Sacred Heart Canossian College
S6 Mock Examination 2019-2020
Mathematics Paper 2

Time allowed: 1 hour and 15 minutes

1. Answer **ALL** questions.
   All the answers should be marked with an HB pencil on the answer sheet provided.
2. For each question, mark only one answer.
   Two or more answers will score no marks.
3. All questions carry equal marks.
   No marks will be deducted for wrong answers.

There are 30 questions in Section A and 15 questions in Section B.
The diagrams in this paper are not necessarily drawn to scale.

Section A

1. $(-8)^{111} \cdot (-3)^{666} =$

$(-2)^{333} \cdot 9^{333}$

$= -18^{333}$

A. -18^{222}.
✓B. -18^{333}.
C. 24^{111}.
D. 24^{666}.

2. If $\dfrac{x+y}{x} = \dfrac{1-y}{y}$, then $x =$

A. $\dfrac{y^2}{1+2y}$.
✓B. $\dfrac{y^2}{1-2y}$.
C. $\dfrac{1+2y}{y^2}$.
D. $\dfrac{1-2y}{y^2}$.

$xy + y^2 = x - xy$

$y^2 = x - 2xy$

$y^2 = x(1-2y)$

$x = \dfrac{y^2}{1-2y}$

3. If $A(x-1)^2 + Bx - 3 \equiv 2x^2 - 5x + C$, then $B =$

$LHS = A(x^2 - 2x + 1) + Bx - 3$

$= Ax^2 + (B-2A)x + A - 3$

$A = 2$, $B - 4 = -5$

$B = -1$

A. -5.
B. -4.
C. -3.
✓D. -1.

4. Timothy buys a TV set and sells it to Miranda at a profit of 20%. Then, Miranda sells the TV set to Paul at a loss of 10%. If Paul buys the TV set for \$2700, how much does Timothy pay for the TV set?

A. \$2025
B. \$2376
✓C. \$2500
D. \$2916

$2700 \div (1-10\%) \div (1+20\%)$
$= 2500$

5. If α is a root of the equation $4x^2 - 7x - 1 = 0$, then $(4\alpha - 5)(1 - 2\alpha) =$ $4\alpha - 8\alpha^2 - 5 + 10\alpha$

A. -9.
✓B. -7.
C. -5.
D. -3.

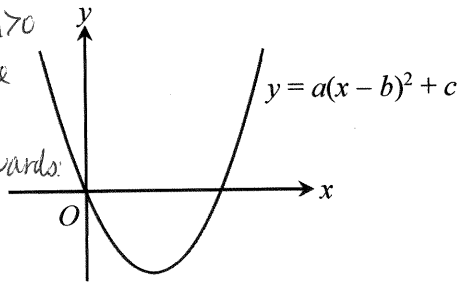
$4\alpha^2 - 7\alpha - 1 = 0$
$-8\alpha^2 + 14\alpha + 2 = 0$
$-8\alpha^2 + 14\alpha = -2$

$= -8\alpha^2 + 14\alpha - 5$
$= -2 - 5 = -7$

6. The figure shows the graph of $y = a(x - b)^2 + c$, where a, b and c are constants. The graph passes through the origin. Which of the following is true?

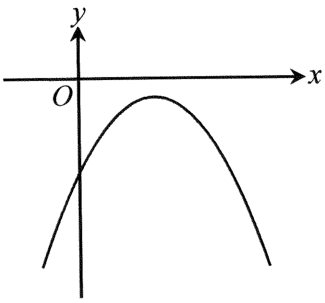
✓A. $a > 0$, $b > 0$ and $c < 0$
B. $a > 0$, $b > 0$ and $c > 0$
C. $a > 0$, $b < 0$ and $c < 0$
D. $a < 0$, $b > 0$ and $c > 0$

open upwards : $a > 0$
translate to the right : $b > 0$
translate downwards : $c < 0$



7. If $a < 0$ and $b < 0$, which of the following may represent the graph of $y = ax^2 + 2x + b$?

open downward, y-intercept < 0

✓A.


