HKUGA College

MOCK EXAMINATION (2021/2022)

Mathematics Compulsory Part

Paper 1

Question-Answer Book

TOTAL MARKS: 105

Time allowed: 2 hours 15 minutes Form: 6

Name: _____ Class (No.): _____ ( )

Teacher: CC / HC / JY / MS / MY / SKC / WC

INSTRUCTIONS

1. This paper consists of 24 pages including this cover page. The words "End of Paper" should appear on the last page.

2. Do not open this exam paper until instructed to do so.

3. This paper consists of **THREE** sections, A(1), A(2) and B. Each section carries 35 marks.

4. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.

5. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class and class number on each sheet.

6. Unless otherwise specified, all working must be clearly shown.

7. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

8. The diagrams in this paper are not necessarily drawn to scale.

9. The use of an HKEAA-approved calculator is permitted.

SECTION A(1) (35 marks)

1. Simplify $\left(x^2y\right)\left(-3xy^{-2}\right)^4$ and express your answer with positive indices. (3 marks)

2. Make x the subject of the formula $\frac{1}{x}+\frac{2}{y}=\frac{3}{z}$. (3 marks)

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3. Factorize

(a) $12x^2 - xy - 6y^2$,

(b) $9x + 6y - 12x^2 + xy + 6y^2$.

(3 marks)

4. (a) Solve the inequality $\frac{20-8x}{3} \leq -2(x-1)$.

(b) Find the number of integers satisfying both inequalities $\frac{20-8x}{3} \leq -2(x-1)$ and $30 - 3x > 0$.

(4 marks)

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5. The marked price of a wallet is 56.25% higher than the cost. The wallet is sold at a discount of 20% on its marked price. After selling the wallet, the profit is $100. Find the cost of the wallet. (4 marks)

6. In a park, the ratio of the number of adults to the number of children is 14 : 9. If 2 adults leave the park and 3 children enter the park, then the ratio of the number of adults to the number of children is 4 : 3. Find the original number of children in the park. (4 marks)

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7. In a polar coordinate system, O is the pole. The polar coordinates of the points A and B are $(6\ ,\ 35°)$ and $(b\ ,\ 125°)$ respectively. It is given that $AB = 10$.

(a) Find b.

(b) Find the perpendicular distance from O to AB.

(4 marks)

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8. The length of a piece of thin metal wire is measured as 8.2 m correct to the nearest 0.1 m.

(a) Is it possible that the actual length of this metal wire exceeds 826 cm? Explain your answer.

(b) Is it possible to cut this metal wire into 28 pieces of shorter metal rods, with each length measured as 30 cm correct to the nearest cm? Explain your answer.

(5 marks)

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9. In Figure 1, AB produced and CD produced meet at the point E. It is given that $\angle ACE = \angle DBE$.

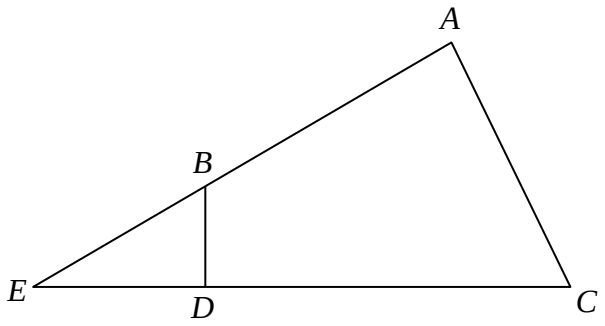

Figure 1

(a) Prove that $\Delta ACE \sim \Delta DBE$.

(b) It is given that $AC = 117$ cm, $EA = 240$ cm, $EC = 267$ cm and $BD = 39$ cm.

(i) Is ΔACE a right-angled triangle? Explain your answer.

(ii) Find the area of ΔBDE.

(5 marks)

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SECTION A(2) (35 marks)

10. The total cost for producing n cars is \$$S$, where n is a positive integer. S is partly constant and partly varies as n. When $n = 2$, $S = 280\,000$. When $n = 6$, $S = 600\,000$.

    (a) Find the total cost for producing 20 cars. (4 marks)

    (b) Peter claims that the total cost for producing $2n$ cars is less than twice the total cost for producing n cars for some positive integer n. Do you agree? Explain your answer. (2 marks)

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11. The following table shows the distribution of the number of mock paper(s) done by a group of students in a certain week.

Number of mock paper(s)	1	2	3	4	5
Number of students	*a*	20	9	8	2

It is given that the mean of the above distribution is 2.75.

(a) Find *a*. (2 marks)

(b) Find the median, the inter-quartile range and the standard deviation of the above distribution. (3 marks)

(c) It is found that the number of mock paper(s) done by one of the students is wrongly recorded. After making the correction, the range of the distribution remains the same. Find the maximum and minimum possible values of the mean after making correction. (2 marks)

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12. A glass container is in a shape of a vertically inverted right circular cone of base radius 48 cm and height 36 cm. Initially, the container is empty. Suppose the water is being added at a constant rate of $x\pi$ cm^3/s . The container will be full after 64 minutes.

(a) Find x. (2 marks)

(b) Find the wet surface area of the container when the water has been added to the container for 27 minutes. (4 marks)

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13. The equation of the circle C is $4x^2+4y^2-40x+96y-6549=0$. Let G be the centre of C. Denote the origin by O.

