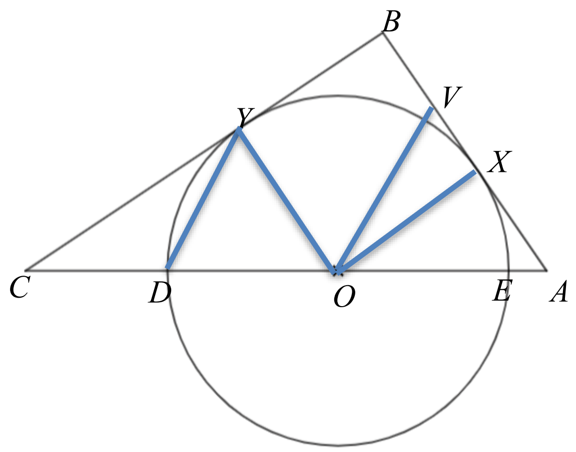
Diocesan Girls' School
Secondary 6 Mock Examinations (2019-2020)
Mathematics (Compulsory Part)
Paper 1 Solutions

1.	$\dfrac{8x^6y^7}{(4x^{-2}y^3)^3}$ $=\dfrac{8x^6y^7}{64x^{-6}y^9}$ $=\dfrac{x^{12}}{8y^2}$
2a.	$8a^2+24ab+18b^2=2\left(4a^2+12ab+9b^2\right)=2\left(2a+3b\right)^2$
2b.	$8a^2+24ab+18b^2-32a-48b$ $=2\left(2a+3b\right)^2-16\left(2a+3b\right)$ $=2\left(2a+3b\right)\left(2a+3b-8\right)$
3.	$a=\dfrac{1}{3}\left(p+2\right)$ $3a=p+2$ $p=3a-2$ $b=3\left(2p+5\right)$ $b=3\left[2\left(3a-2\right)+5\right]=3\left(6a+1\right)$
4a.	650
4b.	648.7
4c.	649
5.	Let $\$x$ be the marked price of the bag $(1-10\%)x-\dfrac{x}{1+25\%}=75$ $\dfrac{9}{10}x-\dfrac{4}{5}x=75$ $\dfrac{1}{10}x=75$ $x=750$ \ The marked price of the bag is $750.
6a.	$\dfrac{2x+5}{3}\geq 3-x$ $2x+5\geq 9-3x$ $5x\geq 4$ $x\geq\dfrac{4}{5}$ $37>5+4x$ $32>4x$ $x<8$ \ the required range is $\dfrac{4}{5}\leq x<8$.
6b.	1

7a.	$x = 7$ $a = 24 - 7 = 17$ $y = 24 + 20 = 44$ $b = 78 - 44 = 34$ $z = 78 + 17 = 95$ $c = 100 - 95 = 5$
7b.	Probability required $= \frac{34+17}{100}$ $= \frac{51}{100}$
8a.	Perpendicular bisector of *AB*.
8b.	Let $P(r, \theta)$ $r = 20\cos 45^\circ$ $= 10\sqrt{2}$ $\theta = 70^\circ + 45^\circ = 115^\circ$ $\therefore P = (10\sqrt{2}, 115^\circ)$
9a.	Let y km be the distance between town B and the place they meet $\frac{y}{74} = \frac{15-2}{120}$ $y = 8\frac{1}{60}$ They meet at a point $8\frac{1}{60}$ km from town B.
	Alternatively, Billy's average speed $= \frac{13}{2}$ = 6.5 km/h Distance required $= 6.5 \times \frac{74}{60}$ $= 8\frac{1}{60}$ km
9b.	Average speed of Billy $= \frac{15-2}{2} = 6.5$ km/h Average speed of Ken $= \frac{15}{2} = 7.5$ km/h > average speed of Billy $\therefore$ The claim is disagreed.

10a.	$\frac{27+29}{2}-\frac{20+c+13}{2}=11$ $c = 1$
10b	$\frac{10+a+322+30+b}{16}=23$ $a+b=6$ Then $a = 0$, $b = 6$ or $a = 1$, $b = 5$ or $a = 2$, $b = 4$. Also, $(30+b)-(10+a)\geq 25$ $b-a\geq 5$ $\therefore a = 0$, $b = 6$.
11a.	$h=k_1a+k_2a^2,\ k_1$ and $k_2\neq 0$ $100=k_1+k_2$..........(1) $280=2k_1+4k_2$........(2) Solving (1) and (2), $k_1=60$ and $k_2=40$ $h=60a+40a^2$
11b.	$\frac{a}{h}=\frac{1}{300}$ $h=300a$ $h=300a=60a+40a^2$ $a=6$ or 0 (rejected) $h=1800$
11c.	$h=60a+40a^2$ $=40\left(a^2+\frac{3}{2}a\right)$ $=40\left(a^2+\frac{3}{2}a+\frac{9}{16}\right)-40\left(\frac{9}{16}\right)$ $=40\left(a+\frac{3}{4}\right)^2-\frac{45}{2}$ Min. value of $h=-\frac{45}{2}$ when $a=-\frac{3}{4}$

