香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2020年香港中學文憑考試
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2020

數學　　必修部分　　試卷一
MATHEMATICS　COMPULSORY PART　PAPER 1

評卷參考
MARKING SCHEME

Hong Kong Diploma of Secondary Education Examination
Mathematics Compulsory Part Paper 1

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles . All fractional answers must be simplified.

Solution	Marks	Remarks
1. $\frac{(mn^{-2})^5}{m^{-4}}$		
$=\frac{m^5n^{-10}}{m^{-4}}$	1M	for $(a^h)^k=a^{hk}$ or $(ab)^l=a^lb^l$
$=\frac{m^{5-(-4)}}{n^{10}}$	1M	for $\frac{c^p}{c^q}=c^{p-q}$ or $d^{-r}=\frac{1}{d^r}$
$=\frac{m^9}{n^{10}}$	1A	
	----------(3)	
2. (a) $\alpha^2+\alpha-6$		
$=(\alpha+3)(\alpha-2)$	1A	or equivalent
(b) $\alpha^4+\alpha^3-6\alpha^2$		
$=\alpha^2(\alpha^2+\alpha-6)$	1M	
$=\alpha^2(\alpha+3)(\alpha-2)$	1A	or equivalent
	----------(3)	
3. (a) 600	1A	
(b) 534.76	1A	
(c) 530	1A	
	----------(3)	
4. $a:b$		
$=6:7$		
$=12:14$	1M	either one
$a:c$		
$=4:3$		
$=12:9$		
$a:b:c$		
$=12:14:9$		
Let $a=12k$, $b=14k$ and $c=9k$, where k is a non-zero constant.		
$\frac{b+2c}{a+2b}$		
$=\frac{14k+2(9k)}{12k+2(14k)}$	1M	
$=\frac{4}{5}$	1A	0.8
	----------(3)	

Solution	Marks	Remarks
5. Let x be the number of female applicants in the recruitment exercise.		
So, the number of male applicants is $(1+28\%)x$.	1A	
$(1+28\%)x-x=91$	1M+1A	1M for getting a linear equation in one unknown
$0.28x=91$		
$x=325$		
$(1+28\%)x=416$	1A	
Thus, the number of male applicants in the recruitment exercise is 416 .		
Let x and y be the number of male applicants and the number of female applicants in the recruitment exercise respectively.		
So, we have $x-y=91$ and $x=(1+28\%)y$.	1A+1A	
Therefore, we have $(1+28\%)y-y=91$.	1M	1M for getting a linear equation in one unknown
$0.28y=91$		
$y=325$		
$x=416$	1A	
Thus, the number of male applicants in the recruitment exercise is 416 .		
The number of male applicants in the recruitment exercise		
$=\dfrac{(1+28\%)(91)}{28\%}$	1M+1A+1A	1M for fraction + 1A for numerator + 1A for denominator
$=416$	1A	
	----------(4)	
6. (a) $3-x>\dfrac{7-x}{2}$		
$6-2x>7-x$		
$-2x+x>7-6$	1M	for putting x on one side
$x<-1$	1A	
$5+x>4$		
$x>-1$		
Thus, we have $x\neq -1$.	1A	$x<-1$ or $x>-1$
(b) -2	1A	
	----------(4)	
7. (a) Since the equation $4x^2+12x+c=0$ has equal roots, we have $\Delta=0$.		
$12^2-4(4)c=0$	1M+1A	
$144-16c=0$		
$c=9$	1A	
(b) y		
$=\text{p}(x)-169$		
$=4x^2+12x-160$ (by (a))	1M	
$=4(x+8)(x-5)$		
Thus, the x-intercepts of the graph of $y=\text{p}(x)-169$ are -8 and 5 .	1A	
	----------(5)	

Solution	Marks	Remarks
8. (a) $\angle AEC$		
$= \angle ADB$	1M	
$= 42^\circ$		
$\angle AEB$		
$= \angle CAE$	1M	
$= 30^\circ$		
$\angle BEC$		
$= \angle AEC - \angle AEB$		
$= 42^\circ - 30^\circ$		
$= 12^\circ$	1A	
(b) $\angle DCE$		
$= \angle BDC$	1M	
$= \theta$		
$\angle CFE$		
$= 180^\circ - \angle BEC - \angle DCE$		
$= 180^\circ - 12^\circ - \theta$		
$= 168^\circ - \theta$	1A	
$\angle DBE$		
$= \angle BEC$	1M	
$= 12^\circ$		
$\angle BFD$		
$= 180^\circ - \angle BDC - \angle DBE$		
$= 180^\circ - \theta - 12^\circ$		
$= 168^\circ - \theta$		
$\angle CFE$		
$= \angle BFD$		
$= 168^\circ - \theta$	1A	
	----------(5)	
9. (a) The mean		
$= 5.4$	1A	
The median		
$= 5.5$	1A	
The standard deviation		
≈ 0.916515139		
≈ 0.917	1A	r.t. 0.917
(b) The new median		
$= 5$		
The decrease in the median		
$= 5.5 - 5$	1M	
$= 0.5$	1A	
	----------(5)	

