2014-DSE
MATH CP
PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2014

MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

8.30 am – 10.45 am (2¼ hours)
This paper must be answered in English

Please stick the barcode label here.

Candidate Number									

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

* A 0 3 0 E 0 0 1 *

SECTION A(1) (35 marks)

1. Simplify $\frac{(xy^{-2})^3}{y^4}$ and express your answer with positive indices. (3 marks)

2. Factorize

(a) $a^2 - 2a - 3$,

(b) $ab^2 + b^2 + a^2 - 2a - 3$.

(3 marks)

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3. (a) Round up 123.45 to 1 significant figure.

(b) Round off 123.45 to the nearest integer.

(c) Round down 123.45 to 1 decimal place.

(3 marks)

4. The table below shows the distribution of the numbers of calculators owned by some students.

Number of calculators	0	1	2	3
Number of students	7	14	15	4

Find the median, the mode and the standard deviation of the above distribution. (3 marks)

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Answers written in the margins will not be marked.

5. Consider the formula $2(3m+n)=m+7$.

(a) Make n the subject of the above formula.

(b) If the value of m is increased by 2, write down the change in the value of n.

(4 marks)

6. The marked price of a toy is $\$255$. The toy is now sold at a discount of 40% on its marked price.

(a) Find the selling price of the toy.

(b) If the percentage profit is 2%, find the cost of the toy.

(4 marks)

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7. Let $f(x) = 4x^3 - 5x^2 - 18x + c$, where c is a constant. When $f(x)$ is divided by $x - 2$, the remainder is -33.

(a) Is $x + 1$ a factor of $f(x)$? Explain your answer.

(b) Someone claims that all the roots of the equation $f(x) = 0$ are rational numbers. Do you agree? Explain your answer.

(5 marks)

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8. The coordinates of the points P and Q are $(-3, 5)$ and $(2, -7)$ respectively. P is rotated anticlockwise about the origin O through $270°$ to P'. Q is translated leftwards by 21 units to Q'.

(a) Write down the coordinates of P' and Q'.

(b) Prove that PQ is perpendicular to $P'Q'$.

(5 marks)

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9. In Figure 1, D is a point lying on AC such that $\angle BAC = \angle CBD$.

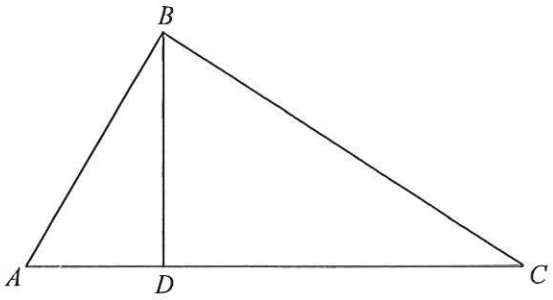

Figure 1

(a) Prove that $\Delta ABC \sim \Delta BDC$.

(b) Suppose that $AC = 25$ cm, $BC = 20$ cm and $BD = 12$ cm. Is ΔBCD a right-angled triangle? Explain your answer.

(5 marks)

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SECTION A(2) (35 marks)

10. Town X and town Y are 80 km apart. Figure 2 shows the graphs for car A and car B travelling on the same straight road between town X and town Y during the period 7:30 to 9:30 in a morning. Car A travels at a constant speed during the period. Car B comes to rest at 8:15 in the morning.

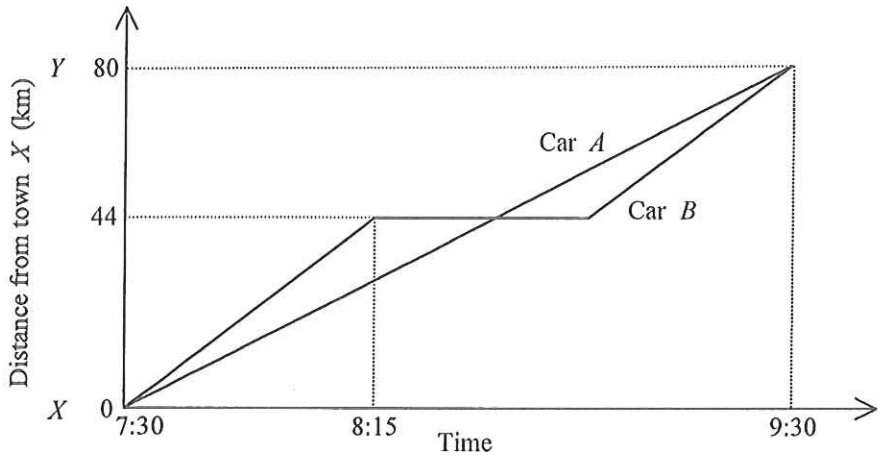

Figure 2

(a) Find the distance of car A from town X at 8:15 in the morning. (2 marks)

(b) At what time after 7:30 in the morning do car A and car B first meet? (2 marks)

(c) The driver of car B claims that the average speed of car B is higher than that of car A during the period 8:15 to 9:30 in the morning. Do you agree? Explain your answer. (2 marks)

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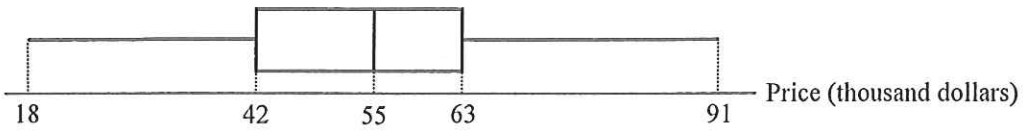
11. There are 33 paintings in an art gallery. The box-and-whisker diagram below shows the distribution of the prices (in thousand dollars) of the paintings in the art gallery. It is given that the mean of this distribution is 53 thousand dollars.