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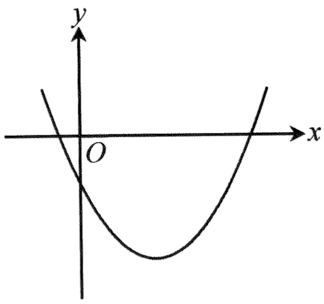

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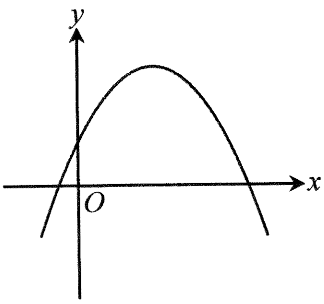
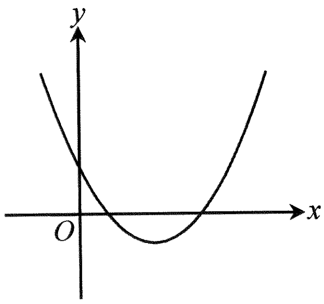
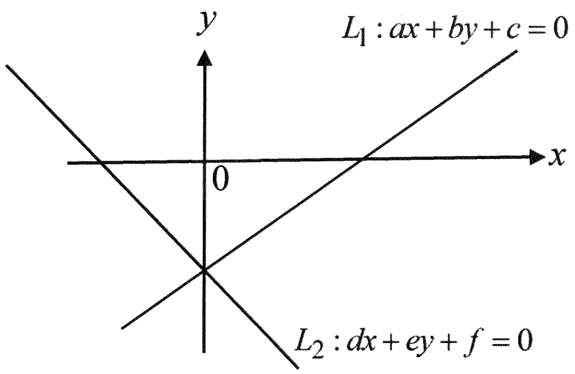


D.


8. In the figure, the straight lines L_1 and L_2 intersect at a point on the negative y-axis. Which of the following must be true?



✓ I. $\frac{a}{b} < \frac{d}{e}$

✗ II. $\frac{c}{a} > \frac{f}{d}$

✓ III. $ce - bf = 0$

$L_1: y = -\frac{a}{b}x - \frac{c}{b}$

$L_2: y = -\frac{d}{e}x - \frac{f}{e}$

slope: $-\frac{a}{b} > -\frac{d}{e}$

$\frac{a}{b} < \frac{d}{e}$

✓ A. I and III only

B. I and II only

C. II and III only

D. I, II and III

x-intercept

$-\frac{c}{a} > -\frac{f}{d}$

$\frac{c}{a} < \frac{f}{d}$

y-intercept:

$-\frac{c}{b} = -\frac{f}{e}$

$ce - bf = 0$

9. A and B are two distinct points on the circle $x^2 + y^2 + 4x + 2ky - 12 = 0$, where k is a constant. P is a moving point such that $AP = BP$. If the equation of the locus of P is $2x + y + 1 = 0$, find k.

locus of P is the ⊥ bisector of AB.

centre of circle lies on the line.

Centre $= (-2, -k)$

$\therefore \quad 2(-2) - k + 1 = 0$

$k = -3$

A. 3

B. 0

✓ C. −3

D. −4

10. The coordinates of the point A are $\left(1, -\frac{15}{4}\right)$ and the equation of the straight line L is $y = -\frac{17}{4}$. Let P be a moving point such that the distance between P and A is equal to the distance from P to L. Find the equation of the locus of P.

Let $P = (x, y)$

$\sqrt{(x-1)^2 + \left(y + \frac{15}{4}\right)^2} = y + \frac{17}{4}$

$x^2 - 2x + 1 + y^2 + \frac{15}{2}y + \frac{225}{16} = y^2 + \frac{17}{2}y + \frac{289}{16}$

$x^2 - 2x - 3 = y$

A. $x = 9$

✓ B. $y = x^2 - 2x - 3$

C. $x^2 - 2x + 16y - 3 = 0$

D. $4y^2 + 30y - 21x - 16 = 0$

11. If the interior angle of a regular n-sided polygon is 36° larger than its exterior angle, which of the following must be true?

