HKUGA College

MOCK EXAMINATION(2021/2022)

Mathematics Compulsory Part

Paper 2

Marking Scheme

TOTAL MARKS: 45

Time allowed: 1 hour 15 minutes

Form: 6

Name: ______________________

Class (No.): ________ (   )

Teacher: CC / HC / JY / MS / MY / SKC / WC

INSTRUCTIONS

1. This paper consists of 33 pages including this cover page. The words "**END OF PAPER**" should appear on the last page.

2. Do not open this exam paper until instructed to do so.

3. All questions carry equal marks.

4. **ANSWER ALL QUESTIONS**. You are advised to use an HB pencil to mark all the answers on the MC Answer Sheet, so that wrong marks can be completely erased with a clean rubber. You must mark the answers clearly; otherwise you will lose marks if the answers cannot be captured.

5. You should mark only **ONE** answer for each question. If you mark more than one answer, you will receive **NO MARKS** for that question.

6. No marks will be deducted for wrong answers.

7. The use of an HKEAA-approved calculator is permitted.

There are 30 questions in Section A and 15 questions in Section B.
The diagrams in this paper are not necessarily drawn to scale.
Choose the best answer for each question.

Section A

1. $\dfrac{2^{111} \times 4^{222}}{8^{333}} =$

   A. 1.

   B. $\dfrac{1}{2^{444}}$.

   C. 2^{444}.

   D. 0.

ANS: B

$$\frac{2^{111} \times 4^{222}}{8^{333}}$$
$$= \frac{2^{111} \times 2^{444}}{2^{999}}$$
$$= \frac{1}{2^{999-111-444}}$$
$$= \frac{1}{2^{444}}$$

2. $3ab + ac - ad - 21b - 7c + 7d =$

   A. $(a+7)(3b-c+d)$.

   B. $(a-7)(3b-c+d)$.

   C. $(a+7)(3b+c-d)$.

   D. $(a-7)(3b+c-d)$.

ANS: D

$$3ab + ac - ad - 21b - 7c + 7d$$
$$= a(3b+c-d) - 7(3b+c-d)$$
$$= (a-7)(3b+c-d)$$

3. $\dfrac{1}{4k+5} - \dfrac{5}{4k-5} =$

A. $\dfrac{-16k-30}{16k^2-25}$.

B. $\dfrac{8k}{16k^2-25}$.

C. $\dfrac{-10}{16k^2-25}$.

D. $\dfrac{16k+20}{16k^2-25}$.

ANS: A

$\dfrac{1}{4k+5} - \dfrac{5}{4k-5}$

$= \dfrac{4k-5-5(4k+5)}{(4k+5)(4k-5)}$

$= \dfrac{-16k-30}{16k^2-25}$

4. If a , b and c are non-zero constants such that $a(2x-1)+b(x+2) \equiv c(x+1)$, then $a:b =$

A. 1 : 3 .

B. 1 : 4 .

C. 3 : 1 .

D. 4 : 1 .

ANS: A

When $x = -1$,

$-3a + b = 0$

$3a = b$

$\dfrac{a}{b} = \dfrac{1}{3}$

$a:b = 1:3$

5. If $f(2x+1)=\dfrac{1}{2x}$, then $f(x-1)=$

A. $\dfrac{1}{x-2}$.

B. $\dfrac{1}{x-1}$.

C. $\dfrac{1}{2x-2}$.

D. $\dfrac{1}{2x-1}$.

ANS: A

$$f(2x+1)=\frac{1}{2x}=\frac{1}{(2x+1)-1}$$

$$f(x)=\frac{1}{x-1}$$

$$f(x-1)$$

$$=\frac{1}{(x-1)-1}$$

$$=\frac{1}{x-2}$$

6. The solution of $8x+2>2(x-2)$ or $9-2x\geq 5$ is

A. $x\geq 2$.

B. $x>-1$.

C. $-1<x\leq 2$.

D. all real numbers.

ANS: D

$8x+2>2(x-2)$ or $9-2x\geq 5$

$8x+2>2x-4$ or $4\geq 2x$

$6x>-6$ or $2x\leq 4$

$x>-1$ or $x\leq 2$

$\therefore$ All real numbers

7. Betty sells two watches for $4500 each. She gains 12.5% on one and loses 10% on the other. After the two transactions, Betty

A. loses $112.5.

B. gains $112.5.

C. gains $225.

D. has no gain and no loss.

ANS: D

Let the cost prices of the two watches be $\$C_1$ and $\$C_2$ respectively.