(a) Find OG. (2 marks)

(b) Does O lie inside C? Explain your answer. (1 mark)

(c) Let P be a moving point in the rectangular coordinate plane such that $OP = GP$. Denote the locus of P by Γ. Suppose that Γ cuts C at the points A and B. Find the area of the quadrilateral $OAGB$. (4 marks)

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14. Let $f(x) = 6x^3 - 11x^2 - 15x - 37$. When $f(x)$ is divided by $3x^2 + ax - 5$, the quotient and the remainder are $bx - 3$ and $cx + d$ respectively, where a , b , c and d are constants.

(a) Find a and b.

(4 marks)

(b) Let $g(x)$ be a polynomial with degree greater than 2. When $g(x)$ is divided by $3x^2 + ax - 5$, the remainder is $cx + d$.

(i) Prove that $f(x) - g(x)$ is divisible by $3x^2 + ax - 5$.

(ii) Edan claims that all the roots of the equation $f(x) - g(x) = 0$ are rational. Do you agree? Explain your answer.

(5 marks)

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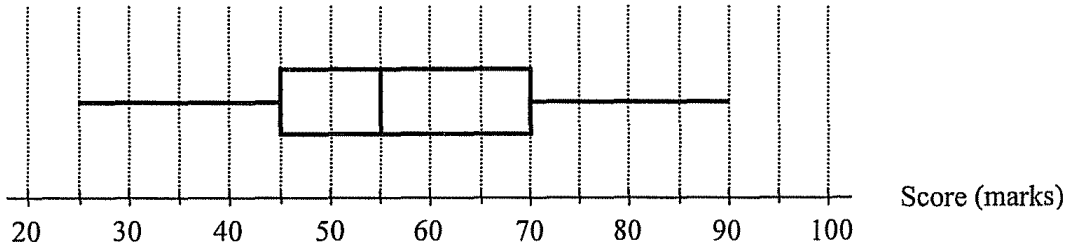
SECTION B (35 marks)

15. The box-and-whisker diagram below shows the distribution of the scores (in marks) of the students of a class in a test. Betty gets the lowest score while Ken gets 73 marks in the test. The standard scores of Betty and Ken in the test are –3 and 1 respectively.



(a) Find the mean of the distribution. (2 marks)

(b) Betty claims that the standard scores of at least half of the students in the test are positive. Do you agree? Explain your answer. (2 marks)

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16. There are 7 boys and 9 girls in a class. 5 students are randomly selected from the class to form a committee.

(a) How many different committee can be formed? (2 marks)

(b) What is the probability that the committee formed only have boys? (2 marks)

(b) What is the probability that the committee formed have members of both genders? (2 marks)

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17. Let α and β be real numbers such that $\begin{cases} \beta = 3\alpha - 4 \\ \beta = \alpha^2 - 5\alpha + 12 \end{cases}$.

(a) Find α and β. (2 marks)

(b) The 1st term and the 2nd term of an arithmetic sequence are $\log \alpha$ and $\log \beta$ respectively. Find the least value of n such that the sum of the first n terms of the sequence is greater than 2022. (4 marks)

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18. Figure 2 shows a geometric model *ABCD* in the form of tetrahedron where ΔBCD lies on a horizontal plane. It is given that AB = 15 cm, AC = 19 cm, $\angle ABD = 90°$, $\angle BAC = 49°$, $\angle BCD = 51°$ and $\angle BDC = 31°$.

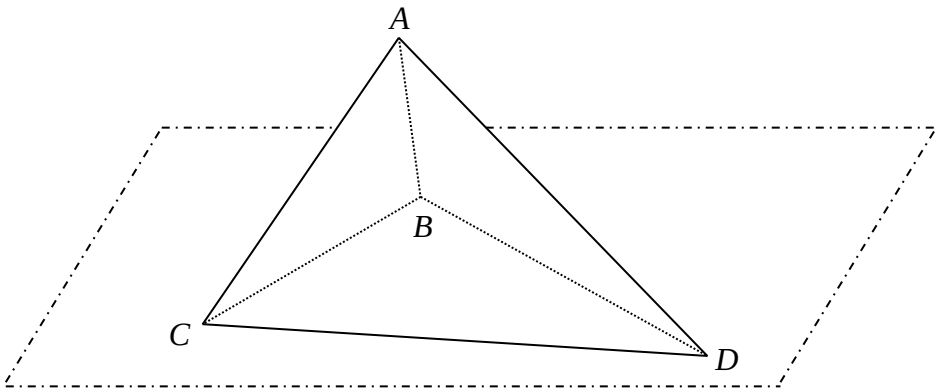

Figure 2

(a) Find *BC* and *AD*. (5 marks)

(b) Let *P* be a movable point on *CD*. Find the greatest possible inclination of *AP*. (4 marks)

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19. Let $f(x) = 2x^2 - 4(k-1)x + 4k^2 - 4k - 8$, where k is a positive constant and $k \neq 1$. P is the vertex of the graph of $y = f(x)$.

(a) Using the method of completing the square, express the coordinates of P in terms of k.

(2 marks)

(b) The graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ in x–axis and then translating the resulting graphs upwards by 8 units. Let Q be the vertex of the graph of $y = g(x)$. Denote the origin by O.

(i) Write down, in terms of k, the coordinates of Q.

(ii) Is it possible that the circumcentre of ΔOPQ lies on the x–axis? Explain your answer.

(iii) The coordinates of the point R are (–5 , 4). It is given that the graph of $y = f(x)$ passes through O. Are P, Q, O and R concyclic? Explain your answer.

(8 marks)

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