$\text{int}\angle = x \quad \text{ext}\angle = x - 36°$

$x + x - 36° = 180°$

$x = 108°$

$\text{ext}\angle = 72°$

$n = \frac{360}{72} = 5$

× I. The value of n is 4.

√ II. The size of each interior angle is 108°.

√ III. The polygon has n reflectional symmetry.

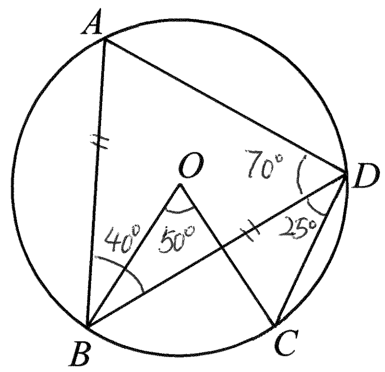
A. I only

B. II only

C. I and II only

√ D. II and III only

12. In the figure, O is the centre of the circle $ABCD$. If $\angle BOC = 50°$, $\angle ABD = 40°$ and $AB = BD$, then $\angle ADC =$



A. 65°.

B. 75°.

C. 85°.

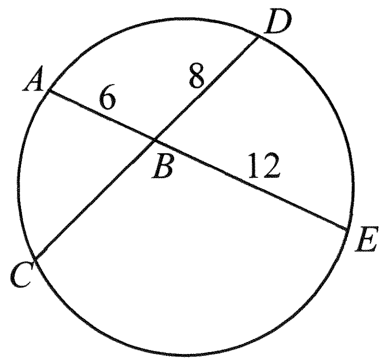
√ D. 95°.

$\angle BAD = \angle BDA = \frac{180° - 40°}{2} = 70°$

$\angle BDC = \frac{1}{2}\angle BOC = 25°$

$\angle ADC = 70° + 25° = 95°$

13. In the figure, ABE and DBC are straight lines, then $BC =$



A. 6 .

B. 7 .

√ C. 9 .

D. 10 .

$\triangle ABC \sim \triangle DBE$

$\frac{BC}{BE} = \frac{AB}{DB}$

$\frac{BC}{12} = \frac{6}{8}$

$BC = 9$

14. The point $A(3, 3)$ is transformed to the point $B(3, -3)$. Which of the following may be the transformation?

√ I. A is reflected about the x-axis.

√ II. A is rotated anticlockwise about the origin through 270°.

√ III. A is translated 6 units downwards.

A. I and II only

B. I and III only

C. II and III only

√ D. I, II and III

15. The table below shows the distribution of the numbers of ball pens owned by some S1 students in a school.

Number of ball pens	10	11	12	13	14
Number of students	40	39	35	10	6

Which of the following is true?

total number of students = 130

median = 11

range = 14 − 10 = 4

upper quartile = 12

lower quartile = 10

IQR = 12 − 10 = 2

A. The median of the distribution is 12.

B. The range of the distribution is 34

C. The upper quartile of the distribution is 11.

√D. The inter-quartile range of the distribution is 2.

16. Which of the following can be obtained from a box-and-whisker diagram?

I. Mean

II. Variance

√III. Inter-quartile range

√A. III only

B. I and II only

C. II and III only

D. I and III only

17. Let $f(x) = x^6 + hx + k$, where h and k are non-zero constants. If $f(x)$ is divisible by $x + h$, find the remainder when $f(x)$ is divided by $x - h$.

$f(-h) = 0$

$h^6 - h^2 + k = 0$

$k = h^2 - h^6$

remainder $= f(h)$

$= h^6 + h^2 + h^2 - h^6$

$= 2h^2$

A. $-2h^2$

B. 0

C. h^2

√D. $2h^2$

18. When $2x^3 - mx^2 - 8x + n$ is divided by $2x + 1$ where m and n are constants, the quotient is $x^2 - 4$ and the remainder is 9. Find the values of m and n.

