Paper 1

Solution	Marks	Remarks
1. $\dfrac{(x^8y^7)^2}{x^5y^{-6}}$		
$=\dfrac{x^{16}y^{14}}{x^5y^{-6}}$	1M	for $(ab)^m=a^mb^m$ or $(a^m)^n=a^{mn}$
$=x^{16-5}y^{14-(-6)}$	1M	for $\dfrac{c^p}{c^q}=c^{p-q}$ or $\dfrac{c^p}{c^q}=\dfrac{1}{c^{q-p}}$
$=x^{11}y^{20}$	1A	
	----------(3)	
2. $Ax=(4x+B)C$		
$Ax=4Cx+BC$	1M	
$Ax-4Cx=BC$	1M	for putting x on one side
$(A-4C)x=BC$		
$x=\dfrac{BC}{A-4C}$	1A	or equivalent
$Ax=(4x+B)C$		
$\dfrac{A}{C}x=4x+B$	1M	
$\dfrac{A}{C}x-4x=B$	1M	for putting x on one side
$\left(\dfrac{A}{C}-4\right)x=B$		
$\left(\dfrac{A-4C}{C}\right)x=B$		
$x=\dfrac{BC}{A-4C}$	1A	or equivalent
	----------(3)	
3. $\dfrac{2}{4x-5}+\dfrac{3}{1-6x}$		
$=\dfrac{2(1-6x)+3(4x-5)}{(4x-5)(1-6x)}$	1M	
$=\dfrac{2-12x+12x-15}{(4x-5)(1-6x)}$	1M	
$=\dfrac{-13}{(4x-5)(1-6x)}$	1A	or equivalent
$=\dfrac{13}{(4x-5)(6x-1)}$		
	----------(3)	

Solution	Marks	Remarks
4. (a) $5m-10n$		
$=5(m-2n)$	1A	
(b) $m^2+mn-6n^2$		
$=(m+3n)(m-2n)$	1A	
(c) $m^2+mn-6n^2-5m+10n$		
$=m^2+mn-6n^2-(5m-10n)$		
$=(m+3n)(m-2n)-5(m-2n)$	1M	for using the results of (a) and (b)
$=(m-2n)(m+3n-5)$	1A	or equivalent
	----------(4)	
5. Let x and y be the number of male members and the number of female members respectively.		
$\begin{cases} x+y=180 \\ x=(1+40\%)y \end{cases}$	1A+1A	
So, we have $1.4y+y=180$.	1M	for getting a linear equation in x or y only
Solving, we have $y=75$ and $x=105$.		
Thus, the difference of the number of male members and the number of female members is 30 .	1A	
Let x be the number of male members. $x=(1+40\%)(180-x)$	1A+1A+1M	1A for $x=(1+40\%)y$ + 1A for $y=180-x$ + 1M for a linear equation in one unknown
Solving, we have $x=105$. Note that $105-(180-105)=30$.		
Thus, the difference of the number of male members and the number of female members is 30 .	1A	
The difference of the number of male members and the number of female members $=\dfrac{(180)(40\%)}{100\%+(100\%+40\%)}$	1A+1A+1M	1A for numerator + 1A for denominator + 1M for fraction
$=30$	1A	
Let d be the difference of the number of male members and number of female members. $\dfrac{180+d}{2}=\left(\dfrac{180-d}{2}\right)(1+40\%)$	1A+1A+1M	1A for $\dfrac{180+d}{2}$ or $\dfrac{180-d}{2}$ + 1A for $\left(\dfrac{180-d}{2}\right)(1+40\%)$ + 1M for a linear equation in one unknown
$d=30$ Thus, the difference of the number of male members and the number of female members is 30 .	1A	
	----------(4)	

