HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2022

MATHEMATICS Compulsory Part
PAPER 2

11:30 am – 12:45 pm (1¼ hours)

INSTRUCTIONS

1. Read carefully the instructions on the Answer Sheet. After the announcement of the start of the examination, you should first stick a barcode label and insert the information required in the spaces provided. No extra time will be given for sticking on the barcode label after the 'Time is up' announcement.

2. When told to open this book, you should check that all the questions are there. Look for the words '**END OF PAPER**' after the last question.

3. All questions carry equal marks.

4. **ANSWER ALL QUESTIONS**. You are advised to use an HB pencil to mark all the answers on the Answer Sheet, so that wrong marks can be completely erased with a clean rubber. You must mark the answers clearly; otherwise you will lose marks if the answers cannot be captured.

5. You should mark only **ONE** answer for each question. If you mark more than one answer, you will receive **NO MARKS** for that question.

6. No marks will be deducted for wrong answers.

Not to be taken away before the end of the examination session

There are 30 questions in Section A and 15 questions in Section B.
The diagrams in this paper are not necessarily drawn to scale.
Choose the best answer for each question.

Section A

1. $\alpha^2-\alpha-\beta^2+\beta=$

   A. $(\alpha+\beta)(\alpha-\beta+1)$.

   B. $(\alpha+\beta)(\alpha-\beta-1)$.

   C. $(\alpha-\beta)(\alpha+\beta+1)$.

   D. $(\alpha-\beta)(\alpha+\beta-1)$.

2. $\dfrac{81^{2n+3}}{(27^{n+1})^2}=$

   A. 3 .

   B. 3^{2n+6} .

   C. 3^{4n+8} .

   D. 3^{10n+14} .

3. If m and n are constants such that $(x+3)^2+mx\equiv(x-n)(x+1)+2n$, then $m=$

   A. -14 .

   B. -8 .

   C. 4 .

   D. 9 .

4. Let c be a constant. Solve the equation $(x-c)(x-4c)=(3c-x)(x-4c)$.

   A. $x=2c$

   B. $x=3c$

   C. $x=c$ or $x=3c$

   D. $x=2c$ or $x=4c$

5. If $\frac{2}{u}+\frac{3}{v}=4$, then $u=$

A. $\frac{2v}{4v-3}$.

B. $\frac{2v}{3-4v}$.

C. $\frac{3v}{4v-2}$.

D. $\frac{3v}{2-4v}$.

6. It is given that x is a real number. If x is rounded down to 3 significant figures, then the result is 345. Find the range of values of x.

A. $344 < x \leq 345$

B. $345 \leq x < 346$

C. $345 < x \leq 345.5$

D. $344.5 \leq x < 345.5$

7. The solution of $3y-5<5y+1\leq 11$ is

A. $-3 < y \leq 2$.

B. $-3 \leq y < 2$.

C. $-2 < y \leq 3$.

D. $-2 \leq y < 3$.

8. Let $f(x)=x^2-x+1$. If k is a constant, which of the following must be true?

A. $f(k)=f(-k)$

B. $f(k)=f(1-k)$

C. $f(k+1)=f(k)+f(1)$

D. $f(k-1)=f(k)-f(1)$

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9. Let $g(x) = x^2 + ax + b$, where a and b are constants. If $g(x)$ is divisible by $x + 2a$, find the remainder when $g(x)$ is divided by $x - 2a$.

A. $-2a^2$

B. 0

C. $2a^2$

D. $4a^2$

10. Let h and k be real constants such that $hk < 0$. Which of the following statements about the graph of $y = (h - x)(k - x)$ are true?

I. The graph opens upwards.
II. The graph has two x-intercepts.
III. The y-intercept of the graph is positive.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

11. A sum of $\$88\,000$ is deposited at an interest rate of 6% per annum for 4 years, compounded monthly. Find the interest correct to the nearest dollar.

