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香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2014年香港中學文憑考試
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2014

數學　必修部分　試卷一
MATHEMATICS　COMPULSORY PART　PAPER 1

評卷參考
MARKING SCHEME

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Hong Kong Diploma of Secondary Education Examination
Mathematics Compulsory Part Paper 1

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Solution	Marks	Remarks
1. $\dfrac{(xy^{-2})^3}{y^4}$		
$=\dfrac{x^3y^{-6}}{y^4}$	1M	for $(ab)^m=a^mb^m$ or $(a^m)^n=a^{mn}$
$=\dfrac{x^3}{y^{4-(-6)}}$	1M	for $c^{-p}=\dfrac{1}{c^p}$ or $\dfrac{c^p}{c^q}=c^{p-q}$
$=\dfrac{x^3}{y^{10}}$	1A	
	----------(3)	
2. (a) a^2-2a-3		
$=(a+1)(a-3)$	1A	or equivalent
(b) $ab^2+b^2+a^2-2a-3$		
$=ab^2+b^2+(a+1)(a-3)$	1M	for using the result of (a)
$=b^2(a+1)+(a+1)(a-3)$		
$=(a+1)(b^2+a-3)$	1A	or equivalent
	----------(3)	
3. (a) 200	1A	
(b) 123	1A	
(c) 123.4	1A	
	----------(3)	
4. The median		
$=1$	1A	
The mode		
$=2$	1A	
The standard deviation		
≈ 0.888819441		
≈ 0.889	1A	r.t. 0.889
	----------(3)	

Solution	Marks	Remarks
5. (a) $2(3m+n)=m+7$		
$6m+2n=m+7$	1M	for expanding
$n=\dfrac{7-5m}{2}$	1A	or equivalent
$2(3m+n)=m+7$		
$3m+n=\dfrac{m+7}{2}$	1M	for division
$n=\dfrac{7-5m}{2}$	1A	or equivalent
(b) The decrease in the value of n	1M	
$=5$	1M	
	----------(4)	
6. (a) The selling price of the toy		
$=255(1-40\%)$	1M	
$=\$153$	1A	
(b) Let $\$x$ be the cost of the toy.		
$(1+2\%)x=153$	1M	
$x=150$	1A	
Thus, the cost of the toy is $\$150$.		
	----------(4)	
7. (a) $\mathrm{f}(2)=-33$	1M	
$4(2)^3-5(2)^2-18(2)+c=-33$		
$c=-9$		
$\mathrm{f}(-1)$	1M	
$=4(-1)^3-5(-1)^2-18(-1)-9$		
$=0$		
Thus, $x+1$ is a factor of $\mathrm{f}(x)$.	1A	f.t.
(b) $\mathrm{f}(x)=0$		
$4x^3-5x^2-18x-9=0$		
$(x+1)(4x^2-9x-9)=0$	1M	for $(x+1)(px^2+qx+r)=0$
$(x+1)(x-3)(4x+3)=0$		
$x=-1$, $x=3$ or $x=\dfrac{-3}{4}$		
Note that -1 , 3 and $\dfrac{-3}{4}$ are rational numbers.		
Thus, the claim is agreed.	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
8. (a) The coordinates of P' are $(5, 3)$.	1A	
The coordinates of Q' are $(-19, -7)$.	1A	
(b) The slope of PQ		
$=\dfrac{5+7}{-3-2}$	1M	either one
$=\dfrac{-12}{5}$	1A	either one
The slope of $P'Q'$		
$=\dfrac{3+7}{5+19}$		
$=\dfrac{5}{12}$		
So, the product of the slope of PQ and the slope of $P'Q'$ is -1 .	1	
Thus, PQ is perpendicular to $P'Q'$.		
	----------(5)	
9. (a) In ΔABC and ΔBDC ,		
$\angle BAC = \angle DBC$ (given)		[已知]
$\angle ACB = \angle BCD$ (common $\angle$)		[公共角]
$\angle ABC = \angle BDC$ ($\angle$ sum of Δ)		[Δ內角和]
$\Delta ABC \sim \Delta BDC$ (AAA)		(AA) (equiangular) [等角]

Marking Scheme:	
Case 1 Any correct proof with correct reasons.	2
Case 2 Any correct proof without reasons.	1

