2023-DSE
MATH CP
PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2023

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

8:30 am – 10:45 am (2¼ hours)
This paper must be answered in English

Please stick the barcode label here.

Candidate Number									

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of THREE sections, A(1), A(2) and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(7) The diagrams in this paper are not necessarily drawn to scale.

(8) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

* A 0 3 0 E 0 0 1 *

SECTION A(1) (35 marks)

1. Make h the subject of the formula $\frac{5}{h+k}=\frac{k}{h-3}$. (3 marks)

2. Simplify $\frac{x^{-8}y}{(x^7y^9)^{-6}}$ and express your answer with positive indices. (3 marks)

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3. A packet of cheese is termed *regular* if its weight is measured as 220 g correct to the nearest 10 g . Someone claims that the total weight of 250 *regular* packets of cheese can be measured as 53.6 kg correct to the nearest 0.1 kg . Is the claim correct? Explain your answer. (3 marks)

4. Consider the compound inequality

$$3x+2>\frac{4x-5}{2} \text{ and } 3x-2<7 \quad \ldots\ldots\ldots\ldots (*) .$$

(a) Solve (*) .

(b) How many negative integers satisfy (*) ?

(4 marks)

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5. On a ferry, the number of female passengers is 40% more than the number of male passengers. If 24 female passengers leave the ferry, then the number of male passengers is 40% more than the number of female passengers. Find the number of male passengers on the ferry. (4 marks)

6. Let a, b and c be non-zero numbers such that $7a = 6b$ and $\frac{4a-3c}{2b-c} = 9$. Find

(a) $a:b:c$,

(b) $\frac{5a+8b}{7b+3c}$.

(4 marks)

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7. In Figure 1, PR is a diameter of the circle $PQRS$. Denote the point of intersection of PR and QS by T.

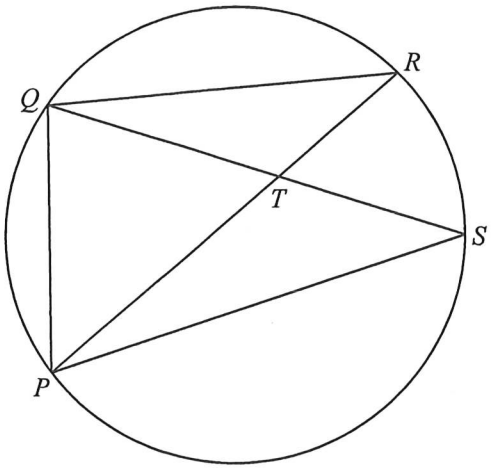

Figure 1

If $\angle PSQ = 41°$ and $\angle PTQ = 68°$, find $\angle RQS$ and $\angle PQS$. (4 marks)

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8. In Figure 2, AB and CD intersect at the point E. It is given that $AC // DB$.

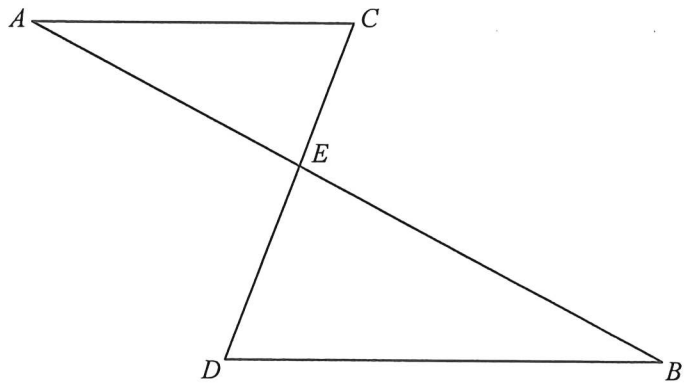

Figure 2

(a) Prove that $\Delta ACE \sim \Delta BDE$.

(b) Suppose that $AB = 20\text{ cm}$, $AC = 10\text{ cm}$, $BD = 15\text{ cm}$ and $CE = 7\text{ cm}$. Is ΔBDE a right-angled triangle? Explain your answer.

(5 marks)

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9. The stem-and-leaf diagram below shows the distribution of the numbers of working hours of a group of workers in a week.

Stem (tens)	Leaf (units)								
2	*a*	5	5	6	6	8	8		
3	3	3	3	4	5	5	9	9	
4	0	1	4	4	5	6	7	7	9

The range of the distribution is 27 .

(a) Find the mean and the mode of the distribution.

(b) If a worker is randomly selected from the group, find the probability that the number of working hours of the selected worker in the week exceeds the mode of the distribution.

(5 marks)

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SECTION A(2) (35 marks)

10. It is given that A and B are two distinct points in a rectangular coordinate plane. Let P be a moving point in the rectangular coordinate plane such that P is equidistant from A and B. Denote the locus of P by Γ.

(a) Describe the geometric relationship between Γ and AB. (1 mark)

(b) Suppose that the coordinates of A are $(2, -4)$ and the equation of Γ is $3x+y-12=0$. Find

(i) the equation of the straight line which passes through A and B,

(ii) the equation of the circle with AB as a diameter.