A. $\$21\,120$

B. $\$23\,098$

C. $\$23\,803$

D. $\$23\,825$

12. Let x, y and z be non-zero numbers. If $x : y = 8 : 5$ and $2x = 4z - 3y$, then $y : z =$

A. $16 : 17$.

B. $17 : 16$.

C. $20 : 31$.

D. $31 : 20$.

13. If u varies directly as the square root of v and inversely as w, which of the following are true?

I. u^2 varies directly as v and inversely as the square of w.
II. v varies directly as w and inversely as the square root of u.
III. w varies directly as the square root of v and inversely as u.

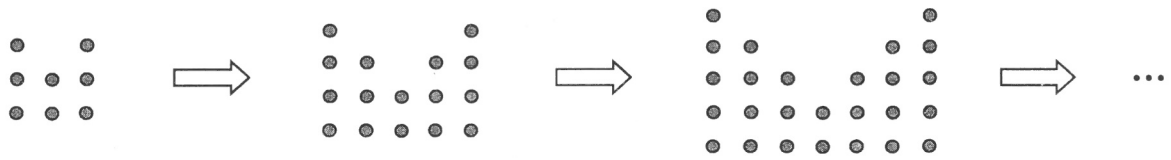
A. I and II only

B. I and III only

C. II and III only

D. I, II and III

14. In the figure, the 1st pattern consists of 8 dots. For any positive integer n, the $(n+1)$th pattern is formed by adding $(2n+6)$ dots to the nth pattern. Find the number of dots in the 7th pattern.



A. 52

B. 68

C. 86

D. 106

15. The radius of a solid hemisphere and the base radius of a solid right circular cylinder are equal. If the height of the circular cylinder is equal to its base diameter, then the ratio of the total surface area of the hemisphere to the total surface area of the circular cylinder is

A. 1:2.

B. 1:3.

C. 2:3.

D. 2:5.

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16. The diameter of a circle is 10 cm . The circle is divided into a major segment and a minor segment by a chord of length 8 cm . Find the area of the major segment correct to the nearest cm^2 .

A. 11 cm^2

B. 23 cm^2

C. 55 cm^2

D. 67 cm^2

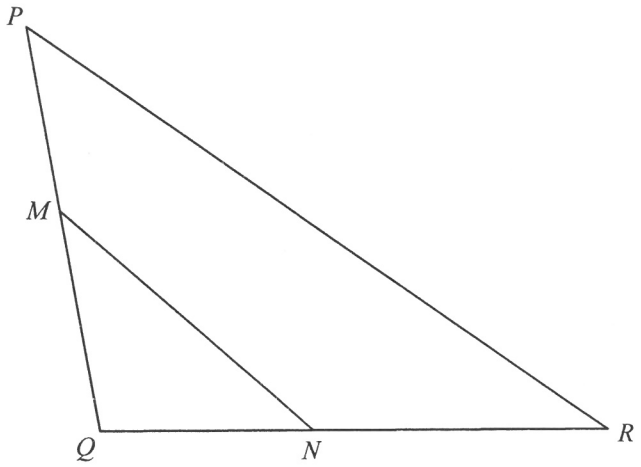
17. In the figure, M and N are points lying on PQ and QR respectively such that $PM : MQ = 5 : 6$ and $QN : NR = 3 : 4$. If the area of the quadrilateral $MNRP$ is 59 cm^2 , then the area of ΔMNQ is