Solution	Marks	Remarks
6. (a) $x+6<6(x+11)$		
$x+6<6x+66$		
$x-6x<66-6$	1M	for putting x on one side
$-5x<60$		
$x>-12$	1A	
Therefore, we have $x>-12$ or $x\le -5$.		
Thus, the solutions of (*) are all real numbers.	1A	
(b) -1	1A ----------(4)	
7. (a) $\angle AOB$		
$=135^\circ-75^\circ$		
$=60^\circ$	1A	
(b) Since $AO=BO$, we have $\angle OAB=\angle OBA$.		
Note that $\angle OAB+\angle OBA+60^\circ=180^\circ$.		
Therefore, we have $\angle OAB=\angle OBA=60^\circ$.		
So, ΔAOB is an equilateral triangle.	1M	can be absorbed
The perimeter of ΔAOB		
$=3(12)$		
$=36$	1A	
(c) 3	1A ----------(4)	
8. (a) Let $\mathrm{f}(x)=hx+kx^2$, where h and k are non-zero constants.	1A	
So, we have $3h+9k=48$ and $9h+81k=198$.	1M	for either substitution
Solving, we have $h=13$ and $k=1$.	1A	
Thus, we have $\mathrm{f}(x)=13x+x^2$.		
(b) $\mathrm{f}(x)=90$		
$13x+x^2=90$		
$x^2+13x-90=0$	1M	
$(x-5)(x+18)=0$		
$x=5$ or $x=-18$	1A ----------(5)	

Solution	Marks	Remarks
9. (a) x		
$= 2 + 4$		
$= 6$	1A	
y		
$= 37 - 15$		
$= 22$	1A	
z		
$= 37 + 3$		
$= 40$	1A	
(b) The required probability		
$= \frac{22-6}{40}$	1M	for $\frac{y-x}{z}$
$= \frac{2}{5}$	1A	0.4
Note that $b = 7$ and $c = 9$.		
The required probability		
$= \frac{7+9}{40}$	1M	for $\frac{b+c}{z}$
$= \frac{2}{5}$	1A	0.4
Note that $a = 2$.		
The required probability		
$= \frac{40-2-4-15-3}{40}$	1M	for $\frac{z-a-4-15-3}{z}$
$= \frac{2}{5}$	1A	0.4
	----------(5)	