(5 marks)

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11. The table below shows the distribution of the numbers of calculators owned by a class of students.

Number of calculators owned	1	2	3	4
Number of students	8	5	n	1

The mean of the distribution is 2 .

(a) Find the median, the inter-quartile range and the variance of the distribution. (5 marks)

(b) Two students now withdraw from the class. It is found that the mean of the distribution remains unchanged. Is there any change in the range of the distribution due to the withdrawal of the two students? Explain your answer. (2 marks)

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12. It is given that $f(x)$ is partly constant and partly varies as x^2. Suppose that $f(10) = 62$ and $f(15) = 122$.

(a) Find $f(5)$. (3 marks)

(b) Suppose that $U(0, u)$ and $V(5, v)$ are points lying on the graph of $y = f(x)$. The horizontal line passing through V cuts the y-axis at the point W. Denote the circle which passes through U, V and W by C. Express the circumference of C in terms of π. (4 marks)

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13. Define $g(x) = x^3 + 5x^2 - 12x - 1$. Let $h(x) = 3x^4 + ax^3 - 16x^2 + bx + c$, where a, b and c are constants. When $h(x)$ is divided by $g(x)$, the quotient and the remainder are equal.

(a) Find the quotient when $h(x)$ is divided by $g(x)$. (3 marks)

(b) How many rational roots does the equation $h(x) = 0$ have? Explain your answer. (4 marks)

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14. The base radius and the curved surface area of a solid metal right circular cone are 14 cm and 700π cm^2 respectively.

(a) Find the height of the circular cone. (3 marks)

(b) The circular cone is divided into a right circular cone X and a frustum Y by a plane which is parallel to its base. The curved surface area of Y is 15 times the curved surface area of X.

(i) Express the volume of Y in terms of π.

(ii) If Y is melted and recast into 2 identical solid spheres, find the diameter of each sphere. (5 marks)

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SECTION B (35 marks)

15. In a box, there are 5 red balls and 4 black balls. From the box, 2 balls are randomly chosen at the same time.

(a) Find the probability that the 2 balls chosen are red. (2 marks)

(b) In a bag, there are 8 red balls. The 2 balls chosen from the box are put into the bag and then 3 balls are randomly chosen at the same time from the bag. Find the probability that the 3 balls chosen are of the same colour. (2 marks)

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16. (a) Let a and b be real constants. If the roots of the equation $x^2+ax+b=0$ are p and $5p$, prove that $5a^2=36b$. (2 marks)

(b) Denote the circle $x^2+y^2-6x-12y+20=0$ by C. Find the constant m such that the straight line $y=mx$ cuts C at the points Q and R with $OQ:QR=1:4$, where O is the origin. (3 marks)

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17. (a) It is given that WXY is a triangle, where $WX = 6$ cm , $XY = 5$ cm and $\angle WYX = 70°$. Find $\angle XWY$. (2 marks)

(b) Figure 3 shows the pyramid $WXYZ$, where $WZ = XZ = YZ$. The base of this pyramid is the triangle WXY described in (a).

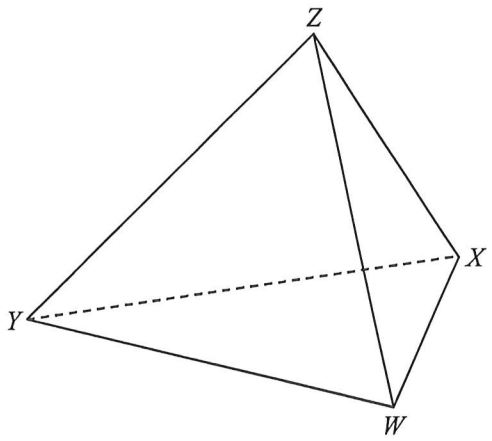

Figure 3

It is given that the angle between WZ and the triangle WXY is $30°$. Does the angle between the triangles WXY and XYZ exceed $45°$? Explain your answer. (4 marks)

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18. Suppose that $\alpha, 7, \beta$ is a geometric sequence, where $1 < \alpha < \beta$.

(a) Express $\log_7 \alpha$ in terms of $\log_7 \beta$. (3 marks)

(b) If $\log_\beta \alpha, \log_7 \beta, \log_\alpha \beta$ is an arithmetic sequence, find the common difference of the arithmetic sequence. (5 marks)

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19. The coordinates of the points P and Q are $(50, 0)$ and $(32, t)$ respectively, where $t > 0$. Denote the origin by O. Let R be a point such that OQ is a median of ΔOPR. Suppose that G and H are the circumcentre and the orthocentre of ΔOPR respectively.

(a) Express the coordinates of G and H in terms of t. (5 marks)

(b) Let S be a point lying on OP such that QS is perpendicular to OP. It is given that $\angle PQS = \angle POQ$.

(i) By considering $\tan\angle PQS$, prove that $t = 24$.

(ii) Are O, G and Q collinear? Explain your answer.

(iii) Denote the in-centre of ΔOPR by I. Find the ratio of the area of ΔGHR to the area of ΔIPQ.

(7 marks)

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