A. 17 cm^2 .

B. 18 cm^2 .

C. 19 cm^2 .

D. 20 cm^2 .



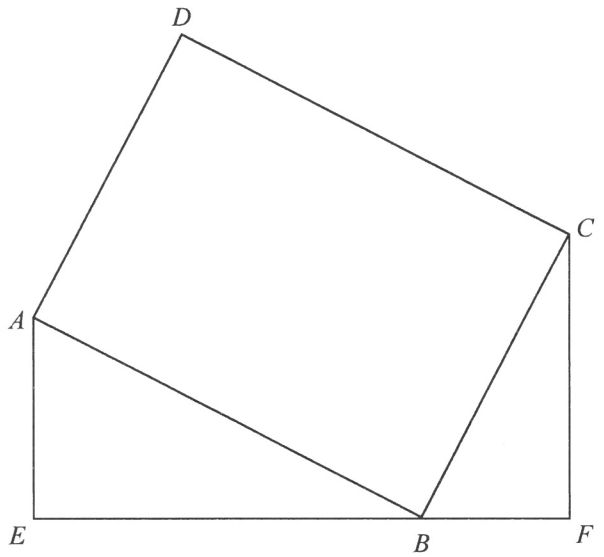
18. In the figure, the perimeter of the rectangle $ABCD$ is 170 cm . It is given that EBF is a straight line and $\angle AEB = \angle BFC = 90°$. If $AE = 24\text{ cm}$ and $BC = 34\text{ cm}$, then $EF =$

A. 45 cm .

B. 51 cm .

C. 61 cm .

D. 75 cm .



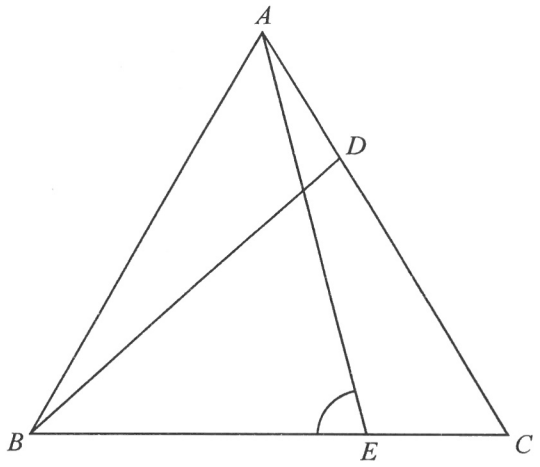
19. In the figure, ABC is an equilateral triangle. Let D and E be points lying on AC and BC respectively such that $AD = CE$. If $\angle CBD = 38°$, then $\angle AEB =$

A. $73°$.

B. $75°$.

C. $78°$.

D. $82°$.



20. The figure shows a parallelogram. Which of the following must be true?

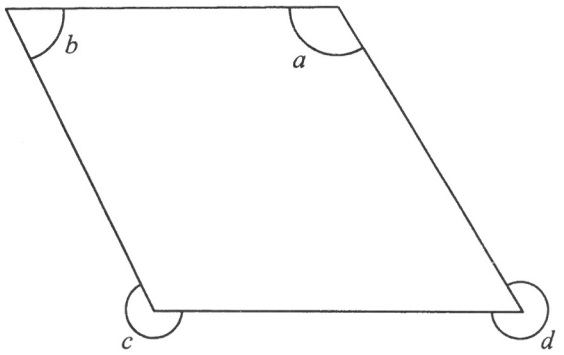
I. $a + b = 180°$
II. $b + c = 360°$
III. $c + d = 540°$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III



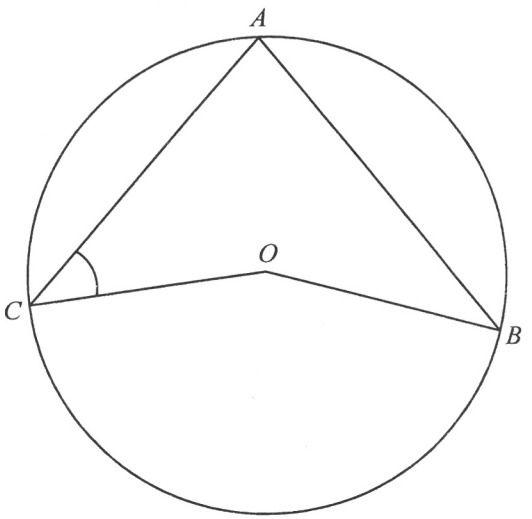
21. In the figure, O is the centre of the circle ABC. If $\angle ABO = 36°$ and $\angle BOC = 164°$, then $\angle ACO =$