(a) Find the range and the inter-quartile range of the above distribution. (3 marks)

(b) Four paintings of respective prices (in thousand dollars) 32 , 34 , 58 and 59 are now donated to a museum. Find the mean and the median of the prices of the remaining paintings in the art gallery. (3 marks)

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12. The circle C passes through the point $A(6, 11)$ and the centre of C is the point $G(0, 3)$.

(a) Find the equation of C. (2 marks)

(b) P is a moving point in the rectangular coordinate plane such that $AP = GP$. Denote the locus of P by Γ.

(i) Find the equation of Γ.

(ii) Describe the geometric relationship between Γ and the line segment AG.

(iii) If Γ cuts C at Q and R, find the perimeter of the quadrilateral $AQGR$. (5 marks)

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13. It is given that $f(x)$ is the sum of two parts, one part varies as x^2 and the other part is a constant. Suppose that $f(2) = 59$ and $f(7) = -121$.

(a) Find $f(6)$. (4 marks)

(b) $A(6, a)$ and $B(-6, b)$ are points lying on the graph of $y = f(x)$. Find the area of ΔABC, where C is a point lying on the x-axis. (4 marks)

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14. Figure 3 shows a vessel in the form of a frustum which is made by cutting off the lower part of an inverted right circular cone of base radius 72 cm and height 96 cm . The height of the vessel is 60 cm . The vessel is placed on a horizontal table. Some water is now poured into the vessel. John finds that the depth of water in the vessel is 28 cm .

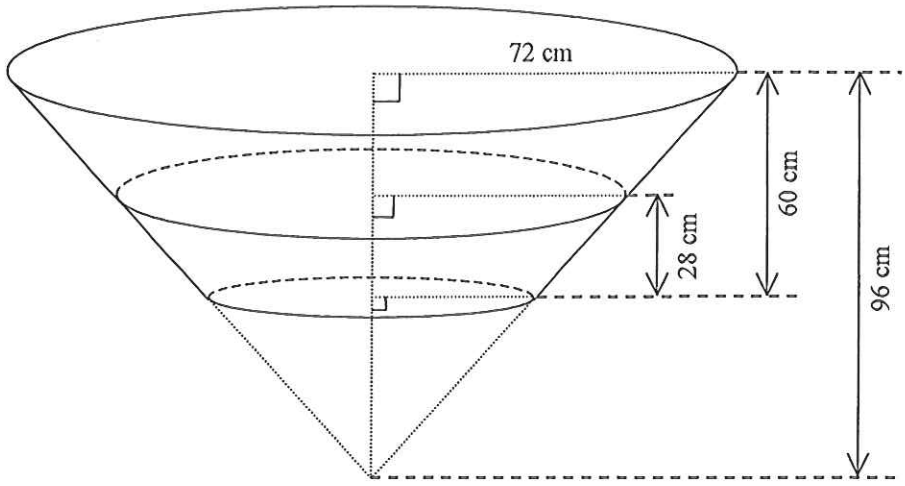

Figure 3

(a) Find the area of the wet curved surface of the vessel in terms of π. (4 marks)

(b) John claims that the volume of water in the vessel is greater than $0.1\,m^3$. Do you agree? Explain your answer. (4 marks)

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SECTION B (35 marks)

15. The graph in Figure 4 shows the linear relation between $\log_4 x$ and $\log_8 y$. The slope and the intercept on the horizontal axis of the graph are $\frac{-1}{3}$ and 3 respectively. Express the relation between x and y in the form $y = Ax^k$, where A and k are constants. (3 marks)

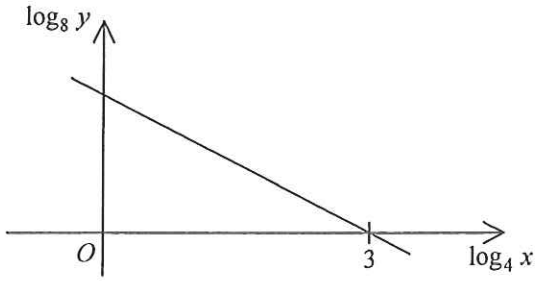

Figure 4

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16. In Figure 5, the 1st pattern consists of 3 dots. For any positive integer n, the $(n+1)$th pattern is formed by adding 2 dots to the nth pattern. Find the least value of m such that the total number of dots in the first m patterns exceeds 6 888. (4 marks)

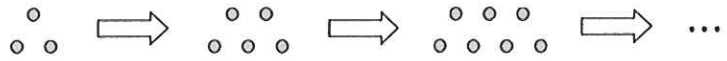

Figure 5

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17. Figure 6(a) shows a solid pyramid $VABCD$ with a rectangular base, where $AB = 18$ cm, $BC = 10$ cm, $VB = VC = 30$ cm and $\angle VAB = \angle VDC = 110°$.

