

Paper 1

Solution	Marks	Remarks
<p>1. <math>\frac{x^{20}y^{13}}{(x^5y)^6}</math></p> <p><math>\frac{x^{20}y^{13}}{x^{30}y^6}</math></p> <p><math>\frac{x^{20-30}y^{13-6}}{x^{30-20}}</math></p> <p><math>= \frac{y^7}{x^{10}}</math></p>	<p>1M</p> <p>1M</p> <p>1A</p> <p>------(3)</p>	<p>for <math>(ab)^m = a^m b^m</math> or <math>(a^m)^n = a^{mn}</math></p> <p>for <math>\frac{c^p}{c^q} = c^{p-q}</math> or <math>\frac{c^p}{c^q} = \frac{1}{c^{q-p}}</math></p>
<p>2. <math>\frac{3}{h} - \frac{1}{k} = 2</math></p> <p><math>\frac{1}{k} = \frac{3}{h} - 2</math></p> <p><math>\frac{1}{k} = \frac{3-2h}{h}</math></p> <p><math>k = \frac{h}{3-2h}</math></p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>for putting <math>k</math> on one side</p> <p>or equivalent</p>
<p><math>\frac{3}{h} - \frac{1}{k} = 2</math></p> <p><math>\left(\frac{3}{h} - \frac{1}{k}\right)hk = 2hk</math></p> <p><math>3k - h = 2hk</math></p> <p><math>3k - 2hk = h</math></p> <p><math>k(3-2h) = h</math></p> <p><math>k = \frac{h}{3-2h}</math></p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>for putting <math>k</math> on one side</p> <p>or equivalent</p>
<p>3. (a) <math>4m^2 - 25n^2</math></p> <p><math>= (2m)^2 - (5n)^2</math></p> <p><math>= (2m-5n)(2m+5n)</math></p> <p>(b) <math>4m^2 - 25n^2 + 6m - 15n</math></p> <p><math>= (2m-5n)(2m+5n) + 6m - 15n</math></p> <p><math>= (2m-5n)(2m+5n) + 3(2m-5n)</math></p> <p><math>= (2m-5n)(2m+5n+3)</math></p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>------(3)</p>	<p>or equivalent</p> <p>for using the result of (a)</p> <p>or equivalent</p>

Solution	Marks	Remarks
4. Let \$x\$ be the price of a pear and \$y\$ be the price of an orange. $\begin{cases} 7x + 3y = 47 \\ 5x + 6y = 49 \end{cases}$ So, we have $2(7x) - 5x = 2(47) - 49$ . Solving, we have $x = 5$ . Thus, the price of a pear is \$5.	$\left. \begin{array}{l} \\ \\ \end{array} \right\} 1A+1A$ 1M 1A	for getting a linear equation in $x$ or $y$ only
Let \$x\$ be the price of a pear. $5x + 6\left(\frac{47 - 7x}{3}\right) = 49$ Solving, we have $x = 5$ . Thus, the price of a pear is \$5.	1M+1A+1A 1A	$\left\{ \begin{array}{l} 1A \text{ for } y = \frac{47 - 7x}{3} \\ + 1M \text{ for } 5x + 6y \end{array} \right.$
The price of a pear $= \frac{2(47) - 49}{2(7) - 5}$ $= \$5$	1M+1A+1A 1A	$\left\{ \begin{array}{l} 1M \text{ for fraction} + 1A \text{ for numerator} \\ + 1A \text{ for denominator} \end{array} \right.$
-----(4)		
5. (a) $\frac{19 - 7x}{3} > 23 - 5x$ $19 - 7x > 69 - 15x$ $-7x + 15x > 69 - 19$ $8x > 50$ $x > \frac{25}{4}$	1M 1A	for putting $x$ on one side $x > 6.25$
(b) $18 - 2x \geq 0$ $x \leq 9$ By (a), we have $\frac{25}{4} < x \leq 9$ . Thus, the required integers are 7, 8 and 9.	1A 1A	for all correct
-----(4)		

