Diocesan Girls' School
Secondary 6 Mock Examinations (2019-2020)
Mathematics (Compulsory Part)
Paper 1

Feb 2020

Time Allowed: 2 hours 15 minutes Total marks: 105

Name:_____( ) Class:_____ Set:_____

Instructions:

1. This paper consists of THREE sections, A(1), A(2) and B.
2. Attempt ALL questions. Write your answers in the spaces provided in this Question-Answer Book.
3. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class and class number on each sheet, and staple them INSIDE this book.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale.

Question No.	Marks
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	

Question No.	Marks
10.	
11.	
12.	
13.	
14.	

Question No.	Marks
15.	
16.	
17.	
18.	
19.	

Total Marks:

Section A(1) (35 marks)

1. Simplify $\frac{8x^6y^7}{(4x^{-2}y^3)^3}$ and express your answer with positive indices. (3 marks)

2. Factorize

   (a) $8a^2 + 24ab + 18b^2$,

   (b) $8a^2 + 24ab + 18b^2 - 32a - 48b$.

   (4 marks)

3. If $a = \frac{1}{3}(p+2)$ and $b = 3(2p+5)$, express b in terms of a. (3 marks)

4. (a) Round up 643.742 to 2 significant figures.

   (b) Round down 648.752 to 1 decimal place.

   (c) Round off 648.742 to the nearest integer.

   (3 marks)

5. The marked price of a bag is 25% above its cost. A profit of $75 is made by selling the bag at a discount of 10% on its marked price. Find the marked price of the bag. (4 marks)

6. (a) Find the range of values of x which satisfy both $\frac{2x+5}{3} \geq 3-x$ and $37 > 5+4x$.

(b) Write down the smallest integer satisfying the inequalities in (a).

(5 marks)

7. The frequency distribution table and the cumulative frequency distribution table below show the distribution of the weights of newborn babies in a hospital.

Weight (kg)	Frequency
2.1 - 2.4	7
2.5 - 2.8	a
2.9 - 3.2	20
3.3 - 3.6	b
3.7 - 4.0	17
4.1 - 4.4	c

Weight less than (kg)	Cumulative frequency
2.45	x
2.85	24
3.25	y
3.65	78
4.05	z
4.45	100

(a) Find a, b and c.

(b) If a baby is randomly selected from the hospital, find the probability that the weight of the selected baby is less than 4.05 kg but not less than 3.25 kg.

(5 marks)

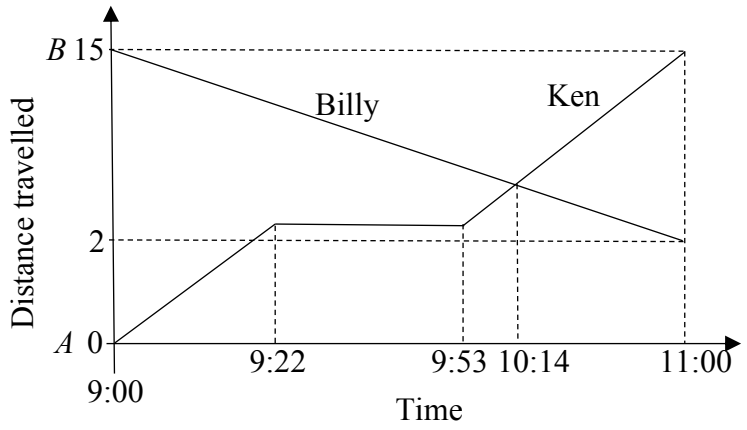
8. In a polar coordinate system, O is the pole. The polar coordinates of the points A and B are $(20, 70°)$ and $(20, 160°)$ respectively. P is point on AB such that OP is the axis of reflectional symmetry of ΔOAB.

(a) Describe the geometric relationship between OP and AB.

(b) Find the polar coordinates of P. (Give the answer in surd form if necessary.)

(4 marks)

9. Town A and town B are 15 km apart. The figure shows the graphs for Ken and Billy running on the same straight road between town A and town B during the period 9:00 to 11:00 in the morning. They start at 9 a.m. and Billy runs at a constant speed. Ken comes to rest at 9:22.



(a) How far are Ken and Billy from town B when they meet?

(b) Billy claims that he runs faster than Ken on average. Do you agree? Explain your answer.