$C_1 \times (1 + 12.5\%) = 4500$

$C_1 = 4000$

$C_2 \times (1 - 10\%) = 4500$

$C_2 = 5000$

Total cost

= 4000 + 5000

= $9000

= $4500 × 2

= Total selling price

8. A sum of $85 000 is deposited at an interest rate of 8% per annum for 10 years, compounded quarterly. Find the interest correct to the nearest dollar.

A. $98 509

B. $102 683

C. $103 669

D. $187 683

ANS: B

Interest

$$= 85000 \times \left(1 + \frac{8\%}{4}\right)^{10 \times 4} - 85000$$

= $102 683 (cor. To the nearest dollar)

9. Let a_n be the nth term of a sequence. If $a_3 = 11$, $a_6 = 47$ and $a_{n+2} = a_n + a_{n+1}$ for any positive integer n , then $a_1 =$

A. 4 .

B. 7 .

C. 18 .

D. 29 .

ANS: A

$$11 + a_4 = a_5 --- (1)$$
$$a_4 + a_5 = 47 --- (2)$$
$$Sub\ (1)\ \text{into}\ (2),$$
$$2a_4 + 11 = 47$$
$$a_4 = 18$$
$$a_5 = 29$$
$$a_2 + 11 = 18$$
$$a_2 = 7$$
$$a_1 + 7 = 11$$
$$a_1 = 4$$

10. Let p(x) be a polynomial. When p(x) is divided by $2x + 1$, the remainder is 2. If p(x) is divisible by $1 - 2x$, find the remainder when p(x) is divided by $4x^2 - 1$.

A. $2x + 1$

B. $2x - 1$

C. $-2x + 1$

D. $-2x - 1$

ANS: C

Let $p(x) = (4x^2 - 1)q(x) + Ax + B$, where A and B are constants.

$\text{p}\left(\frac{-1}{2}\right) = 2$, we have

$$\frac{-A}{2} + B = 2$$

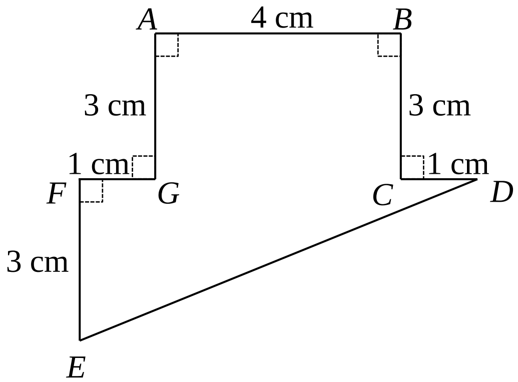
$\text{p}\left(\frac{1}{2}\right) = 0$, we have

$$\frac{A}{2} + B = 0$$

By solving, we have $A = -2$ and $B = 1$

∴ The required remainder is $-2x + 1$

11. In the figure, *ABCDEFG* is a 7-sided polygon, where all the measurements are correct to the nearest cm. Let x cm^2 be the actual area of the polygon. Find the range of values of x.