Solution	Marks	Remarks								
6. (a) $L$ is the angle bisector of $\angle AOB$ .	1M									
(b) Let $S(r, \theta)$ be the point of intersection of $L$ and $AB$ .										
$r$ $= 26 \cos 60^\circ$ $= 13$	1M 1A									
$\theta$ $= 70^\circ$ <p>Thus, the required polar coordinates are <math>(13, 70^\circ)</math> .</p>	1A									
-----	(4)									
<p>7. (a) In <math>\triangle ABC</math> and <math>\triangle DCB</math> ,</p> <table style="width: 100%; border: none;"> <tr> <td style="padding-right: 20px;"><math>\angle BCE = \angle CBE</math></td> <td>( base <math>\angle</math>s, isos. <math>\triangle</math> )</td> </tr> <tr> <td><math>\angle BAC = \angle BDC</math></td> <td>( given )</td> </tr> <tr> <td><math>BC = BC</math></td> <td>( common side )</td> </tr> <tr> <td><math>\triangle ABC \cong \triangle DCB</math></td> <td>( AAS )</td> </tr> </table>	$\angle BCE = \angle CBE$	( base $\angle$ s, isos. $\triangle$ )	$\angle BAC = \angle BDC$	( given )	$BC = BC$	( common side )	$\triangle ABC \cong \triangle DCB$	( AAS )		
$\angle BCE = \angle CBE$	( base $\angle$ s, isos. $\triangle$ )									
$\angle BAC = \angle BDC$	( given )									
$BC = BC$	( common side )									
$\triangle ABC \cong \triangle DCB$	( AAS )									
<b>Marking Scheme:</b>										
<b>Case 1</b> Any correct proof with correct reasons.	2									
<b>Case 2</b> Any correct proof without reasons.	1									
(b) (i) There are 3 pairs of congruent triangles.	1A									
(ii) There are 4 pairs of similar triangles.	1A									
-----	(4)									

Solution	Marks	Remarks
<p>8. (a) The maximum absolute error = 0.5 g</p> <p>The least possible weight <del>= 100 - 0.5</del> = 99.5 g</p> <p>(b) The least possible total weight of 32 <i>regular</i> packs of sea salt = (99.5)(32) = 3 184 g = 3.184 kg = 3.2 kg (correct to the nearest 0.1 kg ) Thus, it is impossible that the total weight of 32 <i>regular</i> packs of sea salt is measured as 3.1 kg correct to the nearest 0.1 kg .</p>	<p>1M 1A</p> <p>1M 1A</p> <p>1A</p>	<p>f.t.</p>
<p>Note that <math>\frac{3.15}{32}</math> kg = 0.0984375 kg = 98.4375 g &lt; 99.5 g Thus, it is impossible that the total weight of 32 <i>regular</i> packs of sea salt is measured as 3.1 kg correct to the nearest 0.1 kg .</p>	<p>1M</p> <p>1A</p> <p>1A</p>	<p>f.t.</p>
------(5)		
<p>9. (a) The mean <math>= \frac{7}{2}</math></p> <p>The inter-quartile range <del>= 4 - 2</del> = 2</p> <p>The standard deviation = 1.5</p> <p>(b) The new standard deviation <math>\approx 1.451456116</math></p> <p>The decrease in the standard deviation <math>\approx 1.5 - 1.451456116</math> = 0.048543884 <math>\approx 0.0485</math></p>	<p>1A</p> <p>1M 1A</p> <p>1A</p> <p>1A</p>	<p>3.5</p> <p>r.t. 0.0485</p>
------(5)		

Solution	Marks	Remarks
10. (a) The median = 31	1A	
The mode = 23	1A	
-----(2)		
(b) (i) Note that $0 \leq a \leq 5$ and $7 \leq b \leq 9$ . Also note that the range of the distribution is 47.		
Thus, we have $\begin{cases} a=0 \\ b=7 \end{cases}$ , $\begin{cases} a=1 \\ b=8 \end{cases}$ or $\begin{cases} a=2 \\ b=9 \end{cases}$ .	1A+1A	1A for one pair + 1A for all
(ii) The required probability $= \frac{3+3+3+3+2+9+9}{260}$ $= \frac{32}{260}$ $= \frac{8}{65}$	1M  1A	r.t. 0.123
The required probability $= \frac{3+3+2+2+2+2+2+2+2+4+4+4}{260}$ $= \frac{32}{260}$ $= \frac{8}{65}$	1M  1A	r.t. 0.123
-----(4)		
11. (a) Let $W = h\ell + k\ell^2$ where $h$ and $k$ are non-zero constants.	1A	
$\begin{cases} h(1) + k(1^2) = 181 \\ h(2) + k(2^2) = 402 \end{cases}$	1M	for substitution
Solving, we have $h = 161$ and $k = 20$ .	1A	for both correct
The required weight $= 161(1.2) + 20(1.2^2)$ $= 222 \text{ grams}$	1A	
-----(4)		
(b) $20\ell^2 + 161\ell = 594$ $20\ell^2 + 161\ell - 594 = 0$ $(4\ell - 11)(5\ell + 54) = 0$ $\ell = \frac{11}{4}$ or $\ell = \frac{-54}{5}$ (rejected)	1M  1A	2.75
Thus, the perimeter of the tray is $\frac{11}{4}$ metres.		
-----(2)		

