S.6 MATHEMATICS MOCK EXAM 2020 - 2021

MARKING SCHEME

PAPER 1 - CONVENTIONAL QUESTIONS

The ceiling mark deduction for "PP" is **2 marks** (@ 1 mark) and for "U" is **1 mark** (@ 1 mark) for the whole paper.

Solution		
1. $\dfrac{(2a^{-3}b)^4}{4a^2b^{-5}} = \dfrac{16a^{-12}b^4}{4a^2b^{-5}}$ $= 4a^{-12-2}b^{4+5}$ $= \dfrac{4b^9}{a^{14}}$		
2. (a) $2a^2+5ab+2b^2$ $=(2a+b)(a+2b)$ **(b)** $2a^2+5ab+2b^2-6a-12b$ $=(2a+b)(a+2b)-6(a+2b)$ $=(a+2b)(2a+b-6)$		
3. (a) $p=\dfrac{1}{4}(3q-1)$ $4p=3q-1$ $3q=4p+1$ $q=\dfrac{4p+1}{3}$ **(b)** Change in $q=\dfrac{4(p+1)+1}{3}-\dfrac{4p+1}{3}$ $=\dfrac{4p+4+1-4p-1}{3}$ $=\dfrac{4}{3}$		
4. (a) $\dfrac{-5-5x}{6}<3(x+1)$ and $2x-7<0$ $-5-5x<18x+18$ $x>-1$ and $x<\dfrac{7}{2}$ $\therefore -1<x<\dfrac{7}{2}$ **(b)** 0		

		Solution		
5.		Let $5k$ and $9k$ be the original number of read balls and yellow balls respectively. $\frac{5k+500}{9k+200}=\frac{3}{4}$ $20k+2000=27k+600$ $k=200$ ∴ the original number of red balls = 1000.		
6.	**(a)**	Let x cm be the height of John. $x(1+20\%)=150$ $x=\frac{150}{1.2}$ $x=125$ ∴ Height of John is 125 cm.		
	(b)	Height of Alice $=150\times(1-20\%)$ $=120$ cm $\neq 125$ cm ∴ Alice and John are not of the same height.		
7.	**(a)**	The maximum absolute error = 5 g The least possible weight = 460 – 5 = 455 (g)		
	(b)	The least possible total weight of 90 air tablets = 455 × 90 g = 40 950 g = 40.95 kg > 40.85 kg The claim is disagreed		
		Alternative Solution (1) $\frac{40.85}{90}$ = 0.453 888 89 kg = 453.888 89 g < 455 g The claim is disagreed.		
		Alternative Solution (2) $\frac{(40.85)(1000)}{455}$ = 89.7802198 < 90 The claim is disagreed		

	<u>Alternative Solution (3)</u> The least possible total weight of 90 air tablets = 455 × 90 g = 40 950 g = 40.95 kg = 41.0 kg (cor. to the nearest 0.1 kg) ≠ 40.8 kg The claim is disagreed		

		Solution		
8.	**(a)**	*A*' (5, 1) *B*' (−5, 4)		
	(b)	slope of *A'B* = $\frac{4-1}{3-5}=-\frac{3}{2}$ slope of *AB'* = $\frac{4-(-5)}{-5-1}=-\frac{3}{2}$ = slope of *A'B* Therefore, *A'B* parallel to *AB'*.		
9.	**(a)**	$5+a=8+b+1$ $a=b+4$ ……(1) $\frac{10+3a+32+5b+6}{5+a+8+b+1}=3.5$ $48+3a+5b=3.5a+3.5b+49$ $1.5b=0.5a+1$ $3b=a+2$ …… (2) On solving (1) and (2), $a = 7$ and $b = 3$		
	(b)	The probability $=\frac{3+1}{5+7+8+3+1}$ $=\frac{1}{6}$		
10.	**(a)**	Let $C=k_1V+k_2V^2$, where k_1 and k_2 are non-zero constants $\begin{cases}65=50k_1+2500k_2\\135=75k_1+5625k_2\end{cases}$ $\begin{cases}97.5=75k_1+3750k_2\\135=75k_1+5625k_2\end{cases}$ $37.5=1875k_2$ $k_2=0.02$ $65=50k_1+50$ $k_1=0.3$ Therefore $C=0.3V+0.02V^2$ When $V=100$		

$$C = 0.3(100) + 0.02(100)^2 = (\$)\ 230$$

(b)

$$14 = 0.3V + 0.02V^2$$
$$V^2 + 15V - 700 = 0$$
$$V = 20 \text{ or } -35 \text{(rej)}$$