A. $6.375 < x < 36.875$

B. $11.5 < x < 29$

C. $15.625 \leq x < 27.125$

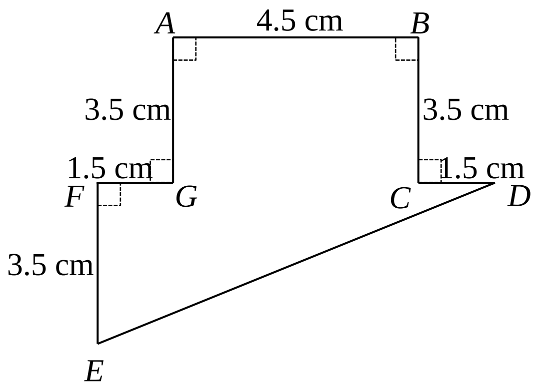
D. $14.375 \leq x < 28.875$

ANS: D

Upper limit of the area

$= (4.5)(3.5) + \frac{1}{2}(1.5 + 4.5 + 1.5)(3.5)$

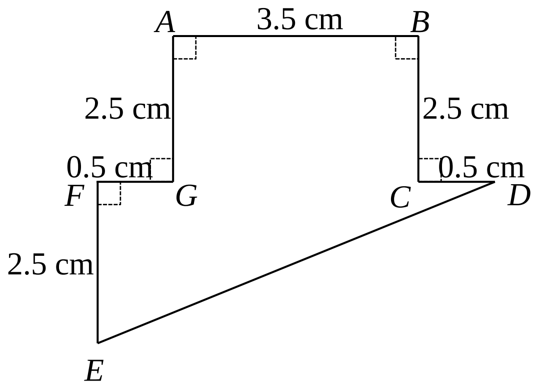
$= 28.875$ cm^2



Lower limit of the area

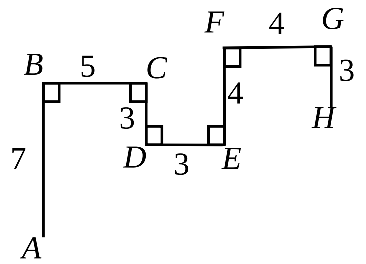
$= (3.5)(2.5) + \frac{1}{2}(0.5 + 3.5 + 0.5)(2.5)$

$= 14.375$ cm^2



$\therefore 14.375 \leq x < 21.125$

12. In the figure, the length of the line segment joining *A* and *H* is



A. 12.

B. 13.

C. 17.

D. 29.

ANS: B

AH

$= \sqrt{(7-3+4-3)^2+(5+3+4)^2}$

= 13

13. If w varies directly as the square root of x and inversely as the cube of y, which of the following must be constant?

I. $\dfrac{x}{w^2y^6}$

II. $\dfrac{wy^3}{x^2}$

III. $\dfrac{wy^3}{\sqrt{x}}+2021$

A. I only

B. I and II only

C. I and III only

D. II and III only

ANS: C

$w=\dfrac{C\sqrt{x}}{y^3}$, where C is a constant.

$C=\dfrac{wy^3}{\sqrt{x}}$

$$\frac{x}{w^2y^6}=\frac{1}{\left(\frac{wy^3}{\sqrt{x}}\right)^2}=\frac{1}{C^2}$$

$\therefore \dfrac{x}{w^2y^6}$ is a variation constant.

$$\frac{wy^3}{\sqrt{x}} + 2021 = C + 2021$$

$$\therefore \frac{wy^3}{\sqrt{x}} + 2021 \text{ is a variation constant.}$$

14. Which of the following statements about the graph of $y = (-x+1)^2 - 2$ is/are true?

I. The graph opens downwards.

II. The y-intercept is –2.

III. The graph passes through the point (4 , 7).

A. I only

B. III only

C. I and II only

D. II and III only

ANS: B

$$y = (-x+1)^2 - 2$$
$$= (x-1)^2 - 2$$

The graph opens upwards.

I is incorrect.

When x = 0,

$$y = (0+1)^2 - 2$$
$$= -1$$
$$\neq -2$$

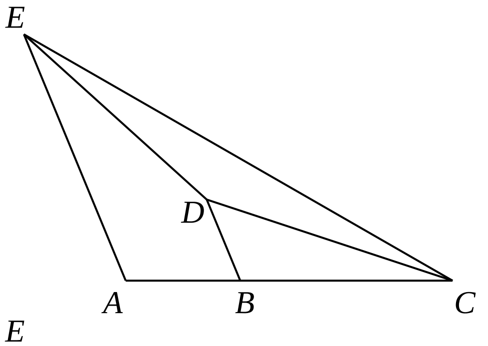
II is incorrect

Sub (4 , 7) into $y = (-x+1)^2 - 2$

R.H.S. = $(-4 + 1)^2 - 2 = 7$ = L.H.S.