Solution	Marks	Remarks
<p>12. (a) By comparing the coefficients of <math>x^3</math> and the constant terms, we have <math>a=3</math> and <math>c=4</math> .  Note that the coefficient of <math>x^2</math> in the expansion of <math>(x-2)(3x^2+bx+4)</math> is <math>b-6</math> .  By comparing the coefficients of <math>x^2</math> , we have <math>b-6=-7</math> .  Thus, we have <math>b=-1</math> .</p>	<p>1A+1A  1M 1A</p>	
<p>Note that <math>x-2</math> is a factor of <math>f(x)</math> .  <math>f(2) = 0</math>  <math>3(2)^3 - 7(2)^2 + 2k - 8 = 0</math>  <math>k = 6</math>  <math>f(x)</math>  <math>= 3x^3 - 7x^2 + 6x - 8</math>  <math>= (x-2)(3x^2 - x + 4)</math>  Thus, we have <math>a=3</math> , <math>b=-1</math> and <math>c=4</math> .</p>	<p>1M  1A+1A+1A</p>	
<p>(b) <math>\Delta</math>  <math>= (-1)^2 - 4(3)(4)</math>  <math>= -47</math>  <math>&lt; 0</math>  So, the equation <math>3x^2 - x + 4 = 0</math> has nonreal roots.  Thus, the claim is disagreed.</p>	<p>------(4)  1M  1M+1A ------(3)</p>	<p>f.t.</p>

Solution	Marks	Remarks
<p>13. (a) (i) <math>\frac{\pi r^2}{\pi R^2} = \frac{1}{9}</math></p> $\left(\frac{r}{R}\right)^2 = \frac{1}{9}$ $\frac{r}{R} = \frac{1}{3}$ $r : R = 1 : 3$ <p>(ii) Let <math>h</math> cm be the height of a larger circular cylinder.</p> $2\pi R^2 h = 27(\pi r^2 (10))$ $h = \frac{270}{2} \left(\frac{r}{R}\right)^2$ $h = \frac{270}{2} \left(\frac{1}{3}\right)^2$ $h = 15$ <p>Thus, the height of a larger circular cylinder is 15 cm .</p> <p>(b) <math>\frac{\text{The base radius of a smaller circular cylinder}}{\text{The base radius of a larger circular cylinder}} = \frac{1}{3}</math></p> $\frac{\text{The height of a smaller circular cylinder}}{\text{The height of a larger circular cylinder}} = \frac{10}{15} = \frac{2}{3}$ $\frac{\text{The base radius of a smaller circular cylinder}}{\text{The base radius of a larger circular cylinder}} \neq \frac{\text{The height of a smaller circular cylinder}}{\text{The height of a larger circular cylinder}}$ <p>Therefore, the two circular cylinders are not similar. Thus, the claim is disagreed.</p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>----- (5)</p> <p>1M</p> <p>1A</p>	<p></p> <p></p> <p></p> <p></p> <p></p> <p>for comparing two ratios</p> <p>f.t.</p>
$\frac{\text{The base area of a smaller circular cylinder}}{\text{The base area of a larger circular cylinder}} = \frac{1}{9}$ $\frac{\text{The curved surface area of a smaller circular cylinder}}{\text{The curved surface area of a larger circular cylinder}} = \frac{2\pi r(10)}{2\pi R(15)} = \frac{2}{9}$ $\frac{\text{The base area of a smaller circular cylinder}}{\text{The base area of a larger circular cylinder}} \neq \frac{\text{The curved surface area of a smaller circular cylinder}}{\text{The curved surface area of a larger circular cylinder}}$ <p>Therefore, the two circular cylinders are not similar. Thus, the claim is disagreed.</p>	<p>1M</p> <p>1A</p>	<p>for comparing two ratios</p> <p>f.t.</p>
$\frac{\text{The base radius of a smaller circular cylinder}}{\text{The base radius of a larger circular cylinder}} = \frac{1}{3}$ $\left(\frac{\text{The base radius of a smaller circular cylinder}}{\text{The base radius of a larger circular cylinder}}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$ $\frac{\text{The volume of a smaller circular cylinder}}{\text{The volume of a larger circular cylinder}} = \frac{2}{27}$ $\frac{\text{The volume of a smaller circular cylinder}}{\text{The volume of a larger circular cylinder}} \neq \left(\frac{\text{The base radius of a smaller circular cylinder}}{\text{The base radius of a larger circular cylinder}}\right)^3$ <p>Therefore, the two circular cylinders are not similar. Thus, the claim is disagreed.</p>	<p>1M</p> <p>1A</p>	<p>for comparing two ratios</p> <p>f.t.</p>
	<p>----- (2)</p>	