The monthly consumption is 20 m^3

		Solution		
11.	**(a)**	Let $f(x) = (x^2 - x + 3)(ax + b)$ $f(-2) = -45$ and $f(1) = 21$ $9(b - 2a) = -45$ $3(a + b) = 21$ $b - 2a = -5$ $a + b = 7$ On solving, $a = 4$ and $b = 3$ The quotient is $4x + 3$		
	(b)	$f(x) = g(x)$ $(x^2 - x + 3)(4x + 3) - (4x^2 + 7x + 3) = 0$ $(x^2 - x + 3)(4x + 3) - (4x + 3)(x + 1) = 0$ $(4x + 3)(x^2 - 2x + 2) = 0$ $x = -\frac{3}{4}$ or $x^2 - 2x + 2 = 0$ Δ of the equation $x^2 - 2x + 2 = 0$ $= (-2)^2 - 4(2)$ $= -4 < 0$ So the quadratic equation $x^2 - 2x + 2 = 0$ does not have real roots. The equation $f(x) = g(x)$ has 1 rational root.		
12.	**(a)**	Let V cm^3 be the volume of the smaller circular cone. $V + \left(\sqrt{\frac{25}{9}}\right)^3 V = (38)(96\pi)$ $V = 648\pi$ $\therefore$ Volume of the smaller circular cone is $648\pi\ cm^3$.		
		<u>Alternative Solution</u> Volume ratio of the smaller cone to the larger cone $= (\sqrt{9})^3 : (\sqrt{25})^3$ $= 27 : 125$ Volume of the smaller cone $= \frac{27}{27+125} \times (38 \times 96\pi)$ $= 648\pi\ (cm^3)$		
	(b)	Height of the smaller circular cone $= \frac{3 \times 648\pi}{6^2 \pi}$ $= 54$ cm		

Curved surface area of the smaller circular cone
$= \pi(6)(\sqrt{6^2 + 54^2})$

Curved surface area of the larger circular cone
$= (\frac{25}{9})\pi(6)(\sqrt{6^2 + 54^2})$
$\approx 2845 \text{ cm}^2$

Solution		
13. **(a)** $(30 + c) - 18 = 21$ $c = 9$ **(b)** $(40 + b) - a < 40$ $b < a$ $\frac{664 + a + (40 + b)}{25} = 28.6$ $a + b = 11$ $\therefore \begin{cases} a = 6 \\ b = 5 \end{cases}$ or $\begin{cases} a = 7 \\ b = 4 \end{cases}$ **(c)** Original mean = 28.6 <u>Case (1) : Original Mode = 39 and New Mode = 42</u> New mean $= \frac{25 \times 28.6 + 2 \times 42}{27} = \frac{799}{27}$ ≈ 29.6 <u>Case (2) : Original Mode = 39 and New Mode = 44</u> New mean $= \frac{25 \times 28.6 + 2 \times 44}{27} = \frac{803}{27}$ ≈ 29.7		

Solution		
14. (a) $\angle AFE = \angle BFD$ (vert. opp. $\angle$s) $\angle AEF = \angle FDB$ ($\angle$s in the same segment) $\angle EAF = \angle FBD$ ($\angle$s in the same segment) / ($\angle$ sum of Δ) $\therefore \Delta BDF \sim \Delta AEF$ (AAA)		
Marking Scheme Correct proof with correct reasons Correct proof with wrong reasons / without reasons		
(b) ΔEDF and ΔECA **(c)** $\dfrac{AE}{FE} = \dfrac{AC}{FD}$ (corr. sides, $\sim\Delta$s) $\dfrac{5\sqrt{2}}{\sqrt{2}} = \dfrac{10}{FD}$ $FD = 2$ $\because \angle DEF = \angle BAF = 90°$ (corr. $\angle$s, $\sim\Delta$s) $ED = \sqrt{2^2 - (\sqrt{2})^2}$ $= \sqrt{2}$ $= FE$ $\therefore \Delta EDF$ is an isos. triangle. $\therefore \Delta ABF$ is an isos. triangle which $AB = AF$. $AB^2 + (AB+2)^2 = 10^2$ $AB = 6$		

		Solution		
15.	**(a)**	The required number $= C_4^{80} - C_4^{48}$ = 1 387 000		
		<u>Alternative Solution</u> $= C_3^{48}C_1^{32} + C_2^{48}C_2^{32} + C_1^{48}C_3^{32} + C_4^{32}$ = 1 387 000		
	(b)	The probability of ordering drinks of the same size $= \frac{16}{80} \times \frac{15}{79} + \frac{40}{80} \times \frac{39}{79} + \frac{24}{80} \times \frac{23}{79}$ $= \frac{147}{395}$		
		<u>Alternative Solution (1)</u> The probability of ordering drinks of the same size $= \frac{C_2^{16} + C_2^{40} + C_2^{24}}{C_2^{80}}$ $= \frac{147}{395}$		
		<u>Alternative Solution (2)</u> The probability of ordering drinks of the same size $= 1 - 2 \times \left(\frac{16}{80} \times \frac{40}{79} + \frac{16}{80} \times \frac{24}{79} + \frac{40}{80} \times \frac{24}{79}\right)$ $= 1 - \frac{248}{395}$ $= \frac{147}{395}$		