Solution	Marks	Remarks
(b) $\dfrac{CD}{BC}=\dfrac{BC}{AC}$	1M	
$\dfrac{CD}{20}=\dfrac{20}{25}$		
$CD = 16$ cm		
$BD^2 + CD^2$	1M	
$= 12^2 + 16^2$		
$= 20^2$		
$= BC^2$		
Thus, ΔBCD is a right-angled triangle.	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
10. (a) The distance of car A from town X at 8:15 in the morning		
$=\frac{45}{120}(80)$	1M	
$=30$ km	1A	
	----------(2)	
(b) Suppose that car A and car B first meet at the time t minutes after 7:30 in the morning.		
$\frac{t}{120}=\frac{44}{80}$	1M	
$t=66$		
Thus, car A and car B first meet at 8:36 in the morning.	1A	
	----------(2)	
(c) During the period 8:15 to 9:30 in the morning, car B travels 36 km while car A travels more than 36 km .	1M	
So, the average speed of car A is higher than that of car B . Thus, the claim is disagreed.	1A	f.t.
The average speed of car A during the period 8:15 to 9:30 in the morning		
$=\frac{80-30}{1.25}$	1M	accept $\frac{80}{2}$
$=\frac{50}{1.25}$		
$=40$ km/h		either one
The average speed of car B during the period 8:15 to 9:30 in the morning		
$=\frac{80-44}{1.25}$		
$=\frac{36}{1.25}$		
$=28.8$ km/h		
Note that $40>28.8$.		
So, the average speed of car A is higher than that of car B .		
Thus, the claim is disagreed.	1A	f.t.
	----------(2)	

Solution	Marks	Remarks
11. (a) The range		
$= 91 - 18$	1M	either one
$= 73$ thousand dollars	1A	
The inter-quartile range		
$= 63 - 42$		
$= 21$ thousand dollars	1A	
	----------(3)	
(b) The mean of the prices of the remaining paintings in the art gallery		
$= \dfrac{(33)(53) - 32 - 34 - 58 - 59}{33 - 4}$	1M	
$= \dfrac{1\,566}{29}$		
$= 54$ thousand dollars	1A	
Note that 32 and 34 are less than 55 .		
Also note that 58 and 59 are greater than 55 .		
The median of the prices of the remaining paintings in the art gallery		
$= 55$ thousand dollars	1A	
	----------(3)	

Solution	Marks	Remarks
12. (a) The radius of C		
$=\sqrt{(6-0)^2+(11-3)^2}$	1M	
$=10$		
Thus, the equation of C is $x^2+(y-3)^2=10^2$.	1A ----------(2)	$x^2+y^2-6y-91=0$
(b) (i) Let (x, y) be the coordinates of P .		
$\sqrt{(x-0)^2+(y-3)^2}=\sqrt{(x-6)^2+(y-11)^2}$	1M	
$3x+4y-37=0$	1A	
Thus, the equation of Γ is $3x+4y-37=0$.		
The slope of AG $=\frac{11-3}{6-0}$ $=\frac{4}{3}$		
Note that the slope of Γ is $\frac{-3}{4}$.	1M	
Also note that the mid-point of AG is $(3, 7)$.		
The equation of Γ is $y-7=\frac{-3}{4}(x-3)$		
$3x+4y-37=0$	1A	
(ii) Γ is the perpendicular bisector of the line segment AG .	1A	
(iii) The perimeter of the quadrilateral $AQGR$		
$=4(10)$	1M	
$=40$	1A ----------(5)	

Solution	Marks	Remarks
13. (a) Let $f(x) = px^2 + q$, where p and q are non-zero constants.	1A	
So, we have $4p + q = 59$ and $49p + q = -121$.	1M	for either substitution
Solving, we have $p = -4$ and $q = 75$.	1A	for both correct
Therefore, we have $f(x) = 75 - 4x^2$.		
Thus, we have $f(6) = -69$.	1A	
	----------(4)	
(b) By (a), we have $a = -69$.	1M	either one
Since $f(x) = 75 - 4x^2$, we have $f(-6) = f(6)$.		
So, we have $b = -69$.		
AB		
$= 6 - (-6)$	1M	can be absorbed
$= 12$		
The area of ΔABC		
$= \dfrac{(12)(69)}{2}$	1M	
$= 414$	1A	
	----------(4)	