Solution	Marks	Remarks
14. (a) The coordinates of $R$ $= (6, 17)$	1A	
-----(1)		
(b) (i) Let $(h, k)$ be the coordinates of $P$ . Since $P$ lies on $L$ , we have $4h + 3k + 50 = 0$ . Note that $RP$ is perpendicular to the straight line $4x + 3y + 50 = 0$ .	1M	
Also note that the slope of $RP$ is $\frac{k-17}{h-6}$ .		
Hence, we have $\left(\frac{k-17}{h-6}\right)\left(\frac{-4}{3}\right) = -1$ .	1M	
So, we have $3h - 4k + 50 = 0$ .		
Solving, we have $h = -14$ and $k = 2$ .		
Therefore, the coordinates of $P$ are $(-14, 2)$ .		
The distance between $P$ and $R$		
$= \sqrt{(-14-6)^2 + (2-17)^2}$	1M	
$= 25$	1A	
(ii) (1) $P, Q$ and $R$ are collinear.	1M	
(2) Note that the radius of the $C$ is $10$ .		
The distance between $Q$ and $R$		
$= 10$	1M	
The distance between $P$ and $Q$		
$= 25 - 10$		
$= 15$	1M	
The required ratio		
$= PQ : QR$		
$= 15 : 10$		
$= 3 : 2$	1A	
-----(8)		



Solution	Marks	Remarks
<p>15. (a) Note that the highest score of the distribution is 90 marks. Let <math>\mu</math> marks and <math>\sigma</math> marks be the mean and the standard deviation of the distribution respectively.</p> $\begin{cases} 90 - \mu = 3\sigma \\ 65 - \mu = 0.5\sigma \end{cases}$ <p>Solving, we have <math>\mu = 60</math> . Thus, the mean of the distribution is 60 marks.</p>	<p>1M 1A</p>	
<p>Note that the highest score of the distribution is 90 marks. Let <math>\sigma</math> marks be the standard deviation of the distribution.</p> $3 - 0.5 = \frac{90 - 65}{\sigma}$ <p><math>\sigma = 10</math> The mean <math>= 90 - 3\sigma</math> <math>= 90 - 3(10)</math> <math>= 60</math> marks</p>	<p>1M 1A</p>	<p>for both</p>
<p>Note that the highest score of the distribution is 90 marks. The mean</p> $= 90 - 3 \left( \frac{90 - 65}{3 - 0.5} \right)$ <p><math>= 60</math> marks</p>	<p>1M 1A</p>	
<p>Note that the highest score of the distribution is 90 marks. The mean</p> $= 65 - 0.5 \left( \frac{90 - 65}{3 - 0.5} \right)$ <p><math>= 60</math> marks</p>	<p>1M 1A</p>	
<p>(b) Note that if the test score of a student is lower than the mean, then the standard score of the student is negative. Also note that the median is 55 marks and the mean is 60 marks. So, the median is less than the mean. Therefore, the test scores of at least half of the students are less than the mean. Thus, the claim is agreed.</p>	<p>------(2) 1M 1A ------(2)</p>	<p>either one f.t.</p>