Solution	Marks	Remarks
10. (a) Let (x, y) be the coordinates of P.		
$\sqrt{(x-5)^2+(y-7)^2}=\sqrt{(x-13)^2+(y-1)^2}$	1M	
$4x-3y-24=0$	1A	or equivalent
Thus, the equation of Γ is $4x-3y-24=0$.		
The slope of AB $=\frac{7-1}{5-13}$ $=\frac{-3}{4}$ The slope of Γ $=\frac{4}{3}$ The mid-point of AB $=\left(\frac{5+13}{2},\frac{7+1}{2}\right)$ $=(9,4)$		
Therefore, the equation of Γ is $y-4=\frac{4}{3}(x-9)$.	1M	
Thus, the equation of Γ is $4x-3y-24=0$.	1A	or equivalent
	----------(2)	
(b) Putting $y=0$ in $4x-3y-24=0$, we have $x=6$. So, the coordinates of H are $(6, 0)$. Putting $x=0$ in $4x-3y-24=0$, we have $y=-8$. Therefore, the coordinates of K are $(0,-8)$.	1M	either one
The diameter of C $=HK$ $=\sqrt{(6-0)^2+(0-(-8))^2}$ $=10$		
The circumference of C $=10\pi$	1M	
≈ 31.41592654 >30		
Thus, the claim is correct.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
11. (a) Let V cm^3 be the final volume of milk in the vessel.		
$\frac{V-444\pi}{V}=\left(\frac{12}{16}\right)^3$	1M+1A	1M for $\left(\frac{12}{16}\right)^3$
$V=768\pi$	1A	
Thus, the final volume of milk in the vessel is 768π cm^3 .		
Let V cm^3 and r cm be the final volume of milk and the final radius of the surface of milk in the vessel respectively.		
$V=\frac{1}{3}\pi r^2(16)$		
$V-444\pi=\frac{1}{3}\pi\left(\frac{12r}{16}\right)^2(12)$		
So, we have $V-444\pi=\frac{1}{3}\pi\left(\frac{12}{16}\right)^2\left(\frac{3V}{16\pi}\right)(12)$.	1M+1A	1M for eliminating r^2
Solving, we have $V=768\pi$.	1A	
Thus, the final volume of milk in the vessel is 768π cm^3 .		
	----------(3)	
(b) Let r cm be the final radius of the surface of milk in the vessel.		
$\frac{1}{3}\pi r^2(16)=768\pi$	1M	
$r=12$		
The final area of the wet curved surface of the vessel		
$=\pi(12)\sqrt{12^2+16^2}$	1M	
$=240\pi$		
≈ 753.9822369 cm^2		
<800 cm^2		
Thus, the claim is disagreed.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
12. (a) $11+a=11+b+4$	1M	
$a=b+4$		
Note that $a>11$ and $4<b<10$.		
Thus, we have $\begin{cases}a=12\\b=8\end{cases}$ or $\begin{cases}a=13\\b=9\end{cases}$.	1A+1A	1A for one pair + 1A for all
	----------(3)	
(b) (i) The median is the greatest when the ages of these four children are 7 , 8 , 9 and 10 .	1M	
The greatest possible median of the ages of the children in the group $=8$	1A	
(ii) The mean is the least when the ages of these four children are 6 , 7 , 8 and 9 .	1M	
By (a), there are two cases.		
Case 1: $a=12$ and $b=8$		
The mean of the ages of the children in the group $=\frac{12(6)+13(7)+12(8)+9(9)+4(10)}{12+13+12+9+4}$ $=7.6$		
Case 2: $a=13$ and $b=9$		
The mean of the ages of the children in the group $=\frac{12(6)+14(7)+12(8)+10(9)+4(10)}{12+14+12+10+4}$ ≈ 7.615384615		
Thus, the least possible mean of the ages of the children in the group is 7.6 .	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
13. (a) In ΔACD and ΔABE ,		
$\angle ADC = \angle AEB$ (given)		
$AD = AE$ (sides opp. equal $\angle$s)		
$CE = BD$ (given)		
$CE + DE = BD + DE$		
$CD = BE$		
$\Delta ACD \cong \Delta ABE$ (SAS)		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
	----------(2)	
(b) (i) Note that $DM = EM = 9\text{ cm}$ and $\angle AMD = \angle AME = 90°$.		
AM		
$= \sqrt{AD^2 - DM^2}$	1M	
$= \sqrt{15^2 - 9^2}$		
$= \sqrt{144}$		
$= 12\text{ cm}$	1A	
(ii) AB^2		
$= AM^2 + BM^2$		
$= 144 + (7+9)^2$		
$= 400$		
By (a), we have $AE = AD = 15\text{ cm}$.	1M	
$AB^2 + AE^2$		
$= 400 + 15^2$		
$= 625$		
$= (7+18)^2$		
$= (BD + DE)^2$		
$= BE^2$	1M	
Thus, ΔABE is a right-angled triangle.	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
14. (a) Note that $p(2)=152+4a+2b+c$ and $p(-2)=40+4a-2b+c$.		
Since $p(2)=p(-2)$, we have $b=-28$.	1M	
By comparing the coefficients of x^4 , we have $l=3$.	1A	
Note that the coefficients of x^3 and x in the expansion of $(3x^2+5x+8)(2x^2+mx+n)$ are $3m+10$ and $8m+5n$ respectively.		
So, we have $3m+10=7$ and $8m+5n=-28$.	1M	
Solving, we have $m=-1$ and $n=-4$.	1A+1A	
	----------(5)	
(b) $p(x)=0$		
$(3x^2+5x+8)(2x^2-x-4)=0$ (by (a))		
$3x^2+5x+8=0$ or $2x^2-x-4=0$		
$5^2-4(3)(8)$	1M	either one
$=-71$	1A	either one
<0		
So, the quadratic equation $3x^2+5x+8=0$ does not have real roots.	1M+1A	either one
$(-1)^2-4(2)(-4)$		
$=33$		
>0		
Therefore, the quadratic equation $2x^2-x-4=0$ has 2 real roots.		
Hence, the equation $(3x^2+5x+8)(2x^2-x-4)=0$ has 2 real roots.		
Thus, the equation $p(x)=0$ has 2 real roots.	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
15. The required probability		
$=\dfrac{C_4^6 4!5!}{(4+5)!}$	1M+1M	1M for denominator + 1M for $4!5!$
$=\dfrac{43\,200}{362\,880}$		
$=\dfrac{5}{42}$	1A	r.t. 0.119
The required probability		
$=\dfrac{4!5!+4!5!(4)(2)+4!5!(3)+4!5!(3)}{(4+5)!}$	1M+1M	1M for denominator + 1M for $4!5!$
$=\dfrac{43\,200}{362\,880}$		
$=\dfrac{5}{42}$	1A	r.t. 0.119
The required probability		
$=\left(\dfrac{4}{4}\right)\left(\dfrac{3}{3}\right)\left(\dfrac{2}{2}\right)\left(\dfrac{1}{1}\right)\left(\dfrac{5}{9}\right)\left(\dfrac{4}{8}\right)\left(\dfrac{3}{7}\right)\left(\dfrac{2}{6}\right)\left(\dfrac{1}{5}\right)(1+(4)(2)+3+3)$	1M+1M	1M for denominator + 1M for $(4)(3)(2)(1)(5)(4)(3)(2)(1)$
$=\dfrac{43\,200}{362\,880}$		
$=\dfrac{5}{42}$	1A	r.t. 0.119
	----------(3)	
16. Let σ marks be the standard deviation of the distribution.		
$\dfrac{22-61}{\sigma}=-2.6$	1M	either one
$\sigma=15$		
The score of Mary		
$=61+1.4\sigma$		
$=61+1.4(15)$		
$=82$ marks	1A	
The difference of the score of Mary and the score of Albert		
$=82-22$		
$=60$ marks		
>59 marks		
Note that the range of the distribution is at least the difference of the score of Mary and the score of Albert. Therefore, the range of the distribution exceeds 59 marks. Thus, the claim is incorrect.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
17. (a) Let d be the common difference of the sequence.		
$555 = 666 + (38-1)d$	1M	
$d = -3$	1A	
The common difference of the sequence		
$= \dfrac{555-666}{38-1}$	1M	
$= -3$	1A	
	----------(2)	
(b) $\dfrac{n}{2}(2(666)+(n-1)(-3)) > 0$	1M+1A	
$1\,335n - 3n^2 > 0$		
$n(n-445) < 0$		
$0 < n < 445$		
Thus, the greatest value of n is 444 .	1A	
	----------(3)	
18. (a) $\mathrm{f}(x)$		
$= \dfrac{-1}{3}x^2 + 12x - 121$		
$= \dfrac{-1}{3}(x^2 - 36x) - 121$		
$= \dfrac{-1}{3}(x^2 - 36x + 18^2 - 18^2) - 121$	1M	
$= \dfrac{-1}{3}(x-18)^2 - 13$		
Thus, the coordinates of the vertex are $(18, -13)$.	1A	
	----------(2)	
(b) $\mathrm{g}(x)$		
$= \mathrm{f}(x) + 13$	1M	
$= \dfrac{-1}{3}(x-18)^2$	1A	accept $\dfrac{-1}{3}x^2 + 12x - 108$
	----------(2)	
(c) Note that $\dfrac{-1}{3}x^2 - 12x - 121 = \mathrm{f}(-x)$.		
Thus, the transformation is the reflection with respect to the y-axis.	1A+1A	1A for reflection + 1A for all correct
Note that $\dfrac{-1}{3}x^2 - 12x - 121 = \mathrm{f}(x+36)$.		
Thus, the transformation is the leftward translation of 36 units.	1A+1A	1A for translation + 1A for all correct
	----------(2)	