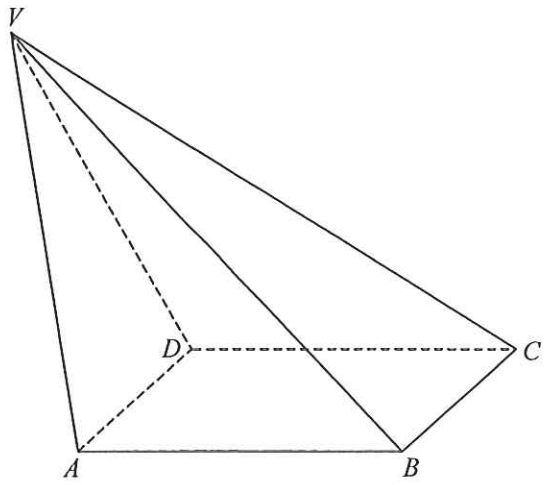

Figure 6(a)

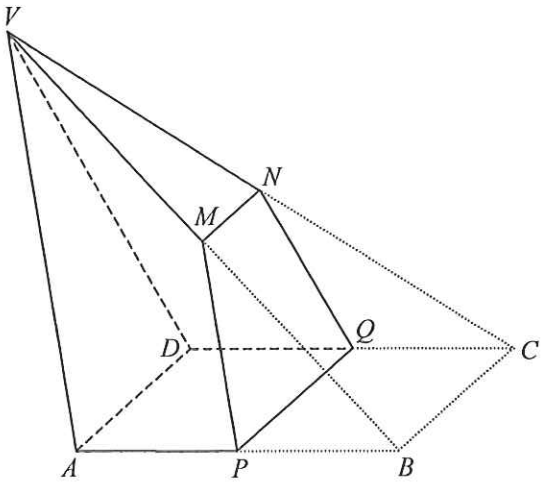

Figure 6(b)

(a) Find $\angle VBA$. (2 marks)

(b) P, Q, M and N are the mid-points of AB, CD, VB and VC respectively. A geometric model is made by cutting off $PBCQNM$ from $VABCD$ as shown in Figure 6(b). A craftsman claims that the area of the trapezium $PQNM$ is less than $70\ cm^2$. Do you agree? Explain your answer. (5 marks)

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18. (a) In Figure 7, the equation of the straight line L_1 is $6x+7y=900$ and the x-intercept of the straight line L_2 is 180. L_1 and L_2 intersect at the point $(45, 90)$. The shaded region (including the boundary) represents the solution of a system of inequalities. Find the system of inequalities. (4 marks)

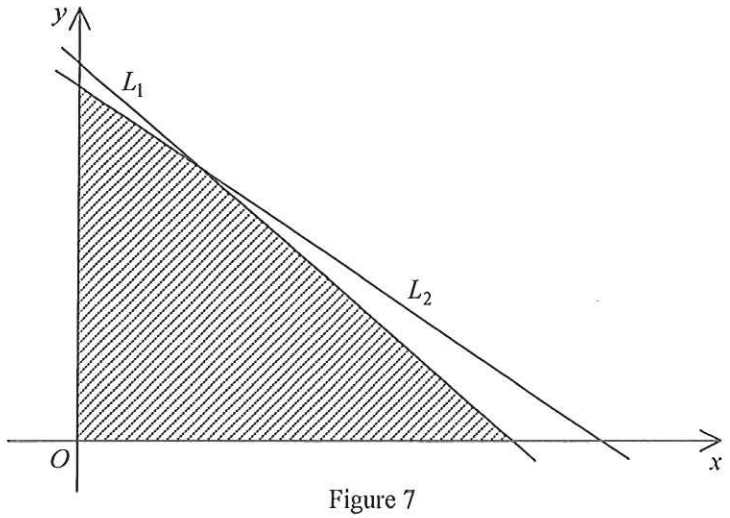

Figure 7

(b) A factory produces two types of wardrobes, X and Y. Each wardrobe X requires 6 man-hours for assembly and 2 man-hours for packing while each wardrobe Y requires 7 man-hours for assembly and 3 man-hours for packing. In a certain month, the factory has 900 man-hours available for assembly and 360 man-hours available for packing. The profits for producing a wardrobe X and a wardrobe Y are \$440 and \$665 respectively. A worker claims that the total profit can exceed \$80 000 that month. Do you agree? Explain your answer. (4 marks)

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19. Ada and Billy play a game consisting of two rounds. In the first round, Ada and Billy take turns to throw a fair die. The player who first gets a number '3' wins the first round. Ada and Billy play the first round until one of them wins. Ada throws the die first.

