2013-DSE
MATH CP
PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2013

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Candidate Number									

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

8.30 am – 10.45 am (2¼ hours)
This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

* A 0 3 0 E 0 0 1 *

SECTION A(1) (35 marks)

1. Simplify $\dfrac{x^{20}y^{13}}{(x^5y)^6}$ and express your answer with positive indices. (3 marks)

2. Make k the subject of the formula $\dfrac{3}{h}-\dfrac{1}{k}=2$. (3 marks)

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3. Factorize

(a) $4m^2 - 25n^2$,

(b) $4m^2 - 25n^2 + 6m - 15n$.

(3 marks)

4. The price of 7 pears and 3 oranges is \$47 while the price of 5 pears and 6 oranges is \$49 . Find the price of a pear. (4 marks)

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5. (a) Solve the inequality $\frac{19-7x}{3} > 23-5x$.

(b) Find all integers satisfying both the inequalities $\frac{19-7x}{3} > 23-5x$ and $18-2x \geq 0$.

(4 marks)

6. In a polar coordinate system, O is the pole. The polar coordinates of the points A and B are $(26, 10°)$ and $(26, 130°)$ respectively. Let L be the axis of reflectional symmetry of ΔOAB .

(a) Describe the geometric relationship between L and $\angle AOB$.

(b) Find the polar coordinates of the point of intersection of L and AB .

(4 marks)

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7. In Figure 1, $ABCD$ is a quadrilateral. The diagonals AC and BD intersect at E. It is given that $BE = CE$ and $\angle BAC = \angle BDC$.

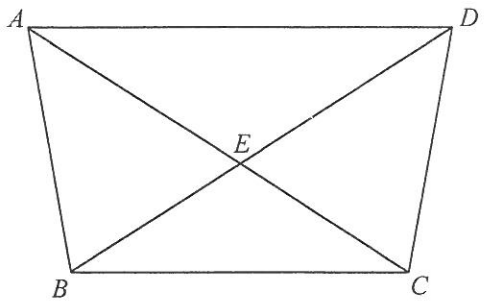

Figure 1

(a) Prove that $\Delta ABC \cong \Delta DCB$.

(b) Consider the triangles in Figure 1.

(i) How many pairs of congruent triangles are there?

(ii) How many pairs of similar triangles are there?

(4 marks)

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8. A pack of sea salt is termed *regular* if its weight is measured as 100 g correct to the nearest g .

(a) Find the least possible weight of a *regular* pack of sea salt.

(b) Is it possible that the total weight of 32 *regular* packs of sea salt is measured as 3.1 kg correct to the nearest 0.1 kg ? Explain your answer.

(5 marks)

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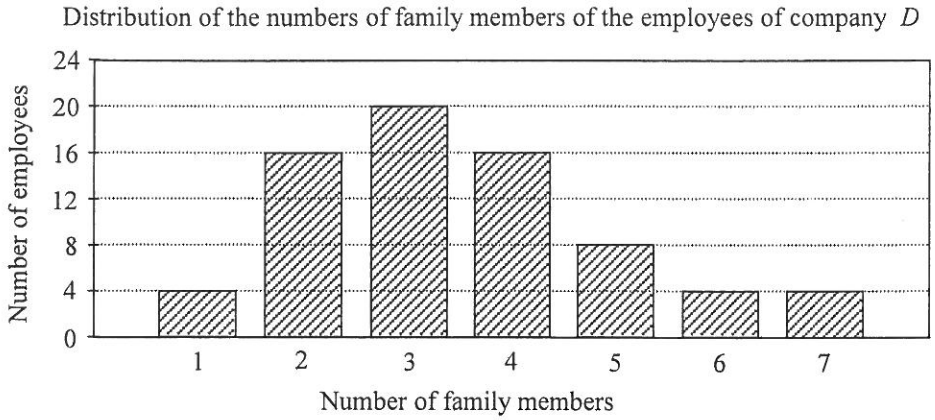
9. The bar chart below shows the distribution of the numbers of family members of the employees of company D.



(a) Find the mean, the inter-quartile range and the standard deviation of the above distribution.

(b) An employee leaves company D. The number of family members of this employee is 7. Find the change in the standard deviation of the numbers of family members of the employees of company D due to the leaving of this employee.

(5 marks)

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SECTION A(2) (35 marks)

10. The ages of the members of Committee A are shown as follows:

17	18	21	21	22	22	23	23	23	31
31	34	35	36	47	47	58	68	69	69

(a) Write down the median and the mode of the ages of the members of Committee A. (2 marks)

(b) The stem-and-leaf diagram below shows the distribution of the ages of the members of Committee B. It is given that the range of this distribution is 47.