III is correct.

15. In the figure, B is a point lying on AC such that $2AB = BC$. It is given that AE // BD and $AE : BD = 3 : 1$. If the area of ΔCDE is 24 cm^2, then the area of the trapezium $ABDE$ is



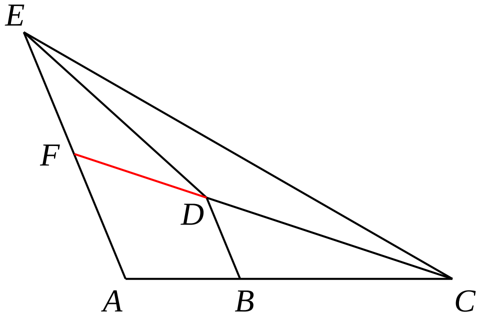
A. 12 cm^2.

B. 20 cm^2.

C. 32 cm^2.

D. 52 cm^2.

ANS: C



Extend CD to F where F lies on AE.

$FD : DC = AB : BC = 1 : 2$

$$\frac{\text{Area of } \Delta EFD}{\text{Area of } \Delta CDE} = \frac{FD}{DC}$$

$$\frac{\text{Area of } \Delta EFD}{24} = \frac{1}{2}$$

Area of $\Delta EFD = 12 \text{ cm}^2$

$DB : FA = BC : AC = 2 : 3$

$AE : BD = 3 : 1$

$DB : AE : FA = 2 : 6 : 3$

We have $EF = FA$

Area of ΔCAF

= Area of ΔBFD

= 12 + 24

= 36 cm^2

Area of ΔBDC

$= \text{Area of } \Delta CAF \times \left(\frac{2}{3}\right)^2$

$= 36 \times \left(\frac{2}{3}\right)^2$

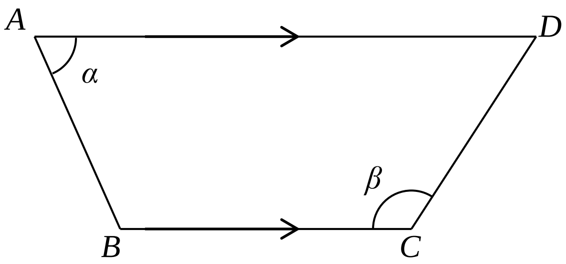
= 16 cm^2

Area of the trapezium $ABDE$

= 72 – 16 – 24

= 32 cm^2

16. In the figure, $CD =$



A. $\dfrac{AB\sin\alpha}{\sin\beta}$.

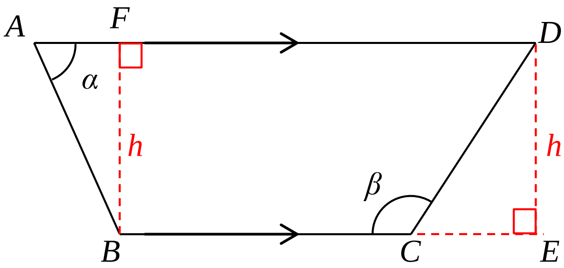
B. $\dfrac{AB\sin\beta}{\sin\alpha}$.

C. $-\dfrac{AB\sin\alpha}{\sin\beta}$.

D. $\dfrac{AB\cos\beta}{\sin\alpha}$.

ANS: A

In ΔABF,



$$\frac{h}{AB} = \sin\alpha$$
$$h = AB\sin\alpha$$

In ΔCDE,

$$\frac{h}{CD} = \sin\angle DEC$$
$$CD = \frac{h}{\sin(180° - \beta)}$$
$$CD = \frac{AB\sin\alpha}{\sin\beta}$$