(a) Find the probability that Ada wins the first round of the game. (3 marks)

(b) In the second round of the game, balls are dropped one by one into a device containing eight tubes arranged side by side (see Figure 8). When a ball is dropped into the device, it falls randomly into one of the tubes. Each tube can hold at most three balls.

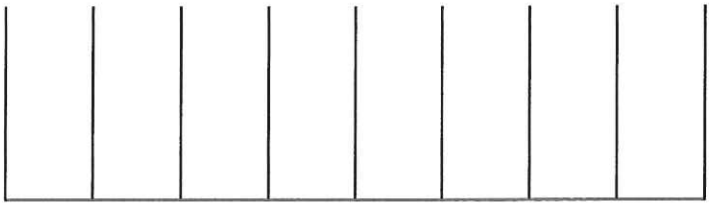

Figure 8

The player of this round adopts one of the following two options.

Option 1: Two balls are dropped one by one into the device. If the two balls fall into the same tube, then the player gets 10 tokens. If the two balls fall into two adjacent tubes, then the player gets 5 tokens. Otherwise, the player gets no tokens.

Option 2: Three balls are dropped one by one into the device. If the three balls fall into the same tube, then the player gets 50 tokens. If the three balls fall into three adjacent tubes, then the player gets 10 tokens. If the three balls fall into two adjacent tubes, then the player gets 5 tokens. Otherwise, the player gets no tokens.

(i) If the player of the second round adopts Option 1, find the expected number of tokens got.

(ii) Which option should the player of the second round adopt in order to maximise the expected number of tokens got? Explain your answer.

(iii) Only the winner of the first round plays the second round. It is given that the player of the second round adopts the option which can maximise the expected number of tokens got. Billy claims that the probability of Ada getting no tokens in the game exceeds 0.9 . Is the claim correct? Explain your answer.

(10 marks)

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HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2014

MATHEMATICS Compulsory Part
PAPER 2

11.30 am – 12.45 pm (1¼ hours)

INSTRUCTIONS

1. Read carefully the instructions on the Answer Sheet. After the announcement of the start of the examination, you should first stick a barcode label and insert the information required in the spaces provided. No extra time will be given for sticking on the barcode label after the 'Time is up' announcement.

2. When told to open this book, you should check that all the questions are there. Look for the words '**END OF PAPER**' after the last question.

3. All questions carry equal marks.

4. **ANSWER ALL QUESTIONS**. You are advised to use an HB pencil to mark all the answers on the Answer Sheet, so that wrong marks can be completely erased with a clean rubber. You must mark the answers clearly; otherwise you will lose marks if the answers cannot be captured.

5. You should mark only **ONE** answer for each question. If you mark more than one answer, you will receive **NO MARKS** for that question.

6. No marks will be deducted for wrong answers.

Not to be taken away before the end of the examination session

There are 30 questions in Section A and 15 questions in Section B.
The diagrams in this paper are not necessarily drawn to scale.
Choose the best answer for each question.

Section A

1. $(2n^3)^{-5} =$

A. $\frac{1}{32n^2}$.

B. $\frac{1}{32n^{15}}$.

C. $\frac{1}{10n^{125}}$.

D. $\frac{1}{10n^{243}}$.

2. $u^2 - v^2 - 5u + 5v =$

A. $(u-v)(u+v-5)$.

B. $(u-v)(u+v+5)$.

C. $(u+v)(u-v-5)$.

D. $(u+v)(u-v+5)$.

3. If p and q are constants such that $px(x-1)+x^2 \equiv qx(x-2)+4x$, then $p =$

A. 1 .

B. 2 .

C. 3 .

D. 4 .

4. Let a be a constant. If the quadratic equation $x^2 + ax + a = 1$ has equal roots, then $a =$

A. -1 .

B. 2 .

C. 0 or -4 .

D. 0 or 4 .

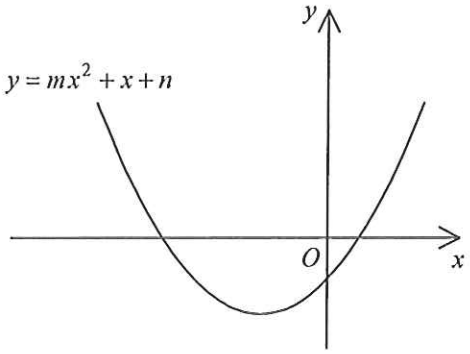
5. The figure shows the graph of $y = mx^2 + x + n$, where m and n are constants. Which of the following is true?