(4 marks)

Section A(2) (35 marks)

10. The stem-and-leaf diagram below shows the time spent on reading (in hours) in a week of the students in a reading club.

Stem (tens)	Leaf (units)								
1	*a*	2	3	3					
2	*c*	2	2	4	6	6	7	7	9
3	0	0	*b*						

It is given that the inter-quartile range of the distribution is 11 hours.

(a) Find the value of c. (2 marks)

(b) It is given that the mean of the distribution is 23 hours and the range of the distribution is not less than 25 hours. Find a and b. (3 marks)

11. It is given that h partly varies directly as a and partly varies directly as a^2. $h = 100$ when $a = 1$ and $h = 280$ when $a = 2$.

(a) Express h in terms of a. (3 marks)

(b) If $a : h = 1 : 300$, find the values of a and h. (2 marks)

(c) Using the method of completing the square, find the minimum value of h and the corresponding value of a. (3 marks)

12. Let $h(x) = 4x^2 + 7x + 3$ and $g(x) = 8x^2 + 26x + 15$. If $12g(x) - 11xh(x)$ is divisible by $x - 3$.

(a) Find the H.C.F. and L.C.M. of $h(x)$ and $g(x)$. (2 marks)

(b) Simplify $\dfrac{14}{g(x)} - \dfrac{1}{h(x)}$. (2 marks)

(c) If $4g(x) - h(x) - 3a$ is divisible by $x - 3$, find the value of a. (2 marks)

13. Figure 1a shows a metal bucket A in the shape of a right cylinder. The curved surface of the bucket is formed by the iron sheet $PQRS$ shown in figure 1b, where $PQ = 10$ cm and area $= 80\pi$ cm^2.

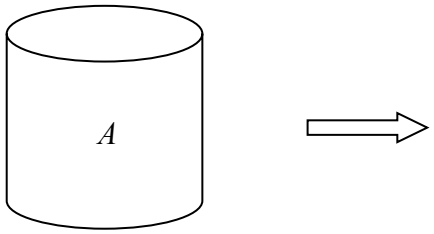


Figure 1a

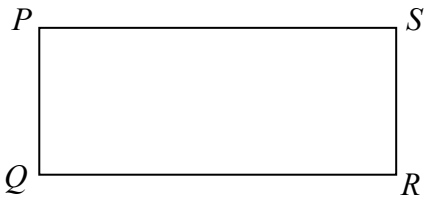
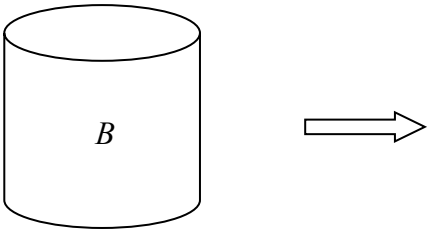


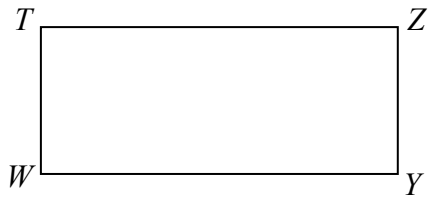
Figure 1b

(a) Find the base radius of bucket A. (2 marks)

(b) Find the capacity of bucket A in terms of π. (2 marks)

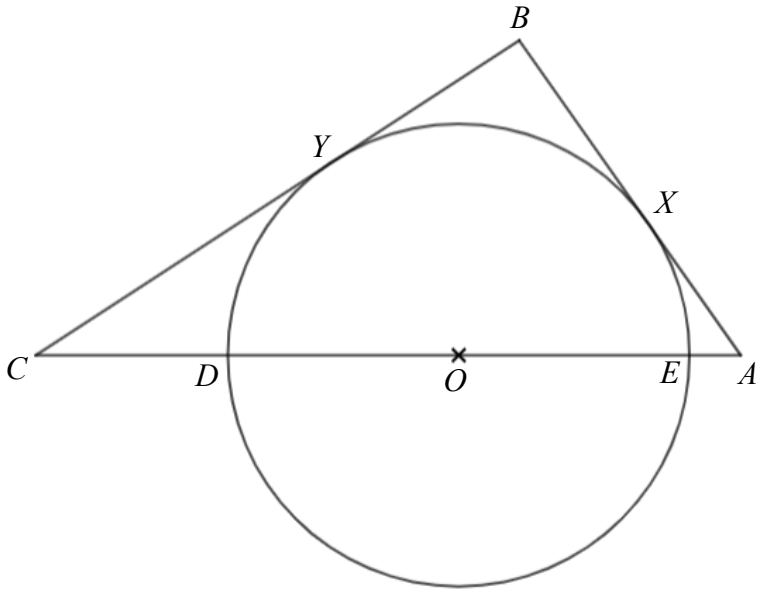
(c) Another metal bucket B in the shape of a right cylinder is shown in the figure below. Its curved surface is formed by the iron sheet $TWYZ$, where $TW = 12$ cm and $TZ = 8\pi$ cm. Are buckets A and B similar? Explain your answer.