17. $[\sin(270° + \theta) + 1][\cos(180° - \theta) - 1] =$

A. $\sin^2\theta$.

B. $-\sin^2\theta$.

C. $\cos^2\theta$.

D. $-\cos^2\theta$.

ANS: B

$$[\sin(270° + \theta) + 1][\cos(180° - \theta) - 1]$$
$$= (-\cos\theta + 1)(-\cos\theta - 1)$$
$$= -(1 - \cos\theta)(1 + \cos\theta)$$
$$= -(1 - \cos^2\theta)$$
$$= -\sin^2\theta$$

18. The sum of the volumes of two solid spheres is 2 340π cm^3. If the ratio of the radius of the larger sphere to the radius of the smaller sphere is 4 : 1, then the sum of the surface areas of the two spheres is

A. 153π cm^2.

B. 306π cm^2.

C. 576π cm^2.

D. 612π cm^2.

ANS: D

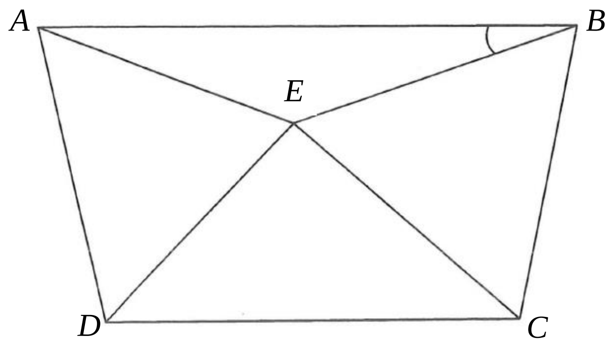
Let the radius of the smaller sphere be *r* cm.

$$\frac{4}{3}\pi(4r)^3 + \frac{4}{3}\pi r^3 = 2340$$
$$r = 3$$

Sum of the surface area of the two spheres

$$= 4\pi(12)^2 + 4\pi(3)^2$$
$$= 612\pi \text{ cm}^2$$

19. In the figure, *BCE* is an equilateral triangle and *ADE* is an isosceles triangle with *AE* = *ED*. If ∠*DAE* = 56° and ∠*EDC* = ∠*ECD* = 41°, then ∠*ABE* =



A. 17°.

B. 19°.

C. 21°.

D. 23°.

ANS: D

$$\angle DEC = 180° - 41° \times 2 = 98°$$
$$\angle AED = 180° - 56° \times 2 = 68°$$
$$\angle CEB = 60°$$
$$\angle AEB = 360° - 98° - 68° - 60° = 134°$$

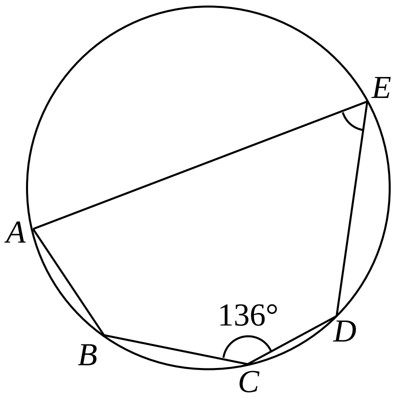
∠*EDC* = ∠*ECD* = 41°

DE = *CE*

AE = *BE*

$$\angle ABE = \frac{180° - 134°}{2} = 23°$$

20. In the figure, *ABCDE* is a circle. If *AB* = *BC* = *CD* and ∠*BCD* = 136°, then ∠*AED* =



A. 44°.

B. 66°.

C. 68°.

D. 72°.

ANS: B

Join *BE* and *CE* .

Let ∠*AEB* = ∠*BEC* =∠*CED* = *x* .

(equal chords, equal ∠s)

∠*BED* + ∠*BCD* = 180° (opp. ∠s, cyclic quad.)