A. $m < 0$ and $n < 0$

B. $m < 0$ and $n > 0$

C. $m > 0$ and $n < 0$

D. $m > 0$ and $n > 0$



6. If $a > b$ and $k < 0$, which of the following must be true?

I. $a^2 > b^2$
II. $a + k > b + k$
III. $\frac{a}{k^2} > \frac{b}{k^2}$

A. I only

B. II only

C. I and III only

D. II and III only

7. The solution of $-3x < 6 < 2x$ is

A. $x > -2$.

B. $x > 0$.

C. $x > 3$.

D. $-2 < x < 3$.

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8. The price of 2 bowls and 3 cups is \$506 . If the price of 5 bowls and the price of 4 cups are the same, then the price of a bowl is

A. \$88 .

B. \$92 .

C. \$110 .

D. \$115 .

9. There are 792 workers in a factory. If the number of male workers is 20% less than that of female workers, then the number of male workers is

A. 352 .

B. 360 .

C. 432 .

D. 440 .

10. If the angle and the radius of a sector are decreased by $x\%$ and 50% respectively so that its area is decreased by 90% , then $x =$

A. 20 .

B. 40 .

C. 60 .

D. 80 .

11. The width and the length of a thin rectangular metal sheet are measured as 8 cm and 10 cm correct to the nearest cm respectively. Let x cm^2 be the actual area of the metal sheet. Find the range of values of x .

A. $71.25 \leq x < 89.25$

B. $71.25 < x \leq 89.25$

C. $79.5 \leq x < 80.5$

D. $79.5 < x \leq 80.5$

12. It is given that $\frac{4}{5a}=\frac{5}{7b}=\frac{7}{9c}$, where a, b and c are positive numbers. Which of the following is true?

A. $a<b<c$

B. $a<c<b$

C. $b<a<c$

D. $b<c<a$

13. If z varies inversely as x and directly as the cube of y, which of the following must be constant?

A. xy^3z

B. x^3yz^3

C. $\frac{y^3}{xz}$

D. $\frac{y}{x^3z^3}$

14. Let a_n be the nth term of a sequence. If $a_2=7$, $a_4=63$ and $a_{n+2}=a_{n+1}+a_n$ for any positive integer n, then $a_5=$

A. 56.

B. 70.

C. 91.

D. 119.

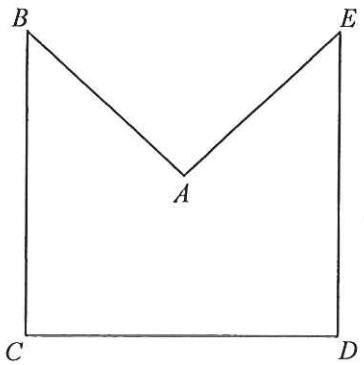
15. In the figure, $AB=AE$ and $\angle BAE=\angle BCD=\angle CDE=90°$. If $BC=CD=DE=16$ cm, then the area of the pentagon $ABCDE$ is

A. 71 cm^2.

B. 128 cm^2.

C. 192 cm^2.

D. 224 cm^2.



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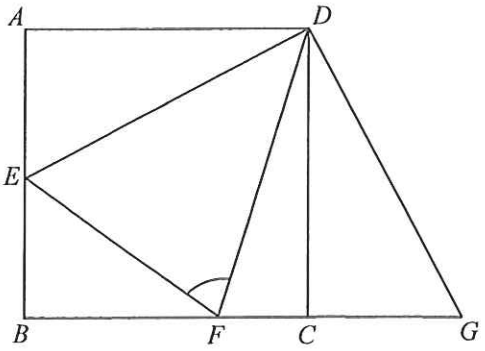
16. In the figure, $ABCD$ is a square. BC is produced to G such that $\angle CDG = 25°$. E is a point lying on AB such that $AE = CG$. If F is a point lying on BC such that $\angle CDF = 20°$, then $\angle DFE =$

A. $60°$.

B. $65°$.

C. $70°$.

D. $73°$.



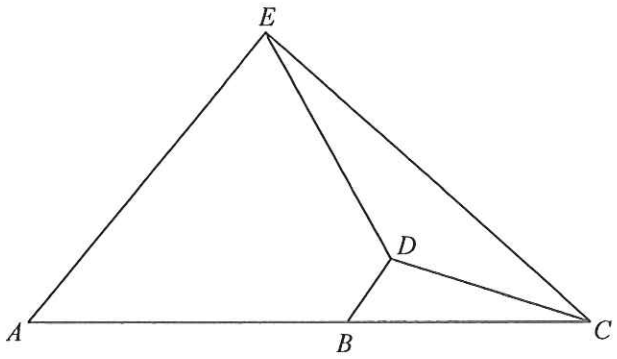
17. In the figure, B is a point lying on AC such that $AB : BC = 3 : 2$. It is given that $AE // BD$. If the area of ΔBCD and the area of ΔCDE are $4\ \text{cm}^2$ and $8\ \text{cm}^2$ respectively, then the area of the trapezium $ABDE$ is