Solution	Marks	Remarks
10. (a) Let $P = a + bh^3$, where a and b are non-zero constants.	1A	
So, we have $a + 27b = 59$ and $a + 343b = 691$.	1M	for either substitution
Solving, we have $a = 5$ and $b = 2$.	1A	can be absorbed
The required price		
$= 5 + 2(4^3)$		
$= \$133$	1A	
	----------(4)	
(b) When $h = 5$, $P = 5 + 2(5^3) = 255$.	1M	
Note that $2(133) = 266$.		
Since $255 < 266$, the claim is not correct.	1A	f.t.
	----------(2)	
11. (a) The range		
$= 50 + w - 11$	1M	
$= (w + 39)$ grams		
The inter-quartile range		
$= 38 - 23$	1M	
$= 15$ grams		
$w + 39 = 3(15)$	1M	
$w = 6$	1A	
	----------(4)	
(b) The mode of the distribution is 38 grams.		
The required probability		
$= \dfrac{6}{20}$	1M	
$= \dfrac{3}{10}$	1A	0.3
	----------(2)	

Solution	Marks	Remarks
12. (a) The volume of the middle part of the circular cone		
$=\frac{1}{3}\pi(15^2)(36)\left(\frac{2^3-1^3}{3^3}\right)$	1M+1M	
$=700\pi\text{ cm}^3$	1A	
Let R cm and r cm be the larger base radius and the smaller base radius of the middle part of the circular cone respectively.		
Therefore, we have $\frac{r}{15}=\frac{12}{36}$ and $\frac{R}{15}=\frac{24}{36}$.	1M	for either one
Solving, we have $r=5$ and $R=10$.		
The volume of the middle part of the circular cone		
$=\frac{1}{3}\pi(10^2)(24)-\frac{1}{3}\pi(5^2)(12)$	1M	for either one
$=700\pi\text{ cm}^3$	1A	
	----------(3)	
(b) The curved surface area of the middle part of the circular cone		
$=\pi(15)\left(\sqrt{15^2+36^2}\right)\left(\frac{2^2-1^2}{3^2}\right)$	1M+1M	
$=195\pi\text{ cm}^2$	1A	
The curved surface area of the middle part of the circular cone		
$=\pi(10)\sqrt{10^2+24^2}-\pi(5)\sqrt{5^2+12^2}$	1M+1M	
$=\pi(10)(26)-\pi(5)(13)$		
$=195\pi\text{ cm}^2$	1A	
	----------(3)	

Solution	Marks	Remarks
13. (a) Let $f(x)=(x^2-1)q(x)+(kx+8)$, where $q(x)$ is a polynomial.	1M	
Since $f(1)=0$, we have $(1^2-1)q(1)+(k+8)=0$.	1M	
Thus, we have $k=-8$.	1A	
	----------(3)	
(b) Let $f(x)=(x-1)(x+3)(ax+b)$, where a and b are constants.	1M	
Since $f(0)=24$, we have $(-1)(3)(b)=24$.	1M	
Solving, we have $b=-8$.	1A	
Note that $f(x)=(x^2-1)q(x)+(-8x+8)$.		
So, we have $f(-1)=((-1)^2-1)q(-1)+((-8)(-1)+8)=16$.		
Therefore, we have $(-1-1)(-1+3)(-a-8)=16$.		
Solving, we have $a=-4$.	1A	
Hence, we have $f(x)=(x-1)(x+3)(-4x-8)$.		
The roots of the equation $f(x)=0$ are 1 , -3 and -2 .		
All the roots of the equation $f(x)=0$ are integers.		
Thus, the claim is correct.	1A	f.t.
Let $f(x)=(x^2-1)(mx+n)+(-8x+8)$, where m and n are constants.	1M	
Since $f(0)=24$, we have $(-1)(n)+8=24$.	1M	
Solving, we have $n=-16$.	1A	
Since $f(-3)=0$, we have $((-3)^2-1)(-3m-16)+((-8)(-3)+8)=0$.		
Solving, we have $m=-4$.	1A	
$f(x)$ $=(x^2-1)(-4x-16)+(-8x+8)$ $=(x-1)(x+1)(-4x-16)-8(x-1)$ $=(x-1)(-4x^2-20x-24)$ $=-4(x-1)(x+2)(x+3)$		
Therefore, the roots of the equation $f(x)=0$ are 1 , -2 and -3 .		
All the roots of the equation $f(x)=0$ are integers.		
Thus, the claim is correct.	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
14. (a) The x-coordinate of G $=\frac{-10+30}{2}$ $=10$	1M	
The radius of C $=\sqrt{(-10-10)^2+(0+15)^2}$ $=25$	1M	
Thus, the equation of C is $(x-10)^2+(y+15)^2=25^2$.	1A	$x^2+y^2-20x+30y-300=0$
The x-coordinate of G $=\frac{-10+30}{2}$ $=10$	1M	
Let $x^2+y^2-20x+30y+F=0$ be the equation of C, where F is a constant.		
Since A lies on C, we have $(-10)^2+0^2-20(-10)+30(0)+F=0$.	1M	
Solving, we have $F=-300$.	1A	
Thus, the equation of C is $x^2+y^2-20x+30y-300=0$.		$(x-10)^2+(y+15)^2=25^2$
	----------(3)	
(b) (i) Γ is parallel to L .	1M	
(ii) The slope of L $=\frac{0+15}{30-10}$ $=\frac{3}{4}$		
So, the slope of Γ is $\frac{3}{4}$ (by (b)(i)).		
The equation of Γ is $y-0=\frac{3}{4}(x-(-10))$	1M	
$3x-4y+30=0$	1A	or equivalent
(iii) $\tan\angle ABG=\frac{3}{4}$	1M	
$\angle ABG\approx 36.86989765°$		
Note that $\angle BAH=\angle ABG$ and $\angle BAG=\angle ABG$.	1M	for either one
$\angle GAH$ $=\angle BAH+\angle BAG$ $=2\angle ABG$		
Since $\angle ABG>35°$, we have $\angle GAH>70°$.		
Thus, the claim is disagreed.	1A	f.t.
	----------(6)	

