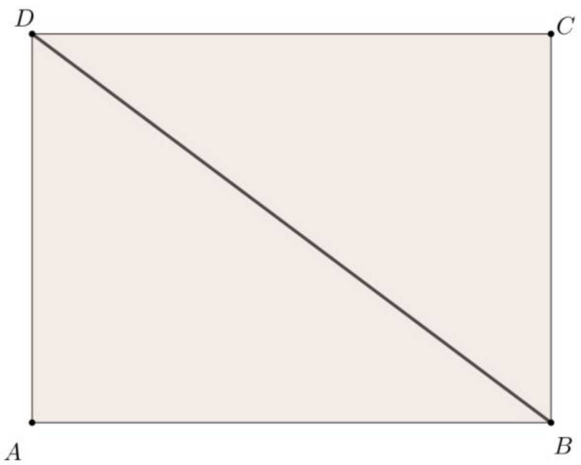

Figure I

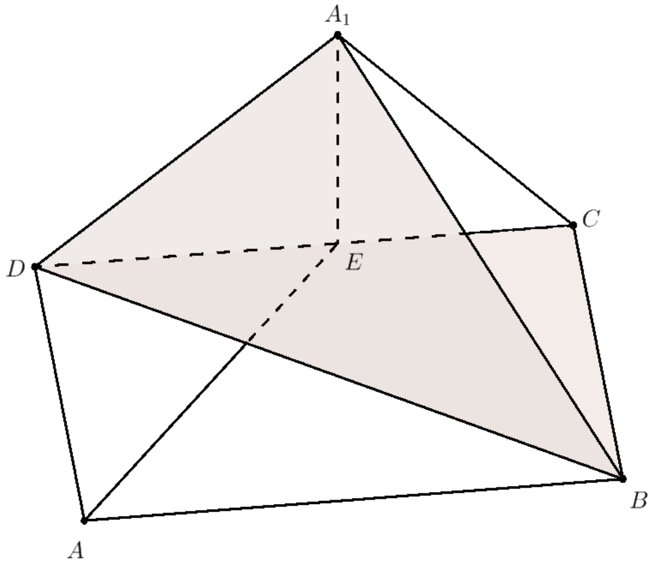

Figure II

(a) Describe the geometric relation between AE and BD. (1 mark)

(b) Find the angle between planes CBD and A_1BD. (5 marks)

(c) (i) Find the area of triangle A_1CB.

(ii) Find the distance between D and plane A_1CB.

(4 marks)

END OF PAPER

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Suggested Solutions

1.	$\dfrac{(2^{-1}a^{-3}b)^3}{ab^2}$ $=\dfrac{2^{-3}a^{-9}b^3}{ab^2}$ $=\dfrac{b}{8a^{10}}$	1M $(ab)^m=a^mb^m$ or $(a^m)^n=a^{mn}$ 1M $\dfrac{c^p}{c^q}=c^{p-q}$ 1A
2.(a)	$18m^2-60mn+50n^2$ $=2(9m^2-30mn+25n^2)$ $=2(3m-5n)^2$	1A
(b)	$18m^2-60mn+50n^2+10n-6m$ $=2(3m-5n)^2-2(3m-5n)$ $=2(3m-5n)(3m-5n-1)$	1M: (a) $-2(3m-5n)$ 1A
3.(a)	$\dfrac{3x-10}{8}<x$ $3x-10<8x$ $-5x<10$ $x>-2$ or $5(8-x)\geq 4(13-2x)$ $40-5x\geq 52-8x$ $3x\geq 12$ $x\geq 4$ $\therefore$ Combining the 2 inequalities, $x>-2$.	1A 1A 1A
(b)	-1	1A
4.(a)	The amount $=(320+650)\times 0.8$ $=\$776$	1A 1A
(b)	Let the marked price $\$y$. $(y+320+650)\times 0.8\times 0.8=776$ $y=242.5$ $\therefore$ The marked price of the extra item is $242.5.	1M L.H.S. = (a) 1A