$2x^3 - mx^2 - 8x + n = (2x+1)(x^2-4) + 9$

$-m = 1$

$m = -1$

$n = -4 + 9 = 5$

√A. $m = -1$ and $n = 5$

B. $m = -1$ and $n = -13$

C. $m = 1$ and $n = 5$

D. $m = 1$ and $n = -13$

19. The scale of the map is 1 : 50 000. If the area of a country park on a map is 9 cm^2, the actual area of the country park is

actual area

A. 4.5×10^5 m^2.
✓B. 2.25×10^6 m^2.
C. 9×10^6 m^2.
D. 1.8×10^7 m^2.

$$= \frac{9 \times 50000^2}{100^2}$$
$$= 2.25 \times 10^6 \text{ m}^2$$

20. It is given that z varies directly as x^2 and inversely as $\sqrt{y}$. If x is increased by 60 % and y is decreased by 36%, then z is

$$z = \frac{kx^2}{\sqrt{y}}$$

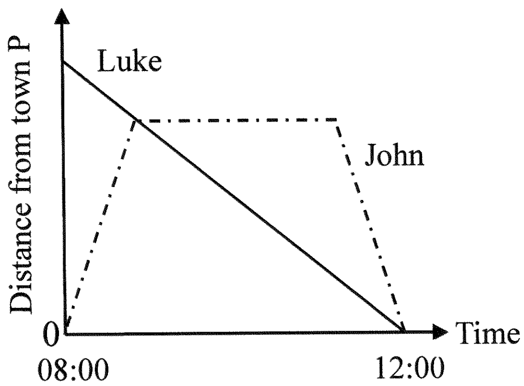
A. decreased by 220%.
B. increased by 104.8%.
✓C. increased by 220%.
D. increased by 320%.

$$z' = \frac{k[(1+60\%)x]^2}{\sqrt{(1-36\%)y}} = 3.2z$$

21. Luke starts to walk from Town Q to Town P and John starts to walk from Town P to Town Q at the same time. John takes a rest for x minutes after meeting Luke when Luke travels $\frac{1}{4}$ of the total distance. After the rest, John goes back to Town P. If John and Luke walk in a uniform speed and they arrive at Town P at the same time, find x.

A. 95
✓B. 120
C. 135
D. 150

Speed of Luke $= S_L$
Speed of John $= S_J$
$S_J = 3S_L$
time taken by John
$$= \left(\frac{3}{4}\cdot 4S_L + \frac{3}{4}4S_L\right)\cdot\frac{1}{3S_L} = \left(\frac{3}{4}+\frac{3}{4}\right)\times\frac{4}{3}$$
$$= 2\text{h} = 120\text{min}$$
$$x = 4\times 60 - 120 = 120.$$



22. There were 3 candies and 4 chocolates in a bag originally. Find the least number of chocolates required to be put into the bag such that the probability of drawing a chocolate from the bag is at least 0.72.

Let x be the number of chocolates

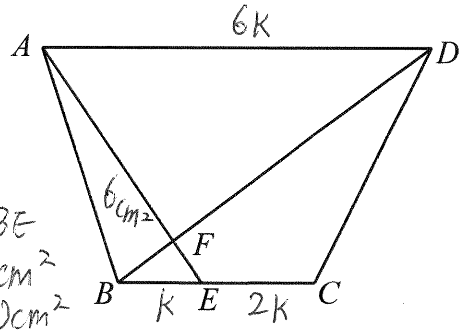
A. 3
✓B. 4
C. 5
D. 6

$$\frac{4+x}{3+4+x} \geq 0.72$$
$$4+x \geq 5.04 + 0.72x$$
$$0.28x \geq 1.04$$
$$x \geq 3.71$$

23. The figure shows a trapezium $ABCD$ with $AD \parallel BC$. AFE, BFD and BEC are straight lines. If $BE : EC = BC : AD = 1 : 2$ and area of $\triangle ABF = 6 \text{ cm}^2$, then the area of quadrilateral $CEFD$ is