Solution	Marks	Remarks
14. (a) The slant height of the circular cone		
$=\sqrt{72^2+96^2}$	1M	
$=120$ cm		
The area of the wet curved surface of the vessel		
$=\pi(72)(120)\dfrac{(96-60+28)^2-(96-60)^2}{96^2}$	1M+1M	
$=\pi(72)(120)\dfrac{64^2-36^2}{96^2}$		
$=2\,625\pi$ cm^2	1A	
Let R cm be the radius of the water surface.		
Then, we have $\dfrac{R}{72}=\dfrac{96-60+28}{96}$.	1M	either one
Therefore, we have $\dfrac{R}{72}=\dfrac{64}{96}$.		
So, we have $R=48$.		
Let r cm be the base radius of the lower part of the inverted right circular cone.		
Then, we have $\dfrac{r}{72}=\dfrac{96-60}{96}$.		
Therefore, we have $\dfrac{r}{72}=\dfrac{36}{96}$.		
So, we have $r=27$.		
The area of the wet curved surface of the vessel		
$=\pi(48)\sqrt{48^2+64^2}-\pi(27)\sqrt{27^2+36^2}$	1M+1M	
$=\pi(48)(80)-\pi(27)(45)$		
$=2\,625\pi$ cm^2	1A	
	----------(4)	
(b) The volume of the circular cone		
$=\dfrac{1}{3}\pi(72)^2(96)$	1M	
$=165\,888\pi$ cm^3		
The volume of water in the vessel		
$=165\,888\pi\left(\dfrac{64^3-36^3}{96^3}\right)$	1M+1A	
$=40\,404\pi$ cm^3		
≈ 0.126932909 m^3		
>0.1 m^3		
Thus, the claim is agreed.	1A	f.t.
The volume of water in the vessel		
$=\dfrac{1}{3}\pi(48)^2(64)-\dfrac{1}{3}\pi(27)^2(36)$	1M+1M+1A	
$=49\,152\pi-8\,748\pi$		
$=40\,404\pi$ cm^3		
≈ 0.126932909 m^3		
>0.1 m^3		
Thus, the claim is agreed.	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
15. $\log_8 y - 0 = \frac{-1}{3}(\log_4 x - 3)$	1M	
$\log_8 y = \frac{-1}{3}\log_4 x + 1$		
$\log_8 y = \log_4 x^{\frac{-1}{3}} + \log_4 4$		
$\log_8 y = \log_4 4x^{\frac{-1}{3}}$		
$\frac{\log_2 y}{\log_2 8} = \frac{\log_2 4x^{\frac{-1}{3}}}{\log_2 4}$	1M	
$\log_2 y = \frac{3}{2}\log_2 4x^{\frac{-1}{3}}$		
$\log_2 y = \log_2 8x^{\frac{-1}{2}}$		
$y = 8x^{\frac{-1}{2}}$	1A	
$\log_8 y - 0 = \frac{-1}{3}(\log_4 x - 3)$	1M	
$\log_8 y = \frac{-1}{3}\log_4 x + 1$		
$\log_8 y = \log_4 x^{\frac{-1}{3}} + \log_4 4$		
$\log_8 y = \log_4 4x^{\frac{-1}{3}}$		
$y = 8^{\log_4 4x^{\frac{-1}{3}}}$		
$y = 4^{\frac{3}{2}\log_4 4x^{\frac{-1}{3}}}$	1M	
$y = 4^{\log_4 8x^{\frac{-1}{2}}}$		
$y = 8x^{\frac{-1}{2}}$	1A	
	----------(3)	
16. Note that the numbers of dots in the patterns form an arithmetic sequence.		
The total number of dots in the first m patterns		
$= 3 + 5 + 7 + \cdots + (2m+1)$		
$= \frac{m}{2}(3 + (2m+1))$	1M+1A	accept $\frac{m}{2}((2)(3) + (m-1)(2))$
$= m^2 + 2m$		
$m^2 + 2m > 6\,888$		
$m^2 + 2m - 6\,888 > 0$	1M	
$(m-82)(m+84) > 0$		
$m < -84$ or $m > 82$		
Thus, the least value of m is 83 .	1A	
	----------(4)	