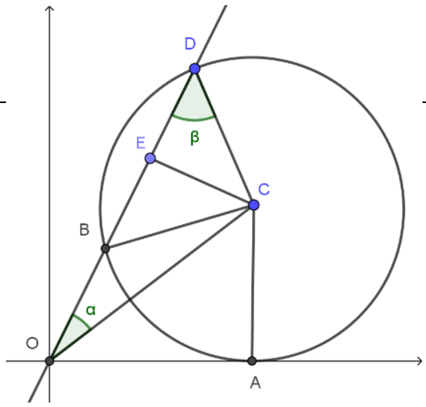
5.(a)	R (4, −8), R'' (−8, 4)	1A + 1A
(b)	$RR'' = \sqrt{(8-4)^2+(4+8)^2} = \sqrt{160}$ $RR'' = \sqrt{(4+8)^2+(-8-4)^2} = \sqrt{288}$ $R'R''$ = 8 + 8 = 16 $RR' \neq RR'' \neq R'R''$ $\therefore$ $\Delta RR'R''$ is not an isosceles triangle.	1M 1A f.t.
6.(a)	$\angle AOB = 316° - 136° = 180°$ A, O and B are collinear.	 1A
(b)	$\angle AOC = 226° - 136° = 90°$ Area $= \dfrac{(9+4)\times k}{2} = 65$ $\dfrac{(9+4)\times k}{2} = 65$ $k = 10$	 1A 1A
(c)	(10, 46°) or other possible answer	1A
7.(a)	$8750 - 10 \leq$ actual total weight $<$ $8750 + 10$ $8740 \leq$ actual total weight $<$ 8760 $8.74g \leq$ actual weight of one pill $<$ $8.76g$	1A : lower / upper limit 1A
(b)	maximum number of pills per week = $\dfrac{70}{8.76}$ = 7.99 (*corr. to 3 sig. fig.*) Therefore at most 7 pills should be taken.	1M: $\dfrac{70}{\text{upper limit}}$ 1A
8.	$\angle CED = \angle CAD = 22.5°$ ($\angle s$ in the same segment) $\angle BEC = \angle CED = 22.5°$ (eq. arcs, eq. $\angle s$) $\angle BED = 45°$ $\angle AED = 90°$ ($\angle$ in semi-circle) $\angle AEB = 45°$ $\because$ $\angle BED = \angle AEB = 45°$ $\therefore$ BE is the angle bisector of $\angle AED$.	1A 1A 1A 1A
9.(a)	a = 18 – 8 = 10 $\dfrac{8}{c} = \dfrac{2}{9}$ c = 36 b = 36 – 33 = 3 $\therefore$ a = 10, b = 3, c = 36	1A 1A 1A
(b)	Mean = 9.31 min Standard deviation = 4.59 min	1A 1A

10.(a)	Since the slope of the line segment during 8:20 – 8:50 is the greatest, Daniel walks at the fastest speed during 8:20 – 8:50.	1M 1A
(b)	Let the required distance be x km. $\frac{x-4}{40}=\frac{10-4}{150}$ $x = 5.6$ $\therefore$ The required distance 5.6 km.	1M (ratio 40:150) + 1A 1A
(c)	Average speed of Daniel = $\frac{3}{2}$ = 1.5 km/h Average speed of Cathy = $\frac{6}{2.5}$ = 2.4 km/h $\neq$ 1.5 km/h $\therefore$ The claim is disagreed.	1M (either) 1A
	Alternative solution: During the period, distance walked by Daniel = 3 km distance walked by Cathy is more than 3 km $\therefore$ The claim is disagreed.	Alternative: 1M 1A
11.(a)	Let $f(x)=k_1x+\frac{k_2}{x}$, where k_1, $k_2\neq 0$. $f(3)=7$ $3k_1+\frac{k_2}{3}=7 \quad \rightarrow \quad 9k_1+k_2=21$ ------- (1) $f(-1)=-5$ $-k_1-k_2=-5$ ------- (2) Solving (1) and (2), $k_1=2$, $k_2=3$ $\therefore f(x)=2x+\frac{3}{x}$.	1A 1M (either) 1A
(b)	$f(\log_2 x)=7$ $2\log_2 x+\frac{3}{\log_2 x}=7$ $2\log_2{}^2 x+3=7\log_2 x$ $2\log_2{}^2 x-7\log_2 x+3=0$ $\log_2 x=3$ or $\log_2 x=\frac{1}{2}$ $x=8$ or $x=\sqrt{2}$.	 1M (sub) 1M (quad. eqn.) 2A