(2 marks)

14. In the figure, O is the centre of the circle and $AEODC$ is a straight line. AB and BC are tangents to the circle at X and Y respectively and $AB \perp BC$. $AB = 12$ and $BC = 16$.



(a) Find the radius of the circle, (3 marks)

(b) (i) Without finding the angles, prove that $\angle YOD = 2\angle CYD$.
(ii) Hence, find $\angle CYD$. (4 marks)

(c) V is a point between X and B such that $YD//VO$. Find VX. (3 marks)

Section B (35 marks)

15. In a box there are 3 green balls, 6 red balls and 7 blue balls. 4 balls are drawn randomly from the box at the same time.

(a) Find the probability that exactly 2 green balls, 1 blue ball and 1 red ball are drawn.

(2 marks)

(b) Find the probability that at least one ball of each colour is drawn.

(2 marks)

16. The 3rd term and the 5th term of a geometric sequence are 900 and 400 respectively.

(a) Find the first term of the sequence. (2 marks)

(b) If the common ratio is positive, find the least value of n such that the difference between the $(n+1)^{\text{th}}$ term and the $(2n+1)^{\text{th}}$ term is less than 2×10^{-5}.

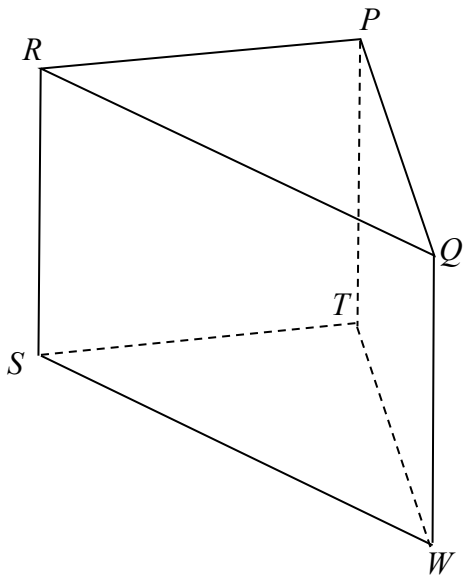
(4 marks)

17. It is given that AB is a line segment of length 5 and P is a moving point in the rectangular coordinate plane such that the area of ΔABP is 20. Denote the locus of P by Γ. Suppose that A and B are the points (0, 3) and (4, 6).

(a) Find the equation of Γ. (3 marks)

(b) Find the coordinates of P when ΔABP is an isosceles triangle. (3 marks)

18. The figure shows a triangular prism in which the bases are equilateral triangles PQR and TWS. $PRST$, $PQWT$ and $RQWS$ are squares. It is given that $PR = 2$ cm and M is the mid-point of TW.



(a) Find $\angle MPS$. (2 marks)

(b) Find the angle between the plane MPS and the plane $PTSR$. (4 marks)

(c) If D is a moving point on ΔPQR, is the area of ΔDSW minimum when D is at P? Explain your answer. (2 marks)

19. It is given that $A(2, 6)$ is the centre of a circle C with radius r.

(a) Write down the equation of the circle C in terms of r. (1 mark)

A circle C' is obtained by reflecting the circle C with respect to the y-axis and then translated vertically by c units. $P(a, b)$ and $Q(d, e)$ are the points of intersection of C and C'. The slope of PQ is $-\frac{1}{2}$.

(b) (i) Find the value of c and determine whether the reflected circle should be translated upward or downward.

(ii) Find the equation of PQ.

(iii) Hence find, in terms of r, the value of $(a-d)^2$.

(8 marks)

(c) A student claims that when $PQ = 4\sqrt{5}$, $B(-1, 1)$ lies inside C. Do you agree? Explain your answer. (2 marks)

End of Paper