A. $41°$.

B. $46°$.

C. $52°$.

D. $64°$.



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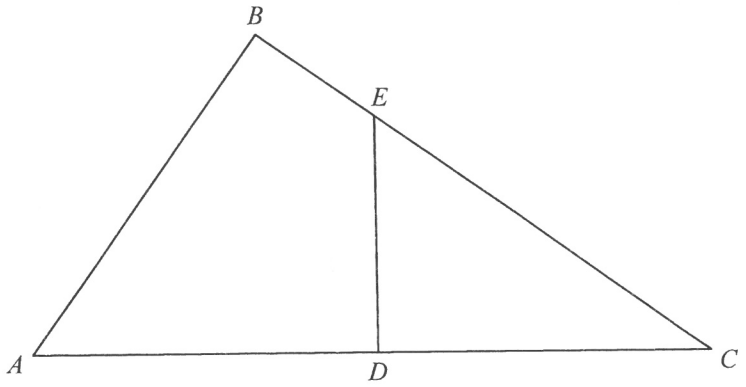
22. In the figure, ABC is a right-angled triangle with $\angle ABC = 90°$. Let D and E be points lying on AC and BC respectively such that $ABED$ is a cyclic quadrilateral. If $AB = 660$ cm, $AD = 572$ cm and $BE = 275$ cm, then $CD =$

A. 429 cm.

B. 715 cm.

C. 728 cm.

D. 845 cm.



23. It is given that $PQRS$ is a rhombus. Let T be the point of intersection of PR and QS. If $\angle QRT = \theta$, then $\frac{PQ}{ST} =$

A. $\sin\theta$.

B. $\cos\theta$.

C. $\frac{1}{\sin\theta}$.

D. $\frac{1}{\cos\theta}$.

24. The figure shows the graph of the straight line $mx + ny = 3$. Which of the following are true?

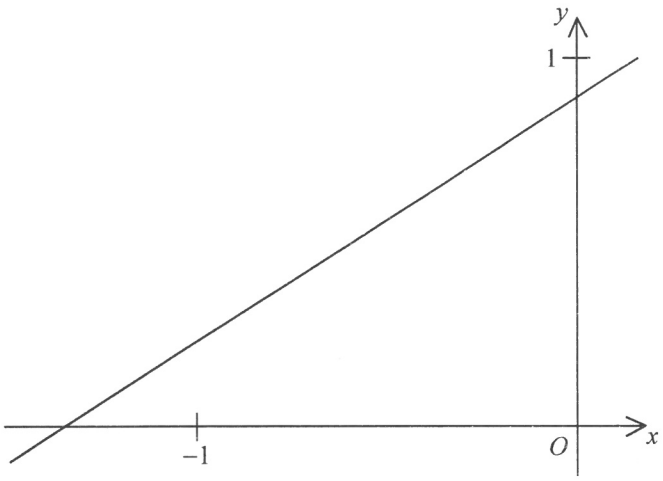
I. $m < 0$
II. $n > 3$
III. $m + n = 0$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III



25. The rectangular coordinates of the point Q are $(4\sqrt{3}, -4)$. If Q is rotated clockwise about the origin through $90°$, then the polar coordinates of its image are

A. $(8, 60°)$.

B. $(8, 120°)$.

C. $(8, 210°)$.

D. $(8, 240°)$.

26. The straight line $12x - 5y = 60$ cuts the x-axis and the y-axis at the points A and B respectively. Let P be a moving point in the rectangular coordinate plane such that $AP = BP$. Find the equation of the locus of P.

A. $10x + 24y + 119 = 0$

B. $15x + 36y + 179 = 0$

C. $x^2 + y^2 - 5x + 12y = 0$

D. $x^2 + y^2 + 12x - 133 = 0$

27. The coordinates of the points P and Q are $(10, -24)$ and $(17, -7)$ respectively. Let C be the circle which passes through the origin, P and Q. Which of the following is true?

A. PQ is a diameter of C.

B. The area of C is 196π.

C. The point $(16, -9)$ lies inside C.

D. The centre of C lies on the straight line $5x + 12y = 0$.

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28. $5\blacklozenge2$ is a 3-digit number, where $\blacklozenge$ is an integer from 0 to 9 inclusive. Find the probability that the 3-digit number is divisible by 7.