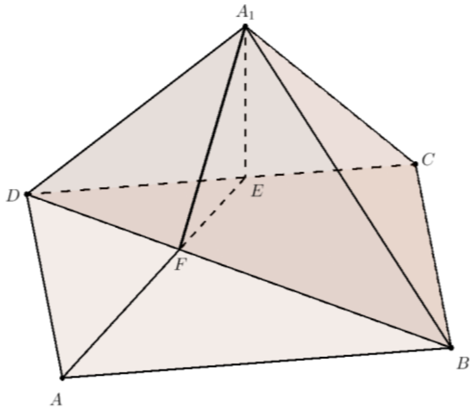
12.(a)	$f(0)=12 \quad \rightarrow \quad b=12$ $f(-1)=0$ $a(-1)^3-9(-1)^2+(-1)+12=0$ $a=2$	1A 1M 1A
(b)	$x(2x^2-7x+16)-(x+10)(2x-5)=38$ $2x^3-7x^2+16x-(2x^2+15x-50)-38=0$ $2x^3-9x^2+x+12=0$ $\because$ $x+1$ is a factor of $f(x)$ $\therefore \quad (x+1)(2x^2-11x+12)=0$ $(x+1)(x-4)(2x-3)=0$ $x=-1$ (*rej.*) or 4 or $\frac{3}{2}$ (*rej.*) $\therefore$ There is only one possible value of x. $\therefore$ The claim is disagreed.	1A 1M 1A 1A (f.t.)
13. (a)(i)	Let r cm be the radius of the water surface. $\frac{r}{18}=\frac{12}{24}$ $r=9$ $\therefore$ The radius of the water surface is 9 cm.	 1A
(a)(ii)	The area of the wet curved surface $=\pi(9)\sqrt{9^2+12^2}$ $=135\pi \text{ cm}^2$	 1M 1A
(b)	Volume of water $=\frac{1}{3}\pi(9)^2(12)=324\pi \text{ cm}^3$ Capacity of the vessel $=\frac{1}{3}\pi(18)^2(24)=2592\pi \text{ cm}^3$ Volume of the empty space before turning upside down $=2592\pi-324\pi=2268\pi \text{ cm}^3$ Let h cm be the height of the empty space after turning upside down. $\left(\frac{h}{24}\right)^3=\frac{2268\pi}{2592\pi}$ $\left(\frac{h}{24}\right)^3=\frac{7}{8}$ $h\approx 22.95517419$ $\therefore$ The new depth of water $=24-22.95517419$ $=1.04$ cm (*corr. to 3 sig. fig.*)	1M (either) 1M 1A