A. 16 cm^2.
✓B. 20 cm^2.
C. 24 cm^2.
D. 28 cm^2.

$\triangle ADF \sim \triangle EBF$
$AD : BE = AF : EF = 6 : 1$
area of $\triangle BEF = \frac{1}{6} \times$ area of $\triangle ABF$
$= 1 \text{ cm}^2$
area of $\triangle BCD = 3 \times$ area of $\triangle ABE$
$= 3 \times (6+1) = 21 \text{ cm}^2$
area of $CEFD = 21 - 1 = 20 \text{ cm}^2$



24. For $0° \le \theta \le 90°$, the greatest value of $\dfrac{1}{3 + 2\sin(90° - \theta)}$ is

A. $\frac{1}{5}$.
✓B. $\frac{1}{3}$.
C. $\frac{1}{2}$.
D. 1.

$\dfrac{1}{3 + 2\cos\theta}$
When $\theta = 90°$, greatest value
$= \dfrac{1}{3 + 2(0)} = \dfrac{1}{3}$

25. In the figure, positive integers are arranged in the pattern such that there is one number in the first row, two numbers in the second row, three numbers in the third row and so on. In which row does the number 175 lie?

A. 17
B. 18
✓C. 19
D. 20

$1 + 2 + \cdots + n \ge 175$
$\dfrac{n(n+1)}{2} \ge 175$
$n^2 + n - 350 \ge 0$
$n \le -19.2$ or $n \ge 18.2$

1
2 3
4 5 6
…
…

26. The solutions of $2 - \dfrac{x}{3} < 1$ or $x \ge 2x - 1$ are

A. $x \le 1$.
B. $x > 3$.
✓C. $x \le 1$ or $x > 3$.
D. All real values of x.

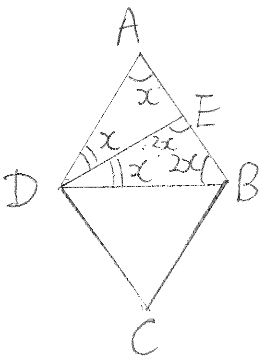
$6 - x < 3$ or $x \le 1$
$x > 3$

27. $\frac{1}{\sqrt{2020}} =$

A. 0.0222 (correct to 3 decimal places)
B. 0.02224 (correct to 4 significant figures)
C. 0.022249 (correct to 5 decimal places)
√D. 0.022250 (correct to 5 significant figures)

28. $ABCD$ is a rhombus. Let E be a point on AB such that DE bisects $\angle ADB$. If $\angle ADE = \angle BAD$, which of the following are true?

√ I. $\angle ADE = 36°$
√ II. $BD = AE$
√ III. $\Delta ABD \sim \Delta DBE$

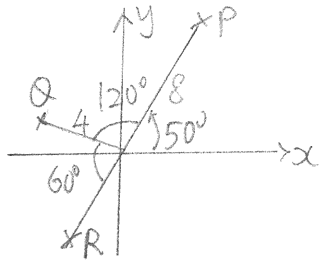


$x + x + x + 2x = 180°$
$x = 36°$

$AE = DE$
$DE = BD$
$\therefore BD = AE$

A. I and II only
B. I and III only
C. II and III only
√D. I, II and III

29. The polar coordinates of the point P, Q and R are $(8, 50°)$, $(4, 170°)$ and $(4, 230°)$ respectively. The area of ΔPQR is

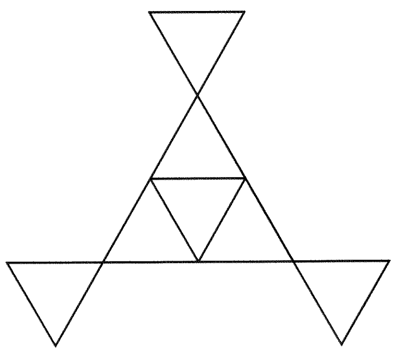


area of ΔPQR
$= \frac{1}{2} \times 4 \times 8 \sin 120° + \frac{1}{2} \times 4 \times 4 \sin 60°$
$= 8\sqrt{3} + 4\sqrt{3}$
$= 12\sqrt{3}$

√A. $12\sqrt{3}$.
B. $24\sqrt{3}$.
C. 12
D. 24.