Solution	Marks	Remarks
16. (a) The required probability $= \frac{C_4^5 C_2^{11} + C_5^5 C_1^{11}}{C_6^{16}}$ $= \frac{286}{8008}$ $= \frac{1}{28}$	1M    1A	for numerator    r.t. 0.0357
The required probability $= 15 \left( \frac{5}{16} \right) \left( \frac{4}{15} \right) \left( \frac{3}{14} \right) \left( \frac{2}{13} \right) \left( \frac{11}{12} \right) \left( \frac{10}{11} \right) + 6 \left( \frac{5}{16} \right) \left( \frac{4}{15} \right) \left( \frac{3}{14} \right) \left( \frac{2}{13} \right) \left( \frac{1}{12} \right) \left( \frac{11}{11} \right)$ $= \frac{1}{28}$	1M  1A	for $15p_1 + 6p_2$  r.t. 0.0357
------(2)		
(b) The required probability $= 1 - \frac{1}{28}$ $= \frac{27}{28}$	1M  1A	for 1-(a)  r.t. 0.964
The required probability $= \frac{C_0^5 C_6^{11} + C_1^5 C_5^{11} + C_2^5 C_4^{11} + C_3^5 C_3^{11}}{C_6^{16}}$ $= \frac{7722}{8008}$ $= \frac{27}{28}$	1M  1A	for considering 4 cases  r.t. 0.964
The required probability $= \left( \frac{11}{16} \right) \left( \frac{10}{15} \right) \left( \frac{9}{14} \right) \left( \frac{8}{13} \right) \left( \frac{7}{12} \right) \left( \frac{6}{11} \right) + 6 \left( \frac{5}{16} \right) \left( \frac{11}{15} \right) \left( \frac{10}{14} \right) \left( \frac{9}{13} \right) \left( \frac{8}{12} \right) \left( \frac{7}{11} \right)$ $+ 15 \left( \frac{5}{16} \right) \left( \frac{4}{15} \right) \left( \frac{11}{14} \right) \left( \frac{10}{13} \right) \left( \frac{9}{12} \right) \left( \frac{8}{11} \right) + 20 \left( \frac{5}{16} \right) \left( \frac{4}{15} \right) \left( \frac{3}{14} \right) \left( \frac{11}{13} \right) \left( \frac{10}{12} \right) \left( \frac{9}{11} \right)$ $= \frac{27}{28}$	1M  1A	for $p_3 + 6p_4 + 15p_5 + 20p_6$  r.t. 0.964
------(2)		

Solution	Marks	Remarks
17. (a) $f(x)$ $= 36x - x^2$ $= -(x^2 - 36x + 18^2) + 18^2$ $= -(x - 18)^2 + 324$ Thus, the coordinates of the vertex are $(18, 324)$ .	1M  1A -----(2)	
(b) (i) $A$ $= x \left( \frac{108 - 3x}{2} \right)$ $= 54x - \frac{3}{2}x^2$  (ii) Note that $A = \frac{3}{2}f(x)$ , where $f(x) = 36x - x^2$ and $0 < x < 36$ . By (a), the greatest value of $A$ is 486. Thus, the claim is disagreed.	1M+1A      1M 1A	1M for $x \left( \frac{108 - mx}{2} \right)$ , $m > 0$    f.t.
Assume that $A > 500$ . Hence, we have $54x - \frac{3}{2}x^2 > 500$ . So, we have $3x^2 - 108x + 1000 < 0$ . $\Delta$ $= (-108)^2 - 4(3)(1000)$ $= -336$ $< 0$ Therefore, we have $3x^2 - 108x + 1000 > 0$ . This is impossible. Thus, the claim is disagreed.	1M      1A	f.t.      f.t.
Note that $A = 54x - \frac{3}{2}x^2$ . Hence, the quadratic equation $3x^2 - 108x + 2A = 0$ has real roots. So, we have $(-108)^2 - 4(3)(2A) \geq 0$ . Therefore, we have $11664 - 24A \geq 0$ . Solving, we have $A \leq 486$ . Thus, the claim is disagreed.	1M    1A	f.t.    f.t.
-----(4)		