A. $\frac{1}{5}$

B. $\frac{1}{7}$

C. $\frac{1}{9}$

D. $\frac{1}{10}$

29. The mean weight of 60 actors and 40 actresses is 57 kg. If the mean weight of the actors is 63 kg, then the mean weight of the actresses is

A. 48 kg.

B. 50 kg.

C. 53 kg.

D. 60 kg.

30. Consider the following positive integers:

$$2 \quad 5 \quad 6 \quad 6 \quad x \quad x \quad x \quad y$$

If both the mean and the median of the above positive integers are 6, which of the following must be true?

I. The mode of the above positive integers is 6.
II. The least possible range of the above positive integers is 6.
III. The greatest possible variance of the above positive integers is 6.

A. I only

B. II only

C. I and III only

D. II and III only

Section B

31. Which of the following is the least?

A. $(-345)^{768}$

B. 453^{-786}

C. $\left(\frac{1}{435}\right)^{867}$

D. $\left(\frac{2}{543}\right)^{876}$

32. It is given that $\log_a y$ is a linear function of x, where $0 < a < 1$. The intercepts on the vertical axis and on the horizontal axis of the graph of the linear function are 6 and 3 respectively. If $y = mn^x$, which of the following is/are true?

I. $m < 1$
II. $n < 1$
III. $mn^3 = 1$

A. I only

B. II only

C. I and III only

D. II and III only

33. If $\log_4 y = 2x - 1$ and $(\log_4 y)^2 = 20x - 31$, then $\log_2 y =$

A. 1 or 2.

B. 2 or 4.

C. 3 or 7.

D. 6 or 14.

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34. $12B00CD000000E_{16} =$

A. $299\times4^{22}+205\times4^{14}+14$.

B. $300\times4^{22}+222\times4^{14}+15$.

C. $299\times4^{24}+205\times4^{16}+14$.

D. $300\times4^{24}+222\times4^{16}+15$.

35. Let $z=4+5i^{10}-ki^{15}+6i^{21}+2ki^{28}$, where k is a real number. If the real part and the imaginary part of z are equal, then the real part of z is

A. 7 .

B. 13 .

C. 17 .

D. 25 .

36. Consider the following system of inequalities:

$$\begin{cases} 2x+y\geq 8 \\ 2x+3y\geq 16 \\ 4x+3y\leq 22 \end{cases}$$

Let R be the region which represents the solution of the above system of inequalities. If (x, y) is a point lying in R, then the least value of $7x+6y$ is

A. 32 .

B. 38 .

C. 41 .

D. 43 .

37. Let a_n be the nth term of a geometric sequence. It is given that $a_1 = 8p^2$, $a_2 = 1$ and $a_3 = 27p$, where p is a real number. Find a_4.

A. $\frac{1}{6}$

B. $\frac{2}{9}$

C. $\frac{9}{2}$

D. $\frac{81}{4}$

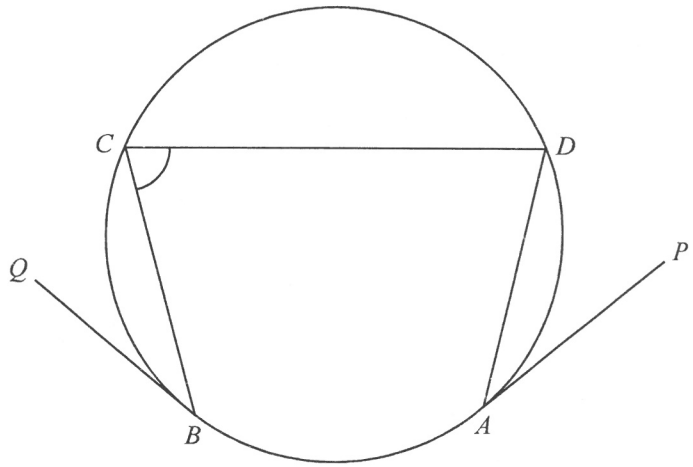
38. In the figure, $ABCD$ is a circle. PA and QB are the tangents to the circle at A and B respectively. If $\angle ADC = 79°$, $\angle CBQ = 39°$ and $\angle DAP = 42°$, then $\angle BCD =$