Solution	Marks	Remarks
17. (a) By sine formula, we have		
$\dfrac{\sin\angle AVB}{AB}=\dfrac{\sin\angle VAB}{VB}$	1M	
$\dfrac{\sin\angle AVB}{18}=\dfrac{\sin 110^\circ}{30}$		
$\angle AVB \approx 34.32008291^\circ$		
$\angle VBA \approx 180^\circ-110^\circ-34.32008291^\circ$		
$\angle VBA \approx 35.67991709^\circ$		
$\angle VBA \approx 35.7^\circ$	1A ----------(2)	r.t. 35.7°
(b) By cosine formula, we have		
$MP^2=BP^2+BM^2-2(BP)(BM)\cos\angle VBA$	1M	
$MP^2\approx 9^2+15^2-2(9)(15)\cos 35.67991709^\circ$		
$MP\approx 9.310329519$ cm		
$MN=\dfrac{BC}{2}$	1M	
$MN=5$ cm		
Note that $MP=NQ$.		
Let h cm be the height of the trapezium $PQNM$.		
$h=\sqrt{MP^2-\left(\dfrac{PQ-MN}{2}\right)^2}$	1M	
$h\approx\sqrt{9.310329519^2-\left(\dfrac{10-5}{2}\right)^2}$		
$h\approx 8.968402074$		
The area of the trapezium $PQNM$		
$=\dfrac{h(MN+PQ)}{2}$	1M	
$\approx\dfrac{(8.968402074)(5+10)}{2}$		
$\approx 67.26301555\text{ cm}^2$		
$<70\text{ cm}^2$		
Thus, the claim is agreed.	1A	f.t.

Solution	Marks	Remarks
By cosine formula, we have $MP^2 = BP^2 + BM^2 - 2(BP)(BM)\cos\angle VBA$ $MP^2 \approx 9^2 + 15^2 - 2(9)(15)\cos 35.67991709°$ $MP \approx 9.310329519$ cm	1M	
$MN = \dfrac{BC}{2}$ $MN = 5$ cm	1M	
$\cos\angle MPQ = \dfrac{\frac{PQ - MN}{2}}{PM}$ $\cos\angle MPQ \approx \dfrac{\frac{10-5}{2}}{9.310329519}$ $\angle MPQ \approx 74.42384466°$	1M	
Note that $MP = NQ$. Let h cm be the height of the trapezium $PQNM$. $\dfrac{h}{MP} = \sin\angle MPQ$ $\dfrac{h}{9.310329519} \approx \sin 74.42384466°$ $h \approx 8.968402074$		
The area of the trapezium $PQNM$ $= h(MN) + \dfrac{1}{2}(MP)(BC - MN)\sin\angle MPQ$ $\approx (8.968402074)(5) + \dfrac{1}{2}(9.310329519)(10-5)\sin 74.42384466°$ $\approx 67.26301555 \text{ cm}^2$ $< 70 \text{ cm}^2$ Thus, the claim is agreed.	1M 1A	 f.t.
	----------(5)	