12a.	$h(x)=4x^2+7x+3=(4x+3)(x+1)$ $g(x)=(4x+3)(2x+5)$ H.C.F. = $4x+3$ L.C.M. = $(4x+3)(2x+5)(x+1)$
12b.	$\frac{14}{g(x)}-\frac{1}{h(x)}$ $=\frac{14}{(4x+3)(2x+5)}-\frac{1}{(4x+3)(x+1)}$ $=\frac{14(x+1)-(2x+5)}{(4x+3)(x+1)(2x+5)}$ $=\frac{12x+9}{(4x+3)(x+1)(2x+5)}$ $=\frac{3(4x+3)}{(4x+3)(x+1)(2x+5)}$ $=\frac{3}{(x+1)(2x+5)}$
12c.	$4g(3)-h(3)-3a=0$ $4(165)-60-3a=0$ $a=200$
13a.	$PS=\frac{80\pi}{10}=8\pi$ Let r cm be base radius $2\pi r=8\pi$ $r=4$ Base radius of A = 4 cm
13b.	Vol. of A = $\pi\left(4^2\right)10=160\pi\ \text{cm}^3$
13c.	$\frac{\text{Curved surface area of A}}{\text{Curved surface area of B}}=\frac{80\pi}{12(8\pi)}=\frac{5}{6}$ $\frac{\text{Height of A}}{\text{Height of B}}=\frac{10}{12}=\frac{5}{6}\neq\sqrt{\frac{\text{Curved surface area of A}}{\text{Curved surface area of B}}}$ $\therefore$ They are not similar.
	Alternatively, $TZ=8\pi$ and $PS=8\pi$ *PQRS* and *TWYZ* are not similar. Therefore, *A* & *B* are not similar.

14a.	Let r be the radius of the circle $OY \perp CB$ and $OX \perp AB$ (tangent $\perp$ radius) $\therefore YOXB$ is a square $OX = OY = BX = BY = r$ $AX = 12 - r$ $\Delta AXO \sim \Delta ABC (AAA)$ $\frac{12-r}{12} = \frac{r}{16}$ $r = \frac{48}{7}$
14bi	ÐYOD = 2ÐYED (Ð at centre twice Ð at circumference) ÐCYD = ÐYED (Ð in alternate segment) \ ÐYOD = 2ÐCYD
14bii	$\tan \angle YOD = \frac{16-r}{r}$ $\angle YOD = 53.13°$ $\angle CYD = 26.6°$
14c	$\angle DYO = \angle YOV$ (alt.$\angle$s YD//VO) $\angle CYO = \angle XOY = 90°$ $\therefore \angle XOV = \angle CYD = 26.6°$ $\frac{VX}{\frac{48}{7}} = \tan 26.6°$ $VX = 3.43$ 

15a.	Probability $=\frac{C_2^3C_1^7C_1^6}{C_4^{16}}$ $=\frac{9}{130}$
15b.	Probability $=\frac{C_2^3C_1^7C_1^6}{C_4^{16}}+\frac{C_2^7C_1^3C_1^6}{C_4^{16}}+\frac{C_2^6C_1^3C_1^7}{C_4^{16}}$ $=\frac{9}{20}$
16a.	$r^2=\frac{400}{900}$ $r=\frac{2}{3}\text{ or }-\frac{2}{3}$ $a\left(\frac{2}{3}\right)^2=900$ $a=2025$ $\therefore$ First term = 2025
16b.	$2025\left(\frac{2}{3}\right)^n-2025\left(\frac{2}{3}\right)^{2n}<2\times10^{-5}$ $2025\left(\frac{2}{3}\right)^{2n}-2025\left(\frac{2}{3}\right)^n+2\times10^{-5}>0$ $\left(\frac{2}{3}\right)^n<9.8765\times10^{-9}$ $n\log\left(\frac{2}{3}\right)<\log 9.8765\times10^{-9}$ OR $\left(\frac{2}{3}\right)^n>1.000$ (*rej.*) $n>45.5$ The least value of *n* is 46.