Solution	Marks	Remarks
15. (a) The required probability $=\frac{C_4^7+C_4^9}{C_4^{19}}$	1M+1M	1M for p_1+p_2 and 1M for denominator
$=\frac{161}{3\,876}$	1A	r.t. 0.0415
The required probability $=\frac{P_4^7+P_4^9}{P_4^{19}}$	1M+1M	1M for p_1+p_2 and 1M for denominator
$=\frac{161}{3\,876}$	1A	r.t. 0.0415
The required probability $=\left(\frac{7}{19}\right)\left(\frac{6}{18}\right)\left(\frac{5}{17}\right)\left(\frac{4}{16}\right)+\left(\frac{9}{19}\right)\left(\frac{8}{18}\right)\left(\frac{7}{17}\right)\left(\frac{6}{16}\right)$	1M+1M	1M for p_1+p_2 and 1M for denominator
$=\frac{161}{3\,876}$	1A	r.t. 0.0415
	----------(3)	
(b) The required probability $=1-\frac{161}{3\,876}$	1M	for $1-$(a)
$=\frac{3\,715}{3\,876}$	1A	r.t. 0.958
	----------(2)	
16. (a) Let a and r be the 1st term and the common ratio of the geometric sequence respectively. Therefore, we have $ar^2=144$ and $ar^5=486$.	1M	for either one
Solving, we have $r=1.5$. So, we have $a=64$.	1A	
Thus, the 1st term of the sequence is 64 .	----------(2)	
(b) $64+64(1.5)+64(1.5^2)+\cdots+64(1.5^{n-1})>8\times10^{18}$		
$\frac{64(1.5^n-1)}{1.5-1}>8\times10^{18}$	1M	
$1.5^n>6.25\times10^{16}+1$		
$\log1.5^n>\log(6.25\times10^{16}+1)$	1M	
$n\log1.5>\log(6.25\times10^{16}+1)$		
$n>95.38167941$		
Thus, the least value of n is 96 .	1A	
	----------(3)	