30. The figure consists of seven identical equilateral triangles. The number of folds of rotational symmetry of the figure is

√A. 3.
B. 4.
C. 6.
D. 7.



Section B

31. Let α and β be the roots of the quadratic equation $x^2 + mx + n = 0$, where m and n are non-zero constants. Which of the following equations has the roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$?

$\alpha+\beta=-m$
$\alpha\beta = n$

A. $x^2 + (m^2 - 2n)x + 1 = 0$
B. $nx^2 + (m^2 - 2n)x + n = 0$
✓C. $nx^2 + (2n - m^2)x + n = 0$
D. $nx^2 - nx - (m^2 - 2n) = 0$

$\text{sum} = \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{m^2-2n}{n}$

$\text{product} = \frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} = 1$

32. The equation of the circle C is $3x^2+3y^2+12x-30y+10=0$. Which of the following is true?

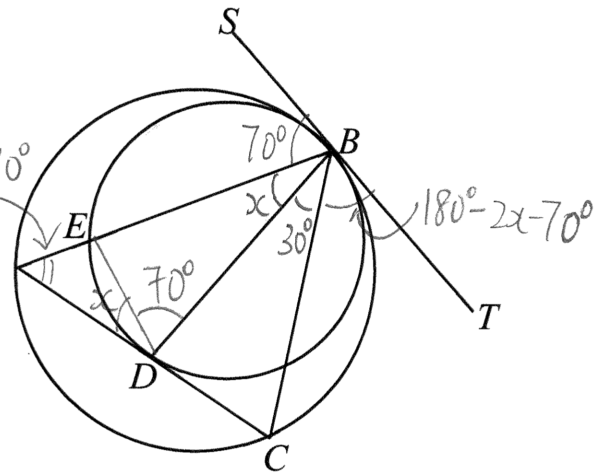
$x^2+y^2+4x-10y+\frac{10}{3}=0$

$\text{centre} = (-2, 5)$
$\text{radius} = \sqrt{(-2)^2+5^2-\frac{10}{3}} = \sqrt{\frac{77}{3}}$
$\text{area} = \pi\left(\sqrt{\frac{77}{3}}\right)^2 = \frac{77\pi}{3}$

✓A. The area of C is $\frac{77}{3}\pi$.
B. The centre of C lies in the first quadrant.
C. The origin lies inside C.
D. The point $(2, 4)$ lies on C.

33. In the figure, SBT is a common tangent to the circles ABC and BDE at the point B. ADC is a tangent to the circle BDE at the point D. If $\angle ABS = 70^\circ$ and $\angle CBD = 30^\circ$, find $\angle ABD$.

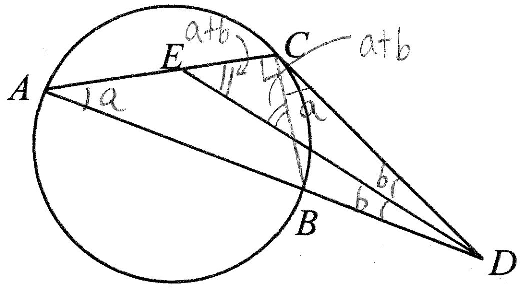
$180^\circ-2x-70^\circ$



$70^\circ+x+30^\circ+180^\circ-2x-70^\circ=180^\circ$
(adj ∠s on st line)
$x=30^\circ$

✓A. 30°
B. 35°
C. 40°
D. 45°

34. In the figure, AB is a diameter of the circle ABC. The tangent to the circle at the point C meets AB produced at the point D. E is a point on AC such that DE bisects $\angle ADC$. Find $\angle CED$.