A. $18\ \text{cm}^2$.

B. $21\ \text{cm}^2$.

C. $27\ \text{cm}^2$.

D. $33\ \text{cm}^2$.



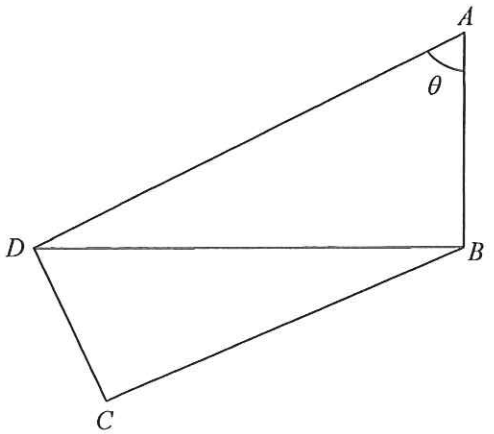
18. In the figure, $\angle ABD = \angle ADC = \angle BCD = 90°$. If $AB = \ell$, then $CD =$

A. $\ell \sin\theta$.

B. $\ell \cos\theta$.

C. $\ell \sin\theta \tan\theta$.

D. $\dfrac{\ell \tan\theta}{\cos\theta}$.



19. $(\cos(90°+\theta)+1)(\sin(360°-\theta)-1)=$

A. $-\cos^2\theta$.

B. $-\sin^2\theta$.

C. $\cos^2\theta$.

D. $\sin^2\theta$.

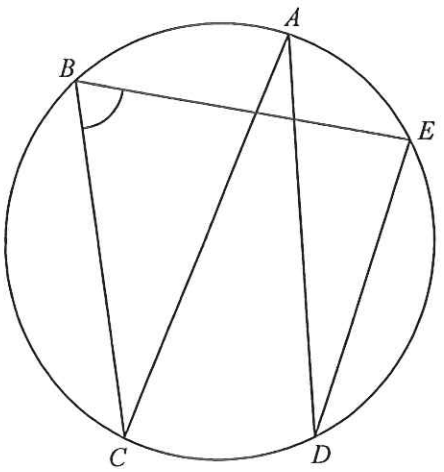
20. In the figure, AC is a diameter of the circle $ABCDE$. If $\angle ADE = 28°$, then $\angle CBE =$

A. $56°$.

B. $62°$.

C. $72°$.

D. $76°$.



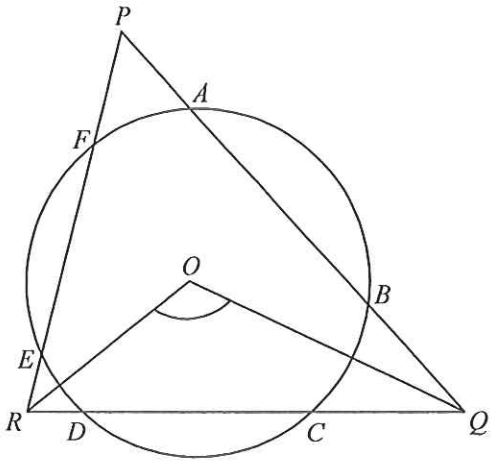
21. In the figure, O is the centre of the circle $ABCDEF$. ΔPQR intersects the circle at A, B, C, D, E and F. If $\angle QPR = 38°$ and $AB = CD = EF$, then $\angle QOR =$

A. $109°$.

B. $117°$.

C. $123°$.

D. $142°$.



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22. If an interior angle of a regular n-sided polygon is greater than an exterior angle by $100°$, which of the following are true?

I. The value of n is 10.
II. Each exterior angle of the polygon is $40°$.
III. The number of axes of reflectional symmetry of the polygon is 9.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

23. The rectangular coordinates of the point P are $(-1, \sqrt{3})$. If P is reflected with respect to the x-axis, then the polar coordinates of its image are

A. $(2, 210°)$.

B. $(2, 240°)$.

C. $(4, 210°)$.

D. $(4, 240°)$.

24. The equations of the straight lines L_1 and L_2 are $2x + 3y = 5$ and $4x + 6y = 7$ respectively. If P is a moving point in the rectangular coordinate plane such that the perpendicular distance from P to L_1 is equal to the perpendicular distance from P to L_2, then the locus of P is a

A. circle.

B. square.

C. parabola.

D. straight line.

25. In the figure, the two straight lines intersect at a point on the positive y-axis. Which of the following are true?

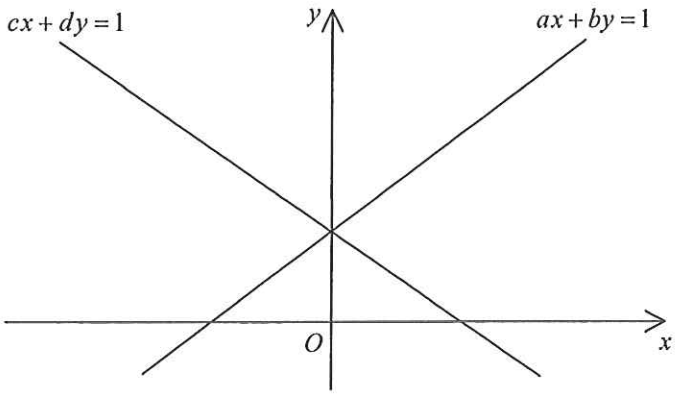
I. $a<0$
II. $c>0$
III. $b=d$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III



26. If a diameter of the circle $x^2+y^2-8x+ky-214=0$ passes through the point $(6,-5)$ and the slope of the diameter is -4, then $k=$

A. -6.

B. -4.

C. 13.

D. 70.

27. A box contains m yellow balls and 20 black balls. If a ball is randomly drawn from the box, then the probability of drawing a yellow ball is $\frac{1}{m}$. Find the value of m.