Stem (tens)	Leaf (units)
2	a 5 6 7
3	3 3 8
4	3
5	1 2 9
6	7 b

(i) Find a and b.

(ii) From each committee, a member is randomly selected as the representative of that committee. The two representatives can join a competition when the difference of their ages exceeds 40. Find the probability that these two representatives can join the competition.

(4 marks)

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11. The weight of a tray of perimeter ℓ metres is W grams. It is given that W is the sum of two parts, one part varies directly as ℓ and the other part varies directly as ℓ^2. When $\ell = 1$, $W = 181$ and when $\ell = 2$, $W = 402$.

(a) Find the weight of a tray of perimeter 1.2 metres. (4 marks)

(b) If the weight of a tray is 594 grams, find the perimeter of the tray. (2 marks)

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12. Let $f(x) = 3x^3 - 7x^2 + kx - 8$, where k is a constant. It is given that $f(x) \equiv (x-2)(ax^2 + bx + c)$, where a, b and c are constants.

(a) Find a, b and c. (4 marks)

(b) Someone claims that all the roots of the equation $f(x) = 0$ are real numbers. Do you agree? Explain your answer. (3 marks)

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13. In a workshop, 2 identical solid metal right circular cylinders of base radius R cm are melted and recast into 27 smaller identical solid right circular cylinders of base radius r cm and height 10 cm . It is given that the base area of a larger circular cylinder is 9 times that of a smaller one.

(a) Find

(i) $r : R$,

(ii) the height of a larger circular cylinder.

(5 marks)

(b) A craftsman claims that a smaller circular cylinder and a larger circular cylinder are similar. Do you agree? Explain your answer. (2 marks)

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14. The equation of the circle C is $x^2 + y^2 - 12x - 34y + 225 = 0$. Denote the centre of C by R.

(a) Write down the coordinates of R. (1 mark)

(b) The equation of the straight line L is $4x + 3y + 50 = 0$. It is found that C and L do not intersect. Let P be a point lying on L such that P is nearest to R.

(i) Find the distance between P and R.

(ii) Let Q be a moving point on C. When Q is nearest to P,

(1) describe the geometric relationship between P, Q and R;

(2) find the ratio of the area of ΔOPQ to the area of ΔOQR, where O is the origin.

(8 marks)

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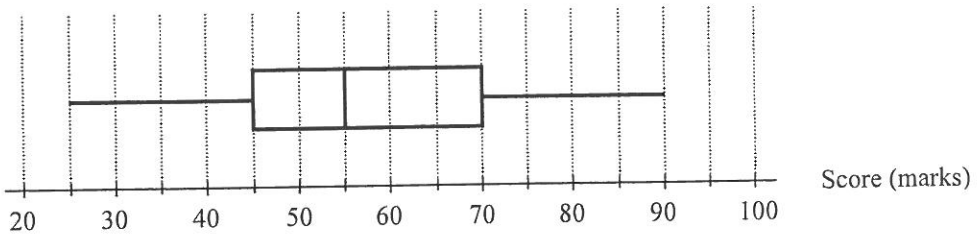
SECTION B (35 marks)

15. The box-and-whisker diagram below shows the distribution of the scores (in marks) of the students of a class in a test. Susan gets the highest score while Tom gets 65 marks in the test. The standard scores of Susan and Tom in the test are 3 and 0.5 respectively.



(a) Find the mean of the distribution. (2 marks)

(b) Susan claims that the standard scores of at least half of the students in the test are negative. Do you agree? Explain your answer. (2 marks)

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16. A box contains 5 white cups and 11 blue cups. If 6 cups are randomly drawn from the box at the same time,

(a) find the probability that at least 4 white cups are drawn; (2 marks)

(b) find the probability that at least 3 blue cups are drawn. (2 marks)

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17. (a) Let $f(x) = 36x - x^2$. Using the method of completing the square, find the coordinates of the vertex of the graph of $y = f(x)$. (2 marks)

(b) The length of a piece of string is 108 m. A guard cuts the string into two pieces. One piece is used to enclose a rectangular restricted zone of area $A\ m^2$. The other piece of length x m is used to divide this restricted zone into two rectangular regions as shown in Figure 2.

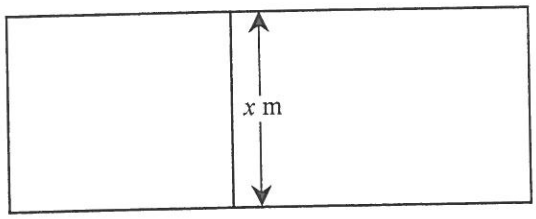

Figure 2

(i) Express A in terms of x.