	Alternative solution: $\frac{\text{Volume of water in the vessel}}{\text{Capacity of the vessel}} = \left(\frac{9}{18}\right)^3 = \frac{1}{8}$ $\frac{\text{Volume of the empty space}}{\text{Capacity of the vessel}} = \frac{8-1}{8} = \frac{7}{8}$ Let h cm be the height of the empty space after turning upside down. $\left(\frac{h}{24}\right)^3 = \frac{7}{8}$ $h \approx 22.95517419$ $\therefore$ The new depth of water $= 24 - 22.95517419$ $= 1.04$ cm (*cor. to 3 sig. fig.*)	1M 1M 1A
14. (a)(i)	$\because \quad AP \perp BP$ $\therefore \quad \frac{y-0}{x-8} \times \frac{y-6}{x+4} = -1$ $y^2 - 6y = -(x^2 - 4x - 32)$ $x^2 + y^2 - 4x - 6y - 32 = 0$ $\therefore$ The equation of Γ is $x^2 + y^2 - 4x - 6y - 32 = 0$ excluding points $A(8, 0)$ and $B(-4, 6)$.	1M 1A
(a)(ii)	Centre of $\Gamma = \left(-\frac{(-4)}{2}, -\frac{(-6)}{2}\right) = (2, 3)$ Γ is a circle with centre D. or D is inside the circle Γ.	1A
(b)(i)	$\begin{cases} x^2 + y^2 - 4x - 6y - 32 = 0 \text{............(1)} \\ x + 2y - 23 = 0 \text{...............(2)} \end{cases}$ From (2), $x = 23 - 2y$ ....(3) Sub (3) into (1) $(23-2y)^2 + y^2 - 4(23-2y) - 6y - 32 = 0$ $529 - 92y + 4y^2 + y^2 - 92 + 8y - 6y - 32 = 0$ $5y^2 - 90y + 405 = 0$ $y = 9$ (repeated) $x = 23 - 2(9) = 5$ $\therefore Q(5, 9)$	1M 1M 1A
(b)(ii)	ΔACD and ΔACQ have the same base AC. Height of $\Delta ACD = 3$ Height of $\Delta ACQ = 9$ The required ratio = 3 : 9 = 1 : 3 $\neq$ 1 : 9 $\therefore$ The claim is disagreed.	1M 1A (f.t.)

15. (a)	Slope of l_3 = $\frac{24-30}{20-0}=-\frac{3}{10}$ Equation of l_3: $y=-\frac{3}{10}x+30$ $3x+10y-300=0$ Thus, the system of inequalities is $\begin{cases}x\geq 0\\ y\geq 0\\ 4x+5y\leq 200\\ 8x+5y\leq 360\\ 3x+10y\leq 300\end{cases}$.	1M (equation of l_3) 1M + 1A
(b)	Let the profits of selling a dozen of mousse and a dozen of cookies be \$2*k* and \$3*k* respectively. Denote the total profit by \$*P*. Then we have $P = 2kx + 3ky$, where $k > 0$. Now the constraints are $\begin{cases}x\geq 0\\ y\geq 0\\ 4x+5y\leq 200\\ 8x+5y\leq 360\\ 3x+10y\leq 300\end{cases}$, so the feasible region is the shaded region in part (a). Sub $(a, 8)$ into $4x+5y=200$ $a = 40$ l_2 intersects the x-axis at (45, 0). At (0, 0), $P = 0$ At (0, 30), $P = 90k$ At (20, 24), $P = 112k$ At (40, 8), $P = 104k$ At (45, 0), $P = 90k$ So the total profit attains its maximum when $\underline{\underline{x = 20, y = 24}}$.	1A (P) 1M (find all vertices or draw a straight line with negative slope on a graph paper) 1M (for testing one point or for sliding the straight line on a graph paper) 1A f.t.
16. (a)	$a = 3$ The equation of the circle is $(x-4)^2+(y-3)^2=9$.	1A 1A
(b)	Let the equation of L be $y = kx$. $\begin{cases}(x-4)^2+(y-3)^2=9\\ y=kx\end{cases}$	