Solution	Marks	Remarks
19. (a) By sine formula,		
$\dfrac{AB}{\sin\angle ADB}=\dfrac{BD}{\sin\angle BAD}$	1M	
$\dfrac{10}{\sin\angle ADB}=\dfrac{15}{\sin 86°}$		
$\angle ADB\approx 41.68560132°$ or $\angle ADB\approx 138.3143987°$ (rejected)		
$\angle ABD=180°-\angle BAD-\angle ADB$		
$\angle ABD\approx 52.31439868°$		
$\angle ABD\approx 52.3°$	1A	r.t. 52.3°
By cosine formula,		
$CD^2=BC^2+BD^2-2(BC)(BD)\cos\angle CBD$	1M	
$CD^2\approx 8^2+15^2-2(8)(15)\cos 43°$		
$CD\approx 10.65246974$		
$CD\approx 10.7$ cm	1A	r.t. 10.7 cm
	----------(4)	
(b) Since $AC^2+BC^2=AB^2$, we have $\angle ACB=90°$.		
By cosine formula,		
$AD^2=AB^2+BD^2-2(AB)(BD)\cos\angle ABD$		
$AD^2\approx 10^2+15^2-2(10)(15)\cos 52.31439868°$		
$AD\approx 11.89964475$		
By cosine formula,		
$AD^2=AC^2+CD^2-2(AC)(CD)\cos\angle ACD$		
$\cos\angle ACD\approx\dfrac{6^2+(10.65246974)^2-(11.89964475)^2}{2(6)(10.65246794)}$		
$\angle ACD\approx 86.46867599°$		
So, $\angle ACD$ is not a right angle.	1M	
Hence, the angle between AB and the face BCD is not $\angle ABC$.		
Thus, the claim is disagreed.	1A	f.t.
Since $AC^2+BC^2=AB^2$, we have $\angle ACB=90°$.		
By cosine formula,		
$AD^2=AB^2+BD^2-2(AB)(BD)\cos\angle ABD$		
$AD^2\approx 10^2+15^2-2(10)(15)\cos 52.31439868°$		
$AD^2\approx 141.6015451$		
$AC^2+CD^2\approx 6^2+(10.65246974)^2$		
$AC^2+CD^2\approx 149.4751116$		
Hence, we have $AD^2\neq AC^2+CD^2$.		
So, $\angle ACD$ is not a right angle.	1M	
Hence, the angle between AB and the face BCD is not $\angle ABC$.		
Thus, the claim is disagreed.	1A	f.t.
	----------(2)	