A. $76°$.

B. $79°$.

C. $81°$.

D. $82°$.



39. For $0° \le x < 360°$, how many roots does the equation $\sin^2 x = 6\cos^2 x$ have?

A. 2

B. 3

C. 4

D. 5

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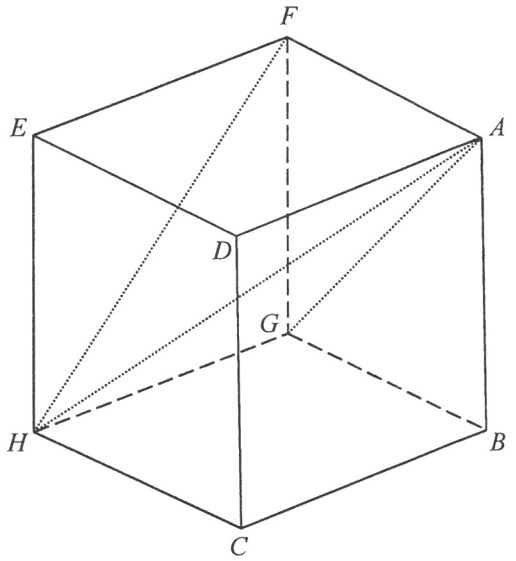
40. In the figure, $ABCDEFGH$ is a cube. Let α be the angle between ΔAFG and ΔAFH while β be the angle between ΔAFH and ΔFGH. Which of the following is true?



A. $\alpha < 60° < \beta$

B. $\alpha < \beta < 60°$

C. $60° < \alpha < \beta$

D. $60° < \beta < \alpha$

41. Let O be the origin. The coordinates of the points A and B are $(a, 0)$ and $(0, b)$ respectively, where a and b are positive numbers. If the circumcentre of ΔOAB lies on the straight line $4x + 16y = 17a$, then $a : b =$

A. $8:15$.

B. $15:8$.

C. $16:47$.

D. $47:16$.

42. If the first five digits and the last two digits of a seven-digit password are formed by a permutation of $1, 3, 5, 7, 9$ and a permutation of $2, 8$ respectively, how many different seven-digit passwords can be formed?

A. 10

B. 240

C. 480

D. 5 040

43. A box contains 2 white balls, 2 yellow balls and 3 red balls. A boy and a girl take turns to draw one ball randomly from the box with replacement until one of them draws a white ball or a yellow ball. The boy draws a ball first. Find the probability that the girl draws a white ball.

A. $\frac{3}{10}$

B. $\frac{3}{20}$

C. $\frac{7}{20}$

D. $\frac{17}{20}$

44. In a test, the median of the test scores of a class of students is 30 marks. All the students fail in the test, so the test score of each student is adjusted such that each score is increased by 50% and then extra 8 marks are added. Let x marks be the median of the test scores of the class of students after the score adjustment. In the test, the standard score of a student before the score adjustment is -2. Denote the standard score of this student after the score adjustment by z. Find x and z.

A. $x = 45$ and $z = -2$

B. $x = 45$ and $z = -1$

C. $x = 53$ and $z = -2$

D. $x = 53$ and $z = -1$

45. It is given that d is a real number. Let S_1 be a group of numbers $\{d-6, d-2, d-1, d+3, d+5, d+7\}$ and S_2 be another group of numbers $\{d-7, d-5, d-3, d+1, d+2, d+6\}$. Which of the following is/are true?

I. The means of S_1 and S_2 are equal.
II. The standard deviations of S_1 and S_2 are equal.
III. The inter-quartile ranges of S_1 and S_2 are equal.

A. I only

B. II only

C. I and III only

D. II and III only

END OF PAPER