A. 4

B. 5

C. 15

D. 25

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28. The mean height of 25 teachers and 140 students is 150 cm . If the mean height of the students is 145 cm , then the mean height of the teachers is

A. 151 cm .

B. 155 cm .

C. 176 cm .

D. 178 cm .

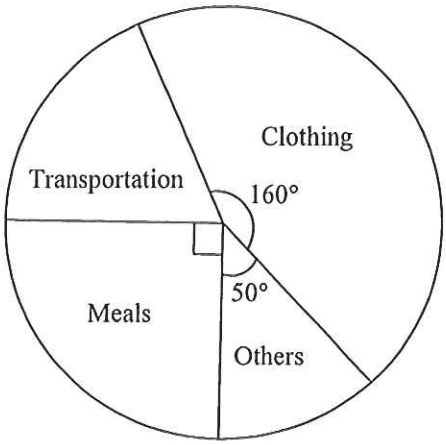
29. The pie chart below shows the expenditure of John in a certain week. John spends \$240 on clothing that week. Find his expenditure on transportation that week.

A. \$40

B. \$60

C. \$90

D. \$135



30. The stem-and-leaf diagram below shows the distribution of the ages of the passengers in a bus.

Stem (tens)	Leaf (units)						
1	h	4	6				
2	3	3	3	4	6	7	7
3	1	2	2	2	6	8	
4	0	k					

If the range of the above distribution is at least 33 , which of the following must be true?

I. $0 \le h \le 3$
II. $3 \le k \le 9$
III. $3 \le k - h \le 5$

A. I only

B. II only

C. I and III only

D. II and III only

Section B

31. The H.C.F. of $3x^4y^2z$, $4xy^5z$ and $6x^2y^3$ is

A. xy^2.

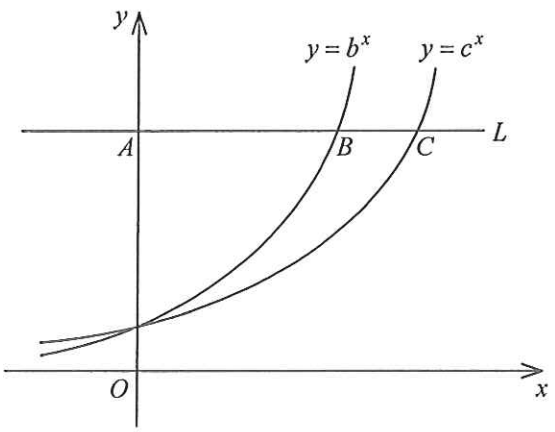
B. xy^2z.

C. $12x^4y^5z$.

D. $12x^7y^9z^2$.

32. The figure shows the graph of $y = b^x$ and the graph of $y = c^x$ on the same rectangular coordinate system, where b and c are positive constants. If a horizontal line L cuts the y-axis, the graph of $y = b^x$ and the graph of $y = c^x$ at A, B and C respectively, which of the following are true?

I. $b < c$
II. $bc > 1$
III. $\frac{AB}{AC} = \log_b c$



A. I and II only

B. I and III only

C. II and III only

D. I, II and III

33. Which of the following is the greatest?

A. 124^{241}

B. 241^{214}

C. 412^{142}

D. 421^{124}

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34. $7\times2^{10}+2^8+5\times2^3-2^3=$

A. 111010100000_2 .

B. 111100010000_2 .

C. 1110100100000_2 .

D. 1111000010000_2 .

35. Let $f(x)=3x^2-6x+k$, where k is a constant. If the y-coordinate of the vertex of the graph of $y=f(x)$ is 7 , then $k=$

A. 1 .

B. 3 .

C. 4 .

D. 10 .

36. If β is a real number, then $\dfrac{\beta^2+4}{\beta+2i}=$

A. $\beta-2i$.

B. $\beta+2i$.

C. $2-\beta i$.

D. $2+\beta i$.

37. If $m>1$, which of the following are geometric sequences?

I. 2^m , 2^{2m} , 2^{3m} , 2^{4m}

II. m , $2m^2$, $3m^4$, $4m^8$

III. $\log m$, $\log m^2$, $\log m^4$, $\log m^8$

A. I and II only

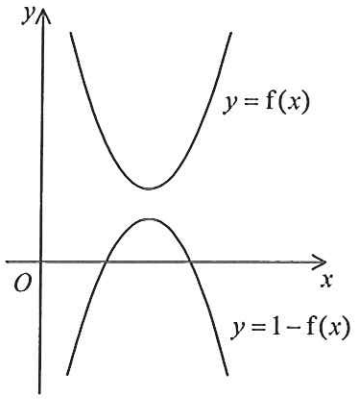
B. I and III only

C. II and III only

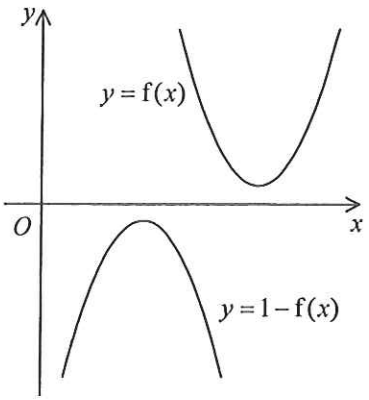
D. I , II and III

38. Which of the following may represent the graph of $y = f(x)$ and the graph of $y = 1 - f(x)$ on the same rectangular coordinate system?