Solution	Marks	Remarks

20. (a) Note that J is the centre of the circle OPQ.

$\angle IPO = \angle IPQ$ (in-centre of Δ)

Also note that P, I and J are collinear.

$\angle JPO = \angle JPQ$

$JO = JP$ (radii)

$\angle JOP = \angle JPO$ (base $\angle$s, isos. Δ)

$JP = JQ$ (radii)

$\angle JPQ = \angle JQP$ (base $\angle$s, isos. Δ)

$\angle JOP = \angle JQP$

$JP = JP$ (common side)

$\Delta JOP \cong \Delta JQP$ (AAS)

Thus, we have $OP = PQ$. (corr. sides, $\cong\Delta$s)

Note that J is the centre of the circle OPQ.

$\angle IPO = \angle IPQ$ (in-centre of Δ)

Also note that P, I and J are collinear.

$\angle JPO = \angle JPQ$

$JP = JQ$ (radii)

$\angle JQP = \angle JPQ$ (base $\angle$s, isos. Δ)

$\quad = \angle JPO$

$2\angle POQ = \angle PJQ$ ($\angle$ at centre twice $\angle$ at circumference)

$\quad = 180° - \angle JPQ - \angle JQP$ ($\angle$ sum of Δ)

$\quad = 180° - \angle JPQ - \angle JPO$

$\quad = \angle POQ + \angle OQP$ ($\angle$ sum of Δ)

$\angle POQ = \angle OQP$

Thus, we have $OP = PQ$. (sides opp. equal $\angle$s)

Note that J is the centre of the circle OPQ.

$\angle IPO = \angle IPQ$ (in-centre of Δ)

Also note that P, I and J are collinear.