	Sub $y = kx$ into $(x-4)^2+(y-3)^2=9$ $(x-4)^2+(kx-3)^2=9$ $x^2-8x+16+k^2x^2-6kx+9=9$ $(1+k^2)x^2-(8+6k)x+16=0 \quad (*)$ Let (x_1, y_1) and (x_2, y_2) be the coordinates of B and D. x_1 , x_2 are the roots of equation (*). $x_1+x_2=\dfrac{8+6k}{1+k^2}$ The x-coordinate of mid-point of BD = $\dfrac{x_1+x_2}{2}=\dfrac{4+3k}{1+k^2}$ Sub $\dfrac{x_1+x_2}{2}=\dfrac{4+3k}{1+k^2}$ into $y = kx$, the y-coordinate of mid-point of BD = $\dfrac{(4+3k)k}{1+k^2}$ $\underline{\underline{\left(\dfrac{4+3k}{1+k^2}, \dfrac{(4+3k)k}{1+k^2}\right)}}$	1M (sub) 1M 1M (sub x into $y = kx$) 1A
(c)	$\dfrac{\sin\alpha}{DC}=\dfrac{\sin\beta}{OC}$ $DC = 3$, $OC = 5$ $\dfrac{\sin\alpha}{3}=\dfrac{\sin\beta}{5}$ $\sin\alpha=\dfrac{3}{5}\sin\beta$  	1M (sine formula) 1A f.t.
	Alternative Method $DC = 3$, $OC = 5$ $CE=\sqrt{\left(\dfrac{4+3k}{1+k^2}-4\right)^2+\left(\dfrac{(4+3k)k}{1+k^2}-3\right)^2}$ $=\sqrt{\left(\dfrac{3k-4k^2}{1+k^2}\right)^2+\left(\dfrac{4k-3}{1+k^2}\right)^2}$ $=\sqrt{\left(\dfrac{3-4k}{1+k^2}\right)^2k^2+\left(\dfrac{4k-3}{1+k^2}\right)^2}$ $=\sqrt{\dfrac{(4k-3)^2}{1+k^2}}$ $=\dfrac{4k-3}{\sqrt{1+k^2}}$ $\sin\alpha=\dfrac{4k-3}{\sqrt{1+k^2}}\div 5$, $\sin\beta=\dfrac{4k-3}{\sqrt{1+k^2}}\div 3$ $\sin\alpha=\dfrac{3}{5}\sin\beta$	1M 1A f.t.

17. (a)	$P(\text{James passes the test}) = \frac{C_3^4 + C_2^4 C_1^6}{C_3^{10}} = \frac{4+6\times 6}{120} = \underline{\underline{\frac{1}{3}}}$	1M + 1A
(b)	$P(\text{Ellen passes} \mid \text{James fails}) = \frac{P(\text{Ellen passes and James fails})}{P(\text{James fails})}$ $= \frac{\frac{C_2^2 C_1^4 + C_1^4 C_1^2 C_1^4 + C_1^4 C_2^2}{C_3^{10}}}{1-\frac{1}{3}}$ $= \underline{\underline{\frac{1}{2}}}$	1M 1A
	Alternative Method $P(\text{Ellen passes} \mid \text{James fails}) = \frac{C_2^2 C_1^4 + C_1^4 C_1^2 C_1^4 + C_1^4 C_2^2}{C_3^{10} - C_3^4 - C_2^4 C_1^6}$ $= \underline{\underline{\frac{1}{2}}}$	1M 1A
18. (a)	Let the number of salmon after *n* years before the spawning season be K_n. $K_1 = K_0(1+20\%) - X = 120 - X$ The minimum number of salmon at the end of the first year is $\underline{\underline{(120 - X)\text{ thousand}}}$.	 1A
(b)	$K_2 = K_1(1+20\%) - X = K_0(1.2)^2 - 1.2X - X$ $K_3 = K_2(1+20\%) - X = K_0(1.2)^3 - 1.2X^2 - 1.2X - X$ $K_n = K_{n-1}(1+20\%) - X = K_0(1.2)^n - 1.2^{n-1}X - 1.2^{n-2}X - \ldots - X$ $= K_0(1.2)^n - X(1.2^{n-1} + 1.2^{n-2} + \ldots + 1)$ $= K_0(1.2)^n - X\frac{1-1.2^n}{1-1.2}$ $K_{50} = K_0(1.2)^{50} - X\frac{1-1.2^{50}}{1-1.2} \geq 2K_0$ $100\left((1.2)^{50} - 2\right) \geq X\frac{1-1.2^{50}}{1-1.2}$ $X \leq 19.997$ The maximum value of X is $\underline{\underline{19.997\ (\textit{corr. to 3 decimal places})}}$.	 1A 1M (sum of GS) 1A (Inequality) 1M (solving inequality) 1A