Solution		
16. **(a)** Let a and r be the first term and the common ratio of the sequence respectively. So we have $ar^2 = 420$ and $ar^3 = 280$ Solving, we have $a = 945$ and $r = \frac{2}{3}$ The 1st term is 945. **(b)** From (a), $r = \frac{2}{3}$ $\frac{945\left[\left(\frac{2}{3}\right)^n - 1\right]}{\frac{2}{3} - 1} > 2834$ $n > \log\left(\frac{2834 \times \left(-\frac{1}{3}\right)}{945} + 1\right) \div \log\left(\frac{2}{3}\right)$ $n > 19.6066124$ $\therefore$ least value of n is 20. **(c)** Sum of all terms in the sequence $< \frac{945}{1 - \frac{2}{3}}$ = 2835 $\therefore$ It cannot exceed 2835. I don't agree.		
17. **(a)** $f(x) = x^2 + 2kx + 4k^2 - 4$ $= x^2 + 2kx + k^2 + 3k^2 - 4$ $= (x + k)^2 + 3k^2 - 4$ The vertex is $(-k, 3k^2 - 4)$ **(b)** $D = (-k + 4, 3k^2 - 4)$ Since $k > 4$ $-k + 4 < 0$ $3k^2 - 4 > 3(4)^2 - 4$ > 0 $E = (3k^2 - 4, k - 4)$ Since $\angle EOD = 90°$, if the in-centre lies on the y-axis, D and E is symmetrical about the y-axis $k - 4 = 3k^2 - 4$ $3k^2 - k = 0$ $k = 0$ or $\frac{1}{3}$ However, since $k > 4$, therefore, it is impossible.		

Solution		

18. (a) (i) Γ is the angle bisector of $\angle DEF$

(ii) Slope of DE = $\dfrac{0-8}{-2-6}=1$

$\therefore$ angle between DE and the x-axis is 45°

Slope of EF = $\dfrac{0-(-7)}{-2-5}=-1$

$\therefore$ angle between EF and the x-axis is 45°

Therefore, the equation of Γ is $y = 0$.

(b) (i) Let G $(g, 0)$ be the centre of circle C.

$GH \perp DE$ (tangent $\perp$ radius)

$\angle GHE = 90°$

$\angle GEH = \angle HGE = 45°$

$EH = HG = \sqrt{(-2-0)^2+(0-2)^2}$

$= \sqrt{8}$

Coordinates of G are (2, 0)

Equation of C is $(x-2)^2+y^2=8$

or $x^2+y^2-4x-4=0$

(ii) Equation of DF :

$$\frac{y-8}{x-6}=\frac{8-(-7)}{6-5}$$

$$y = 15x - 82$$

$$\begin{cases} x^2+y^2-4x-4=0 \\ y=15x-82 \end{cases}$$

$$x^2+(15x-82)^2-4x-4=0$$

$$226x^2-2464x+6720=0$$

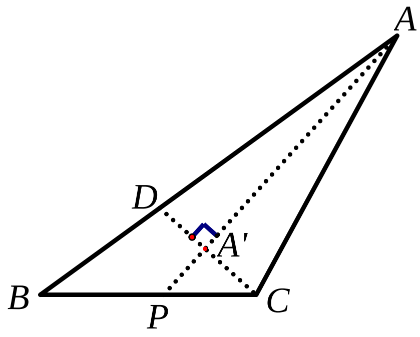
Δ of the equation $226x^2-2464x+6720=0$

$= (-2464)^2 - 4 \times 226 \times 6720$

$= -3584$

< 0

$\therefore$ DF is not tangent to C.

Solution		
19. **(a)** $\dfrac{AC}{\sin(180° - 32° - 68°)} = \dfrac{24}{\sin 32°}$ $AC = 44.60186234$ ≈ 44.6 (cm) $\dfrac{BC}{\sin 100°} = \dfrac{24}{\sin 42°}$ $BC = 35.32253023$ ≈ 35.3 cm		
(b) **(i)** $\cos\angle ACB = \dfrac{BC^2 + AC^2 - 30^2}{2(BC)(AC)}$ $\angle ACB = 42.12400765°$ $\approx 42.1°$		
(ii)  Let *A'* be the foot of perpendicular of *A* to *DC*. *AA'* produced meets *BC* at *P*. The required angle is $\angle AA'P$. $AA' = AC \sin 68°$ $A'C = AC \cos 68°$ $A'P = A'C \tan 38°$ $CP = \dfrac{A'C}{\cos 38°}$ In Figure 4(b), $AP^2 = AC^2 + CP^2 - 2(AC)(CP)\cos\angle ACB$ $AP = 32.18793019$ $\cos\angle AA'P = \dfrac{AA'^2 + A'P^2 - AP^2}{2(AA')(A'P)}$ $\angle AA'P = 38.53811011°$ $\approx 38.5°$		