$\angle JPO = \angle JPQ$

$JO = JP$ (radii)

$\angle JOP = \angle JPO$ (base $\angle$s, isos. Δ)

$JP = JQ$ (radii)

$\angle JPQ = \angle JQP$ (base $\angle$s, isos. Δ)

$\angle JOP = \angle JQP$

$JO = JQ$ (radii)

$\angle JOQ = \angle JQO$ (base $\angle$s, isos. Δ)

$\angle JOP - \angle JOQ = \angle JQP - \angle JQO$

$\angle POQ = \angle OQP$

Thus, we have $OP = PQ$. (sides opp. equal $\angle$s)

Marking Scheme:		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step and one correct reason.	1

----------(3)

Solution	Marks	Remarks
(b) (i) Let $(h, 19)$ be the coordinates of P.		
By (a), we have $h^2+19^2=(40-h)^2+(30-19)^2$.	1M	
Solving, we have $h=17$.		
Let $x^2+y^2+Dx+Ey+F=0$ be the equation of C.		
Since C passes through the origin, we have $F=0$.	1A	
So, we have $17D+19E+650=0$ and $40D+30E+2\,500=0$.	1M	for either one
Solving, we have $D=-112$ and $E=66$.	1A	
Thus, the equation of C is $x^2+y^2-112x+66y=0$.		$(x-56)^2+(y+33)^2=65^2$
(ii) Note that the equations of L_1 and L_2 are in the form $y=\frac{3}{4}x+c$, where c is a constant.		
Putting $y=\frac{3}{4}x+c$ in $x^2+y^2-112x+66y=0$, we have		
$x^2+\left(\frac{3}{4}x+c\right)^2-112x+66\left(\frac{3}{4}x+c\right)=0$.	1M	
$25x^2+(24c-1\,000)x+16c^2+1\,056c=0$		
Since L_1 and L_2 are tangents to C, we have		
$(24c-1\,000)^2-4(25)(16c^2+1\,056c)=0$.	1M	
$16c^2+2\,400c-15\,625=0$		
$(4c-25)(4c+625)=0$		
$c=\frac{25}{4}$ or $c=\frac{-625}{4}$		
Therefore, the equations of L_1 and L_2 are		
$y=\frac{3}{4}x+\frac{25}{4}$ and $y=\frac{3}{4}x-\frac{625}{4}$ respectively.	1M	for either one
Note that the coordinates of S, T, U and V are $\left(\frac{-25}{3},0\right)$, $\left(0,\frac{25}{4}\right)$, $\left(\frac{625}{3},0\right)$ and $\left(0,\frac{-625}{4}\right)$ respectively.		
The area of the trapezium $STUV$		
$=\frac{1}{2}\left(\left(\frac{625}{3}\right)\left(\frac{625}{4}\right)+\left(\frac{625}{4}\right)\left(\frac{25}{3}\right)+\left(\frac{25}{3}\right)\left(\frac{25}{4}\right)+\left(\frac{25}{4}\right)\left(\frac{625}{3}\right)\right)$	1M	$\frac{2(65)}{2}\left(\sqrt{\left(\frac{625}{3}\right)^2+\left(\frac{-625}{4}\right)^2}+\sqrt{\left(\frac{-25}{3}\right)^2+\left(\frac{25}{4}\right)^2}\right)$
$=\frac{105\,625}{6}$		
$\approx 17\,604.16667$		
$>17\,000$		
Thus, the claim is correct.	1A ---------(9)	f.t.

Paper 2

Question No.	Key	Question No.	Key
1.	A (47)	26.	B (37)
2.	A (81)	27.	C (56)
3.	D (65)	28.	C (58)
4.	C (87)	29.	B (69)
5.	A (80)	30.	B (76)
6.	B (76)	31.	C (61)
7.	A (62)	32.	D (40)
8.	C (82)	33.	A (43)
9.	D (46)	34.	B (38)
10.	C (69)	35.	D (47)
11.	D (81)	36.	B (35)
12.	D (67)	37.	A (46)
13.	A (81)	38.	B (49)
14.	C (92)	39.	A (35)
15.	B (45)	40.	D (38)
16.	D (80)	41.	C (45)
17.	A (55)	42.	A (55)
18.	C (79)	43.	D (51)
19.	A (59)	44.	B (52)
20.	C (51)	45.	C (50)
21.	B (57)		
22.	D (54)		
23.	A (82)		
24.	B (64)		
25.	D (35)		

Note: Figures in brackets indicate the percentages of candidates choosing the correct answers.