Solution		Marks	Remarks
18. (a) (i)	<p>Note that <math>\angle ABC = 90^\circ</math>.</p> $\tan \angle BCM = \frac{28}{21}$ $\angle BCM \approx 53.13010235^\circ$ $\angle BCM \approx 53.1^\circ$	1A	r.t. $53.1^\circ$
(ii)	<p>By sine formula,</p> $\frac{CM}{\sin \angle MBC} = \frac{BC}{\sin \angle BMC}$ $\frac{CM}{\sin(180^\circ - 75^\circ - 53.13010235^\circ)} \approx \frac{21}{\sin 75^\circ}$ $CM \approx 17.10154643 \text{ cm}$ $CM \approx 17.1 \text{ cm}$	1M	
		1A	r.t. 17.1 cm
		------(3)	
(b) (i)	<p>By cosine formula,</p> $AC^2 = AM^2 + CM^2 - 2(AM)(CM)\cos \angle AMC$ $AC^2 \approx (35 - 17.10154643)^2 + (17.10154643)^2 - 2(35 - 17.10154643)(17.10154643)\cos 107^\circ$ $AC \approx 28.13898297 \text{ cm}$ $AC \approx 28.1 \text{ cm}$	1M	
		1A	r.t. 28.1 cm
(ii)	<p><math>CN</math></p> $= CM \cos \angle BCM$ $\approx 17.10154643 \cos 53.13010235^\circ$ $\approx 10.26092786$ <p>By cosine formula,</p> $AB^2 = BC^2 + AC^2 - 2(AC)(BC)\cos \angle ACB$ $\cos \angle ACB \approx \frac{21^2 + (28.13898297)^2 - 28^2}{2(28.13898297)(21)}$ $\angle ACB \approx 67.6818202^\circ$ <p>By cosine formula,</p> $AN^2 = CN^2 + AC^2 - 2(CN)(AC)\cos \angle ACB$ $AN^2 \approx (10.26092786)^2 + (28.13898297)^2 - 2(10.26092786)(28.13898297)\cos 67.6818202^\circ$ $AN \approx 26.03453787$ <p>By sine formula,</p> $\frac{AC}{\sin \angle ANC} = \frac{AN}{\sin \angle ACN}$ $\frac{28.13898297}{\sin \angle ANC} \approx \frac{26.03453787}{\sin 67.6818202^\circ}$ $\angle ANC \approx 89.06498097^\circ \text{ or } \angle ANC \approx 90.93501903^\circ$ <p>So, <math>\angle ANC</math> is not a right angle.</p> <p>Hence, <math>\angle ANM</math> is not the angle between the face <math>BCM</math> and the horizontal ground.</p> <p>Thus, the claim is disagreed.</p>	1M	
		1A	f.t.

Solution	Marks	Remarks
<p> <math>CN</math>  <math>= CM \cos \angle BCM</math>  <math>\approx 17.10154643 \cos 53.13010235^\circ</math>  <math>\approx 10.26092786</math> </p> <p>           By cosine formula,  <math>AB^2 = BC^2 + AC^2 - 2(AC)(BC) \cos \angle ACB</math>  <math>\cos \angle ACB \approx \frac{21^2 + (28.13898297)^2 - 28^2}{2(28.13898297)(21)}</math>  <math>\angle ACB \approx 67.6818202^\circ</math> </p> <p>           By cosine formula,  <math>AN^2 = CN^2 + AC^2 - 2(CN)(AC) \cos \angle ACB</math>  <math>AN^2 \approx (10.26092786)^2 + (28.13898297)^2 - 2(10.26092786)(28.13898297) \cos 67.6818202^\circ</math>  <math>AN \approx 26.03453787</math> </p> <p> <math>CN^2 + AN^2 \approx (10.26092786)^2 + (26.03453787)^2</math>  <math>CN^2 + AN^2 \approx 783.0838027</math> </p> <p> <math>AC^2 \approx (28.13898297)^2</math>  <math>AC^2 \approx 791.8023627</math> </p> <p>           Hence, we have <math>CN^2 + AN^2 \neq AC^2</math>.            So, <math>\angle ANC</math> is not a right angle.            Therefore, <math>\angle ANM</math> is not the angle between the face <math>BCM</math> and the horizontal ground.            Thus, the claim is disagreed.         </p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>any one</p> <p>f.t.</p>
<p> <math>CN</math>  <math>= CM \cos \angle BCM</math>  <math>\approx 17.10154643 \cos 53.13010235^\circ</math>  <math>\approx 10.26092786</math> </p> <p>           By cosine formula,  <math>AB^2 = BC^2 + AC^2 - 2(AC)(BC) \cos \angle ACB</math>  <math>\cos \angle ACB \approx \frac{21^2 + (28.13898297)^2 - 28^2}{2(28.13898297)(21)}</math>  <math>\cos \angle ACB \approx 0.379749707</math> </p> <p>           Suppose that <math>N'</math> is the foot of the perpendicular from <math>A</math> to <math>BC</math>.  <math>CN'</math>  <math>= AC \cos \angle ACB</math>  <math>\approx 28.13898297(0.379749707)</math>  <math>\approx 10.68577054</math> </p> <p>           Hence, we have <math>CN' \neq CN</math>.            So, <math>N</math> is not the foot of the perpendicular from <math>A</math> to <math>BC</math>.            Therefore, <math>\angle ANM</math> is not the angle between the face <math>BCM</math> and the horizontal ground.            Thus, the claim is disagreed.         </p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>any one</p> <p>f.t.</p>
	<p>(5)</p>	