<table>
<tr><td>19. (a)</td><td><u>AE is perpendicular to BD.</u></td><td>1A</td></tr>
<tr><td>(b)</td><td>Denote the intersection point of AE and BD by F. Join A_1F.
The required angle is $\angle A_1FE$

$A_1D = AD = 6$
$BD = \sqrt{6^2+8^2} = 10$
$\Delta ADE \sim \Delta BAD$
$\frac{DE}{AD} = \frac{AD}{BA} = \frac{AE}{BD}$
$\frac{DE}{6} = \frac{6}{8} = \frac{AE}{10}$
$DE = 4.5$
$AE = 7.5$
$AF = 4.8$
$FE = 2.7$
$A_1E = \sqrt{6^2 - 4.5^2} = \sqrt{15.75}$
$A_1A = \sqrt{7.5^2 + \sqrt{15.75}^2} = \sqrt{72}$
$A_1F = \sqrt{2.7^2 + 15.75} = 4.8$
$\cos\angle A_1FE = \frac{EF}{A_1F} = \frac{2.7}{4.8} = 0.5625$
$\angle A_1FE = 55.8°$
The required angle is <u>55.8°</u>. (*corr. to 3 sig. fig.*)</td><td>1A (claim the angle)

2A (any two line segments on plane *ABCD*)

1M (A_1E)

1A</td></tr>
<tr><td>(c) (i)</td><td>$EC = 8 - 4.5 = 3.5$
$A_1C = \sqrt{3.5^2 + \sqrt{15.75}^2} = \sqrt{28}$
$BE = \sqrt{3.5^2 + 6^2} = \sqrt{48.25}$
$A_1B = \sqrt{\sqrt{48.25}^2 + \sqrt{15.75}^2} = 8$
$\cos\angle A_1CB = \frac{6^2 + \sqrt{28}^2 - 8^2}{2\cdot\sqrt{28}\cdot 6} = 0$ $\angle A_1CB = 90°$ (optional)
The area of $\Delta A_1CB = \frac{6\times\sqrt{28}}{2} = \underline{\underline{6\sqrt{7}}}$ cm^2.</td><td>1M ($\frac{6\times\sqrt{28}}{2}$ or Heron's formula)

1A</td></tr>
<tr><td></td><td>Alternative Method
$A_1E \perp$ Plane $ABCD$, then $A_1E \perp$ every line in $ABCD$.
$\therefore A_1E \perp BC$
Plus $CD \perp BC$
$\therefore BC \perp$ Plane A_1CD, then $BC \perp$ every line in A_1CD
$\therefore BC \perp A_1C$</td><td>1M ($\frac{6\times\sqrt{28}}{2}$ or Heron's formula)</td></tr>
</table>

	The area of $\mathrm{V}A_1CB = \frac{6\times\sqrt{28}}{2} = \underline{\underline{6\sqrt{7}}}$ cm^2.	1A
(c) (ii)	Let the required distance be d cm. $BC \perp A_1C$ (by c(i)) Volume of the triangular pyramid $= \frac{1}{3}\cdot\frac{6\times8}{2}\cdot\sqrt{15.75} = \frac{1}{3}\cdot\frac{6\times\sqrt{28}}{2}\cdot d$ $8\cdot\sqrt{15.75} = \sqrt{28}\cdot d$ $d = 6$ The required distance is $\underline{\underline{6\text{ cm}}}$.	 1M 1A