Solution	Marks	Remarks
17. (a) $g(x)$		
$= x^2 - 2kx + 2k^2 + 4$		
$= x^2 - 2kx + k^2 + k^2 + 4$	1M	
$= (x-k)^2 + k^2 + 4$		
Thus, the coordinates of the vertex of the graph of $y = g(x)$ are		
(k, k^2+4) .	1A	
	----------(2)	
(b) Note that $D = (k-2, k^2+4)$ and $E = (k+2, -k^2-4)$.	1A	for either one
Denote the point $(0, 3)$ by C .		
CD^2		
$= ((k-2)-0)^2 + ((k^2+4)-3)^2$	1M	either one
$= k^4 + 3k^2 - 4k + 5$		
CE^2		
$= (k+2-0)^2 + ((-k^2-4)-3)^2$		
$= k^4 + 15k^2 + 4k + 53$		
$CD^2 = CE^2$		
$k^4 + 3k^2 - 4k + 5 = k^4 + 15k^2 + 4k + 53$	1M	
$3k^2 + 2k + 12 = 0$		
Note that $2^2 - 4(3)(12) = -140 < 0$.		
So, the equation $3k^2 + 2k + 12 = 0$ has no real roots.		
Thus, there is no point F on the same rectangular coordinate system such that the coordinates of the circumcentre of ΔDEF are $(0, 3)$.	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
18. (a) $\angle TUV = \angle TWU$ ($\angle$ in alt. segment)		[交錯弓形的圓周角]
$\angle UTV = \angle UTW$ (common angle)		[公共角]
$\angle TVU = \angle TUW$ ($\angle$ sum of Δ)		[Δ內角和]
$\Delta UTV \sim \Delta WTU$ (AAA)		(AA) (equiangular) [等角]
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
	----------(2)	
(b) (i) $\dfrac{TW}{TU} = \dfrac{TU}{TV}$ (by (a))	1M	
$\dfrac{TV+VW}{TU} = \dfrac{TU}{TV}$		
$\dfrac{325+VW}{780} = \dfrac{780}{325}$		
$VW = 1\,547$ cm		
Thus, the circumference of C is $1\,547\pi$ cm .	1A	
(ii) By (a), we have $UV : UW = TV : TU = 325 : 780 = 5 : 12$.		
Since VW is a diameter of C, we have $\angle VUW = 90°$.		
So, we have $UV : UW : VW = 5 : 12 : 13$.	1M	
$UV = (1\,547)\left(\dfrac{5}{13}\right) = 595$ cm	1M	either one
$UW = (1\,547)\left(\dfrac{12}{13}\right) = 1\,428$ cm		
The perimeter of ΔUVW $= 595 + 1\,428 + 1\,547$ $= 3\,570$ cm $= 35.7$ m > 35 m		
Thus, the claim is agreed.	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
19. (a) $\dfrac{PR}{\sin \angle PQR} = \dfrac{PQ}{\sin \angle PRQ}$ (by sine formula)	1M	
$\dfrac{PR}{\sin 30^\circ} = \dfrac{60}{\sin 55^\circ}$		
$PR \approx 36.62323766$ cm		
Since $\angle QPR = 95^\circ$, we have $\angle RPS = 25^\circ$.		
$RS^2 = PS^2 + PR^2 - 2(PS)(PR)\cos \angle RPS$ (by cosine formula)	1M	
$RS^2 \approx 40^2 + 36.62323766^2 - 2(40)(36.62323766)\cos 25^\circ$		
$RS \approx 16.90879944$		
$RS \approx 16.9$ cm	1A	r.t. 16.9 cm
Thus, the length of RS is 16.9 cm .		
	----------(3)	
(b) The area of the paper card		
$= \frac{1}{2}(PQ)(PR)\sin \angle QPR + \frac{1}{2}(PR)(PS)\sin \angle RPS$	1M	
$\approx \frac{1}{2}(60)(36.62323766)\sin 95^\circ + \frac{1}{2}(36.62323766)(40)\sin 25^\circ$		
$\approx 1\,404.069236$		
$\approx 1\,400 \text{ cm}^2$	1A	r.t. $1\,400 \text{ cm}^2$
	----------(2)	
(c) (i) Let H be the foot of the perpendicular from P to QR .		
$PH = PQ \sin \angle PQH$	1M	either one
$PH = 60 \sin 30^\circ$		
$PH = 30$ cm		
Denote the projection of P on the horizontal ground by G .		
So, the angle between the paper card and the horizontal ground is $\angle PHG$.	1M	either one
Hence, we have $\angle PHG = 32^\circ$.		
$PG = PH \sin \angle PHG$		
$PG = 30 \sin 32^\circ$		
$PG \approx 15.89757793$		
$PG \approx 15.9$ cm	1A	r.t. 15.9 cm
Thus, the shortest distance from P to the horizontal ground is 15.9 cm .		
(ii) Denote the projection of S on the horizontal ground by K .		
Let T be the point at which PS produced and QR produced meet.		
Then, we have $\Delta SKT \sim \Delta PGT$ and $PT = PQ$.		
So, we have $SK = \left(\dfrac{PT - PS}{PT}\right)PG = \left(\dfrac{PQ - PS}{PQ}\right)PG = \left(\dfrac{60-40}{60}\right)PG = \dfrac{1}{3}PG$.	1M	
By (c)(i), we have $SK = 10 \sin 32^\circ$ cm .		
Note that the angle between RS and the horizontal ground is $\angle SRK$.	1M	
$\sin \angle SRK = \dfrac{SK}{RS}$	1M	
$\sin \angle SRK \approx \dfrac{10 \sin 32^\circ}{16.90879944}$		
$\angle SRK \approx 18.26416068^\circ$		
Therefore, we have $\angle SRK \le 20^\circ$.		
Thus, the claim is correct.	1A	f.t.
	----------(7)	