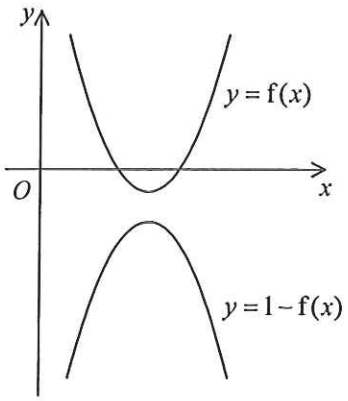
A.



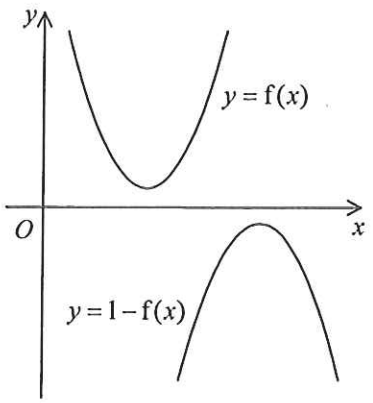
B.



C.



D.



39. For $0° \le x \le 360°$, how many roots does the equation $7\sin^2 x = \sin x$ have?

A. 2

B. 3

C. 4

D. 5

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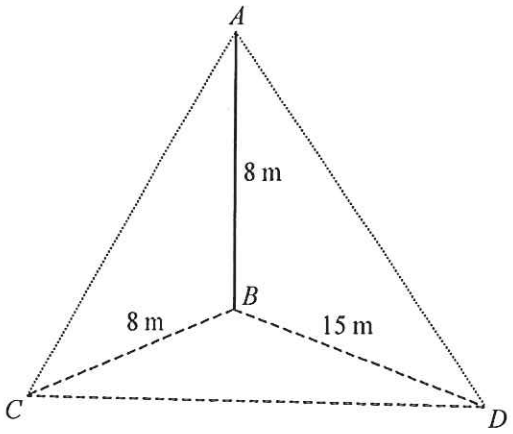
40. In the figure, AB is a vertical pole standing on the horizontal ground BCD, where $\angle CBD = 90°$. If the angle between the plane ACD and the horizontal ground is θ, then $\tan\theta =$

A. $\frac{8}{15}$.

B. $\frac{15}{8}$.

C. $\frac{15}{17}$.

D. $\frac{17}{15}$.



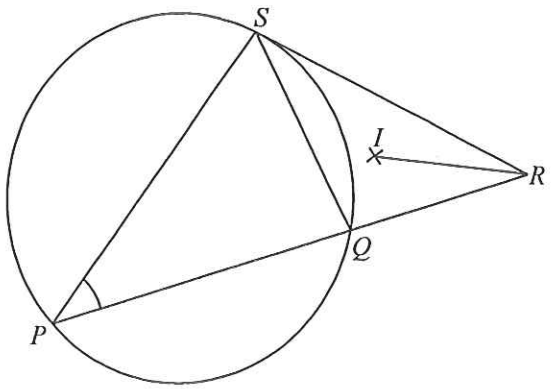
41. In the figure, PQS is a circle. PQ is produced to R such that RS is the tangent to the circle at S. I is the in-centre of ΔQRS. If $\angle IRQ = 12°$ and $\angle PSQ = 70°$, then $\angle QPS =$

A. $24°$.

B. $37°$.

C. $43°$.

D. $62°$.



42. If the straight line $x - y = k$ and the circle $x^2 + y^2 + 2x - 4y - 1 = 0$ intersect at A and B, then the x-coordinate of the mid-point of AB is

A. $1 + k$.

B. $1 - k$.

C. $\frac{1+k}{2}$.

D. $\frac{1-k}{2}$.

43. There are 13 boys and 17 girls in a class. If a team of 2 boys and 3 girls is selected from the class to participate in a voluntary service, how many different teams can be formed?

A. 38 896

B. 53 040

C. 142 506

D. 636 480

44. In an examination, Peter gets 55 marks and his standard score is -3 while Mary gets 95 marks and her standard score is 2. Find the mean of the examination scores.

A. 8 marks

B. 64 marks

C. 75 marks

D. 79 marks

45. If the variance of the four numbers a, b, c and d is 9, then the variance of the four numbers $14-a$, $14-b$, $14-c$ and $14-d$ is

A. 5.

B. 9.

C. 23.

D. 121.

END OF PAPER