17a.	Let θ be the inclination of *AB*. $\tan\theta = \frac{3}{4}$, so $\cos\theta = \frac{4}{5}$. Let c be the y-intercept of the straight line Γ. $\frac{4}{5} = \pm\frac{8}{c-3}$ $c = -7$ or 13 Equation of Γ is $\frac{y+7}{x} = \frac{3}{4}$ or $\frac{y-13}{x} = \frac{3}{4}$ i.e. $3x - 4y - 28 = 0$ or $3x - 4y + 52 = 0$.
17b.	Equation of the perpendicular bisector of *AB* is $\frac{y - \frac{9}{2}}{x-2} = \frac{-4}{3}$ or $8x + 6y - 43 = 0$. Solve $\begin{cases} 3x - 4y - 28 = 0 \\ 8x + 6y - 43 = 0 \end{cases}$ or $\begin{cases} 3x - 4y + 52 = 0 \\ 8x + 6y - 43 = 0 \end{cases}$ P is the point $\left(\frac{34}{5}, -\frac{19}{10}\right)$ or $\left(-\frac{14}{5}, \frac{109}{10}\right)$.
18a.	$PS = \sqrt{2^2 + 2^2} = \sqrt{8}$ (pyth theroem) $PM = \sqrt{2^2 + 1^2} = \sqrt{5}$ (pyth theroem) $PM \perp MS$ $\angle MPS = \cos^{-1}\frac{PM}{PS} = 37.76124391° = 37.8°$
18a.	*Alternatively,* $PS = \sqrt{2^2 + 2^2} = \sqrt{8}$ (pyth theroem) $PM = \sqrt{2^2 + 1^2} = \sqrt{5}$ (pyth theroem) $TW \perp MS$ $MS = \sqrt{ST^2 - TM^2} = \sqrt{2^2 - 1^2} = \sqrt{3}$ (pyth theroem) $\angle MPS = \cos^{-1}\frac{PS^2 + PM^2 - MS^2}{2(PS)(PM)} = 37.76124391° = 37.8°$

18b.	Draw $MA \perp PS$ and $AB \perp PS$ where B lies on TS $\angle$ required = $\angle MAB$ In ΔPMS $MA = MP \sin \angle MPS = 1.36931$ $PA = MP \cos \angle MPS = 1.767767$ $AS = PS - PA = \sqrt{8} - 1.767767 = 1.06066$ In ΔABS $AB = AS \tan \angle ASB = AS \tan 45° = AS = 1.06066$ $AB = AS = 1.06066$ $BS = \sqrt{AS^2 + AB^2} = 1.5$ In ΔSBM $\angle BSM = 30°$ $BM = \sqrt{BS^2 + MS^2 - 2BS(MS)\cos \angle BSM} = \sqrt{0.75}$ In ΔABM $\angle MAB = \cos^{-1} \dfrac{BA^2 + AM^2 - BM^2}{2BA(AM)} = 39.2314° = 39.2°$
18c.	Let E be a point on WS such that $DE \perp WS$ Area of $\Delta DSW = \frac{1}{2}(WS)DE$ $\because WS$ is a constant $\therefore$ area of ΔDSW is min. when DE is min. $\because DE$ is min. when D is on RQ $\therefore$ area of ΔDSW is not min. when D is at P.

19a.	$(x-2)^2+(y-6)^2=r^2$ or $x^2+y^2-4x-12y+40-r^2=0$
19bi.	Let G be the centre of the circle C'. Then G = (–2, 6 + c). Slope of AG = 2. $\frac{6+c-6}{-2-2}=2$ $c = -8$ So the reflected circle should be translated downward by 8 units.
19bii.	Mid-point of AG = (0, 2). Equation of PQ is $\frac{y-2}{x}=-\frac{1}{2}$ or $x+2y-4=0$.
19biii.	Put $y=-\frac{1}{2}x+2$ into the equation of C, we have $(x-2)^2+(-\frac{1}{2}x-4)^2=r^2$ $\frac{5}{4}x^2+20-r^2=0$ Since a and d are the roots of the equation, $a+d=0$ and $ad=\frac{4(20-r^2)}{5}$ $(a-d)^2=(a+d)^2-4ad$ $=-\frac{16(20-r^2)}{5}$ $=\frac{16(r^2-20)}{5}$
19c.	$PQ^2=(a-d)^2+(b-e)^2$ $=(a-d)^2+\frac{1}{4}(a-d)^2$ $=4(r^2-20)$ When $PQ=4\sqrt{5}$, $4(r^2-20)=80$ $r=2\sqrt{10}$ Now, $AB=\sqrt{(2+1)^2+(6-1)^2}=\sqrt{34}<r$. So B lies inside C and the claim is agreed.