Solution	Marks	Remarks
18. (a) The slope of L_2 $=\frac{90-0}{45-180}$ $=\frac{-2}{3}$		
The equation of L_2 is $y-90=\frac{-2}{3}(x-45)$	1M	
$2x+3y-360=0$	1A	
Thus, the system of inequalities is $\begin{cases}6x+7y\le 900\\2x+3y\le 360\\x\ge 0\\y\ge 0\end{cases}$.	1M+1A	or equivalent
	----------(4)	
(b) Let x and y be the numbers of wardrobes X and Y produced that month respectively. Now, the constraints are $6x+7y\le 900$ and $2x+3y\le 360$, where x and y are non-negative integers. Denote the total profit on the production of wardrobes by $\$P$.		
Then, we have $P=440x+665y$.	1A	
Note that the vertices of the shaded region in Figure 7 are the points $(0,0)$, $(0,120)$, $(45,90)$ and $(150,0)$.		
At the point $(0,0)$, we have $P=(440)(0)+(665)(0)=0$.	1M+1M	1M for testing a point+
At the point $(0,120)$, we have $P=(440)(0)+(665)(120)=79\ 800$.		1M for testing all points
At the point $(45,90)$, we have $P=(440)(45)+(665)(90)=79\ 650$.		
At the point $(150,0)$, we have $P=(440)(150)+(665)(0)=66\ 000$.		
So, the greatest possible profit is $\$79\ 800$.		
Thus, the claim is disagreed.	1A	f.t.
Let x and y be the numbers of wardrobes X and Y produced that month respectively. Now, the constraints are $6x+7y\le 900$ and $2x+3y\le 360$, where x and y are non-negative integers. Denote the total profit on the production of wardrobes by $\$P$.		
Then, we have $P=440x+665y$.	1A	
Draw the straight line $88x+133y=k$ on Figure 7, where k is a constant.	1M+1M	1M for sliding straight line+ 1M for straight line with negative slope
It is found that P attains its greatest value at the point $(0,120)$.		
So, the greatest value of P is $\$79\ 800$.		
Thus, the claim is disagreed.	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
19. (a) The required probability		
$=\frac{1}{6}+\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)+\cdots$	1M	
$=\frac{1}{6}+\left(\frac{1}{6}\right)\left(\frac{25}{36}\right)+\left(\frac{1}{6}\right)\left(\frac{25}{36}\right)^2+\cdots$		
$=\dfrac{\frac{1}{6}}{1-\frac{25}{36}}$	1M	
$=\frac{6}{11}$	1A	r.t. 0.545
Let p be the probability that Ada wins the first round of the game. Then, the probability that Billy wins the first round of the game is $\frac{5p}{6}$.	1M	
$p+\frac{5p}{6}=1$	1M	
$\frac{11p}{6}=1$		
$p=\frac{6}{11}$	1A	r.t. 0.545
Thus, the required probability is $\frac{6}{11}$.		
	----------(3)	
(b) (i) Suppose that the player of the second round adopts Option 1.		
The probability of getting 10 tokens		
$=(1)\left(\frac{1}{8}\right)$	1M	accept $\frac{8}{8^2}$
$=\frac{1}{8}$		
The probability of getting 5 tokens		
$=\frac{(7)(P_2^2)}{8^2}$		
$=\frac{7}{32}$	1A	can be absorbed
The expected number of tokens got		
$=(10)\left(\frac{1}{8}\right)+(5)\left(\frac{7}{32}\right)$	1M	
$=\frac{75}{32}$	1A	r.t. 2.34

Solution	Marks	Remarks
(ii) Suppose that the player of the second round adopts Option 2.		
The probability of getting 50 tokens $=(1)\left(\frac{1}{8}\right)\left(\frac{1}{8}\right)$ $=\frac{1}{64}$		
The probability of getting 10 tokens $=\frac{(6)(P_3^3)}{8^3}$ $=\frac{9}{128}$		
The probability of getting 5 tokens $=(2)\left(\frac{1}{8}\right)^2\left(\frac{1}{8}\right)+(6)\left(\frac{1}{8}\right)^2\left(\frac{2}{8}\right)+\left(\frac{7}{32}\right)\left(\frac{2}{8}\right)$ $=\frac{21}{256}$	1M	accept $\frac{(7)(2)(C_2^3)}{8^3}$
The expected number of tokens got $=(50)\left(\frac{1}{64}\right)+(10)\left(\frac{9}{128}\right)+(5)\left(\frac{21}{256}\right)$ $=\frac{485}{256}$		
Note that $\frac{75}{32}>\frac{485}{256}$.	1M	
Thus, the player of the second round should adopt Option 1.	1A	f.t.
(iii) The probability of Ada getting no tokens $=1-\left(\frac{6}{11}\right)\left(\frac{1}{8}+\frac{7}{32}\right)$ $=\frac{13}{16}$ $=0.8125$ <0.9	1M+1M	$\begin{cases}\text{1M for } 1-\text{(a)}p_1 \\ +\text{1M for } p_1=p_2+p_3\end{cases}$
Thus, the claim is incorrect.	1A	f.t.
The probability of Ada getting no tokens $=\left(\frac{6}{11}\right)\left(1-\frac{1}{8}-\frac{7}{32}\right)+\frac{5}{11}$ $=\frac{13}{16}$ $=0.8125$ <0.9	1M+1M	$\begin{cases}\text{1M for } \text{(a)}p_4+1-\text{(a)} \\ +\text{1M for } p_4=1-p_5-p_6\end{cases}$
Thus, the claim is incorrect.	1A	f.t.
	---------(10)	