(ii) The guard claims that the area of this restricted zone can be greater than $500\ m^2$. Do you agree? Explain your answer. (4 marks)

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18. (a) Figure 3(a) shows a piece of triangular paper card ABC with $AB = 28$ cm , $BC = 21$ cm and $AC = 35$ cm . Let M be a point lying on AC such that $\angle BMC = 75°$.

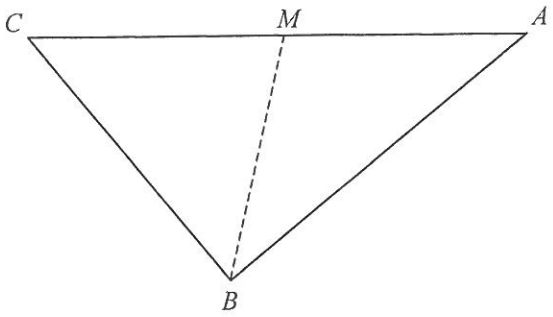

Figure 3(a)

Find

(i) $\angle BCM$,

(ii) CM .

(3 marks)

(b) Peter folds the triangular paper card described in (a) along BM such that AB and BC lie on the horizontal ground as shown in Figure 3(b). It is given that $\angle AMC = 107°$.

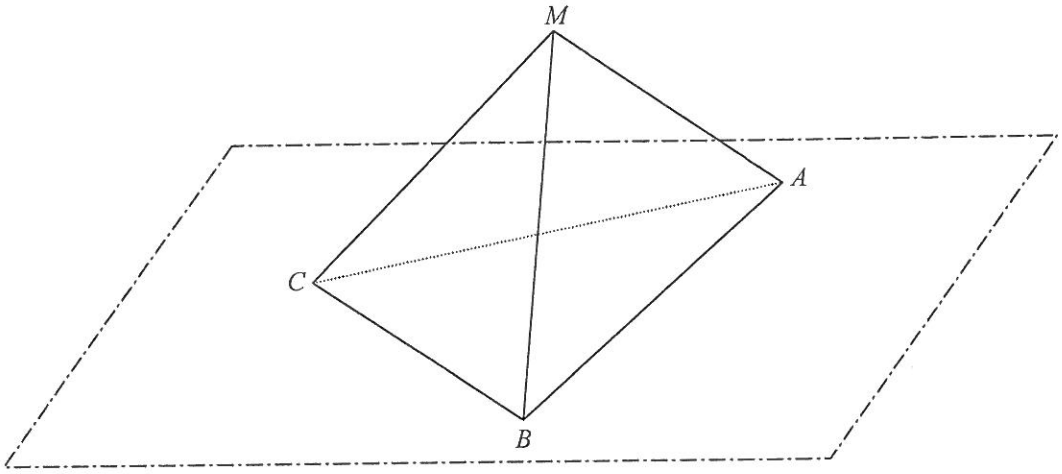

Figure 3(b)

(i) Find the distance between A and C on the horizontal ground.

(ii) Let N be a point lying on BC such that MN is perpendicular to BC . Peter claims that the angle between the face BCM and the horizontal ground is $\angle ANM$. Do you agree? Explain your answer.

(5 marks)

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19. The development of public housing in a city is under study. It is given that the total floor area of all public housing flats at the end of the 1st year is 9×10^6 m^2 and in subsequent years, the total floor area of public housing flats built each year is r % of the total floor area of all public housing flats at the end of the previous year, where r is a constant, and the total floor area of public housing flats pulled down each year is 3×10^5 m^2. It is found that the total floor area of all public housing flats at the end of the 3rd year is 1.026×10^7 m^2.

(a) (i) Express, in terms of r, the total floor area of all public housing flats at the end of the 2nd year.

(ii) Find r.

(4 marks)

(b) (i) Express, in terms of n, the total floor area of all public housing flats at the end of the nth year.

(ii) At the end of which year will the total floor area of all public housing flats first exceed 4×10^7 m^2 ?

(5 marks)

(c) It is assumed that the total floor area of public housing flats needed at the end of the nth year is $(a(1.21)^n+b)$ m^2, where a and b are constants. Some research results reveal the following information:

n	The total floor area of public housing flats needed at the end of the nth year (m^2)
1	1×10^7
2	1.063×10^7

A research assistant claims that based on the above assumption, the total floor area of all public housing flats will be greater than the total floor area of public housing flats needed at the end of a certain year. Is the claim correct? Explain your answer. (4 marks)

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