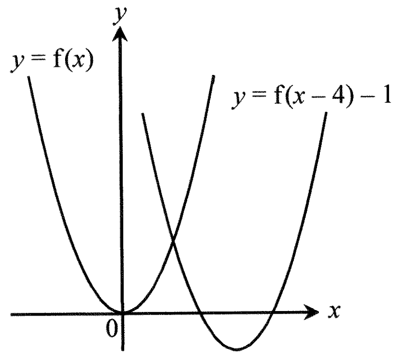


$2(a+b)+90^\circ = 180^\circ$
$\angle CED = a+b = 45^\circ$

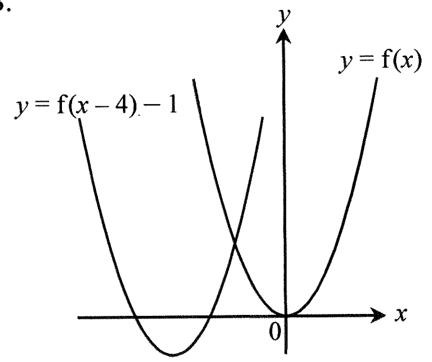
✓A. 45°
B. 52°
C. 60°
D. 65°

35. Which of the following may represent the graph of $y = f(x)$ and the graph of $y = f(x-4) - 1$ on the same rectangular coordinate system?

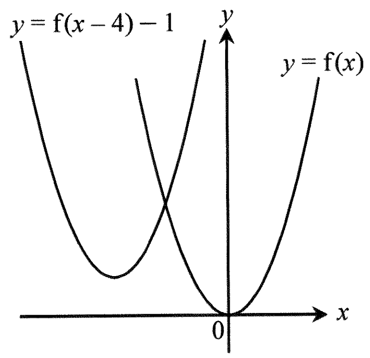
✓ A.



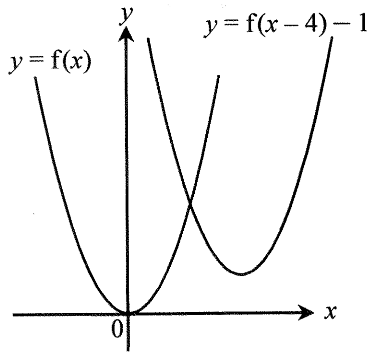
B.



C.



D.



36. The mean, median and inter-quartile range of a set of numbers $\{x_1, x_2, x_3, \ldots\ldots, x_n\}$ are p, q and r respectively. Find the mean, median and inter-quartile range of the set of numbers $\{a-3x_1, a-3x_2, a-3x_3, \ldots\ldots, a-3x_n\}$.

A. mean $= a-3p$ median $= a-3q$ inter-quartile range $= a-3r$

B. mean $= 3p$ median $= 3q$ inter-quartile range $= a+3r$

✓ C. mean $= a-3p$ median $= a-3q$ inter-quartile range $= 3r$

D. mean $= -3p$ median $= -3q$ inter-quartile range $= -3r$

37. Andy, Ben and 9 other boys are arranged to form a queue randomly. Find the probability that Andy is just in front of Ben in the queue.

A. $\frac{1}{3}$

B. $\frac{1}{9}$

✓ C. $\frac{1}{11}$

D. $\frac{1}{33}$

$\frac{10!}{11!} = \frac{1}{11}$

38. Carly and Charles take turns to toss a fair coin until one of them gets a 'Head'. Carly tosses the coin first. Find the probability that Carly gets a 'Head' first.

A. $\frac{1}{8}$

√B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. $\frac{1}{2}$

$\frac{1}{2} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} + \cdots$

$= \frac{1}{2} + \frac{1}{8} + \frac{1}{32}$

$= \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{2}{3}$

39. If k is a real number, then the imaginary part of $\frac{25k}{4+3i^3} - ki^5$ is

A. $-4k$.

B. $-2k$.

√C. $2k$.