Solution	Marks	Remarks
<p>19. (a) (i) The required area  <math>= 9 \times 10^6(1+r\%) - 3 \times 10^5</math>  <math>= (870 + 9r) \times 10^4 \text{ m}^2</math></p> <p>(ii) <math>(9 \times 10^6(1+r\%) - 3 \times 10^5)(1+r\%) - 3 \times 10^5 = 1.026 \times 10^7</math>  <math>150(1+r\%)^2 - 5(1+r\%) - 176 = 0</math>  <math>1+r\% = \frac{11}{10}</math> or <math>1+r\% = \frac{-16}{15}</math> (rejected)  Thus, we have <math>r = 10</math> .</p>	<p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>----- (4)</p>	<p><math>3r^2 + 590r - 6200 = 0</math></p>
<p>(b) (i) The required area  <math>= 9 \times 10^6(1.1)^{n-1} - 3 \times 10^5(1.1)^{n-2} - 3 \times 10^5(1.1)^{n-3} - 3 \times 10^5(1.1)^{n-4} - \dots - 3 \times 10^5</math>  <math>= 9 \times 10^6(1.1)^{n-1} - 3 \times 10^5 \left( \frac{(1.1)^{n-1} - 1}{1.1 - 1} \right)</math>  <math>= 9 \times 10^6(1.1)^{n-1} - 3 \times 10^6((1.1)^{n-1} - 1)</math>  <math>= (6(1.1)^{n-1} + 3) \times 10^6 \text{ m}^2</math></p> <p>(ii) When <math>(6(1.1)^{n-1} + 3) \times 10^6 &gt; 4 \times 10^7</math> , we have <math>(1.1)^{n-1} &gt; \frac{37}{6}</math> .  Hence, we have <math>\log(1.1)^{n-1} &gt; \log\left(\frac{37}{6}\right)</math> .  So, we have <math>(n-1)\log 1.1 &gt; \log\left(\frac{37}{6}\right)</math> .  Therefore, we have <math>n-1 &gt; \frac{\log\left(\frac{37}{6}\right)}{\log 1.1}</math> .  Solving, we have <math>n &gt; 20.08671715</math> .  Thus, the total floor area of all public housing flats will first exceed <math>4 \times 10^7 \text{ m}^2</math> at the end of the 21st year.</p>	<p>1M</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>----- (5)</p>	<p>for sum of geometric sequence</p>
<p>(c) Note that <math>a(1.21)^1 + b = 1 \times 10^7</math> and <math>a(1.21)^2 + b = 1.063 \times 10^7</math> .  Solving, we have <math>a = \frac{300}{121} \times 10^6</math> and <math>b = 7 \times 10^6</math> .  Consider <math>(6(1.1)^{n-1} + 3) \times 10^6 &gt; \left( \frac{300}{121}(1.21)^n + 7 \right) \times 10^6</math> ..... (*) .  So, we have <math>\frac{-300}{121}(1.21)^n - 7 + 6(1.1)^{n-1} + 3 &gt; 0</math> .  Therefore, we have <math>75((1.1)^n)^2 - 165(1.1)^n + 121 &lt; 0</math> .  <math>\Delta</math>  <math>= (-165)^2 - 4(75)(121)</math>  <math>= -9075</math>  <math>&lt; 0</math>  Since <math>75 &gt; 0</math> , we have <math>75((1.1)^n)^2 - 165(1.1)^n + 121 &gt; 0</math> for all <math>n</math> .  Hence, there is no solution for (*) .  Thus, the claim is incorrect.</p>	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>----- (4)</p>	<p><math>3((1.1)^{n-1})^2 - 6(1.1)^{n-1} + 4 &lt; 0</math></p> <p>f.t.</p>