D. $4k$.

$\frac{25k}{4-3i} - ki = \frac{25k(4+3i)}{16+9} - ki$

$= 4k + 3ki - ki$

$= 4k + 2ki$

40. For $0° \le \theta \le 360°$, how many roots does the equation $7\sin^2\theta = 6\sin\theta + 1$ have?

A. 1

B. 2

√C. 3

D. 4

$7\sin^2\theta - 6\sin\theta - 1 = 0$

$(7\sin\theta + 1)(\sin\theta - 1) = 0$

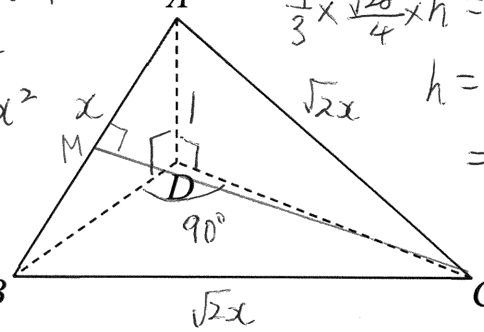
41. The figure shows a tetrahedron $ABCD$ with the base BCD lying on the horizontal ground, where $\angle BDC = 90°$. It is given that A is vertically above D and $AB : BC : AC = 1 : \sqrt{2} : \sqrt{2}$. If $AD = 1$ cm, find the distance from D to the plane ABC.

A. $\frac{2\sqrt{6}}{7}$ cm

B. $\frac{4\sqrt{6}}{7}$ cm

√C. $\frac{\sqrt{21}}{7}$ cm

D. $\frac{2\sqrt{21}}{7}$ cm



$BD^2 = x^2 - 1$

$CD^2 = (\sqrt{2}x)^2 - 1 = 2x^2 - 1$

$BD^2 + CD^2 = BC^2$

$x^2 - 1 + 2x^2 - 1 = 2x^2$

$x^2 = 2$

$x = \sqrt{2}$

$CM = \sqrt{2^2 - \left(\frac{\sqrt{2}}{2}\right)^2}$

$= \frac{\sqrt{14}}{2}$

area of $\triangle ABC = \frac{1}{2}\times\sqrt{2}\times\frac{\sqrt{14}}{2} = \frac{\sqrt{28}}{4}$

Let h cm be the distance from D to $\triangle ABC$

$\frac{1}{3}\times\frac{\sqrt{28}}{4}\times h = \frac{1}{3}\times\frac{1}{2}\times 1\times\sqrt{3}\times 1$

$h = \frac{\sqrt{3}}{2}\times\frac{4}{\sqrt{28}}$

$= \sqrt{\frac{3}{4}\times\frac{16}{28}}$

$= \sqrt{\frac{3}{7}}$

$= \frac{\sqrt{21}}{7}$

42. Find the sum of the first 36 terms of the sequence $\log_4 2$, $\log_4 2^6$, $\log_4 2^{11}$, $\log_4 2^{16}$, ...

A. 1584
✓B. 1593
C. 1638
D. 3186

$\log_4 2 + \log_4 2^6 + \log_4 2^{11}$ to 36 terms
$= \log_4 (2 \times 2^6 \times 2^{11} \times \cdots)$
$= \log_4 2^{1+6+11+\cdots \text{ to 36 terms}}$
$= \frac{36}{2}[2(1) + (36-1)(5)] \log_4 2$
$= 1593$

43. The solutions of $x(x-2) \geq x-2$ are

A. $x \leq 1$.
B. $x \geq 1$.
C. $1 \leq x \leq 2$.
✓D. $x \leq 1$ or $x \geq 2$.

$x(x-2)-(x-2) \geqslant 0$
$(x-2)(x-1) \geqslant 0$

44. $2^{12} + 5 \times 2^6 + 6 =$

A. 100101000110_2.
B. 100010100110_2.
C. 1000010100110_2.
✓D. 1000101000110_2.

$6 = 110_2$

2^{12} 2^6

45. If $2^m = 3$ and $9^n = 7$, then $\log_2 7 =$

A. $m + 2n$.
B. $2m + 2n$.
C. $\frac{1}{2}mn$.
✓D. $2mn$.

$m = \frac{\log 3}{\log 2}$, $n = \frac{\log 7}{\log 9} = \frac{\log 7}{2\log 3}$

$\log_2 7 = \frac{\log 7}{\log 2}$
$= \frac{\log 3}{\log 2} \cdot \frac{\log 7}{2\log 3} \cdot 2$
$= 2mn$

END OF PAPER