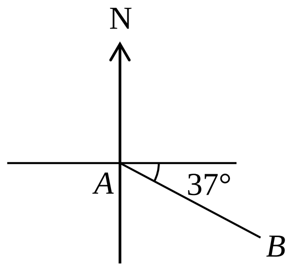
$2x + 136° = 180°$

$x = 22°$

∠*AED* = 3*x*

= <u>66°</u>

21. In the figure, the bearing of *A* from *B* is



A. N 37° W .

B. N 53° W .

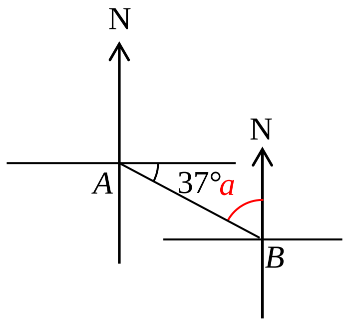
C. S 37° E .

D. S 53° E .

ANS: B

a = 90° – 37° = 53°

The bearing of *A* from *B* is N37°W



22. If the interior angle of a regular *n*-sided polygon is greater than the exterior angle by 160°, which of the following are true?

I. The value of *n* is 18.

II. Each exterior angle is 10°.

III. The number of axes of reflectional symmetry of the polygon is 36.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

ANS: C

Let the size of each exterior angle be x.

Then the size of each interior angle is $160° + x$.

$x + 160° + x = 180°$

$x = 10°$

II is true

$n \times 10° = 360°$

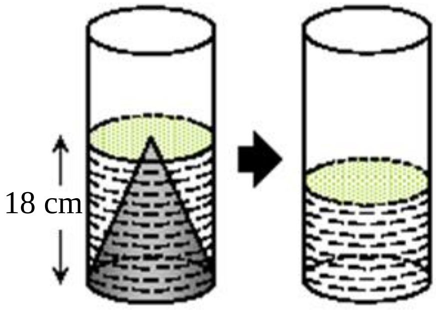
$n = 36$

I is false

A regular 36-sided polygon has 36 axes of reflectional symmetry.

III is true

23. In the figure, a solid right circular cone of height of 18 cm is put into a cylinder which has the same internal radius as the base radius of the cone. Water is then poured into the cylinder until water level just reaches the tip of the cone. If the cone is removed, what is the height of water in the cylinder?



A. 6 cm

B. 9 cm

C. 12 cm

D. 15 cm

ANS: C

Let the base radius of the cone be r cm. Let the water level in the cylinder be h cm.

$$\pi r^2(18)=\frac{1}{3}\pi r^2(18)+\pi r^2 h$$

$$12\pi r^2=\pi r^2 h$$

$$h=12$$

24. A point *A* is rotated anticlockwise about the origin through 270° to the point *B*. *B* is then translated upwards by 3 units to the point *C*. If the coordinates of *C* are (7 , –2), find the *y*-coordinate of *A*.

    A. –7
    B. 5
    C. 7
    D. 9

    ANS: C

    Let the coordinates of *A* be (*a* , *b*)

    Coordinates of *B* = (*b* , –*a*)

    Coordinates of *C* = (*b* , –*a* + 3)

    Since coordinates of *C* are (7 , –2), we have

    a = 5, *b* = 7

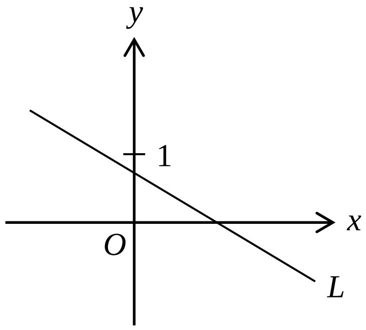
    y-coordinate of *A* = *b* = 7

25. *A* and *B* are two fixed points in the rectangular coordinate plane. If *P* is a moving point such that *PA* is perpendicular to *PB*, then the locus of *P* is a

    A. circle.
    B. straight line.
    C. parabola.
    D. triangle.

    ANS: A

26. In the figure, the equation of the straight line L is $x - ay - b = 0$. Which of the following must be true?



I. $a < 0$

II. $b < 0$

III. $a + b > 0$

A. I only

B. I and II only

C. I and III only

D. II and III only

ANS: A

$x - ay - b = 0$

$y = \frac{x}{a} - \frac{b}{a}$

Slope is negative, we have

$\frac{1}{a} < 0$

$a < 0$

I is true

y-intercept > 0, we have

$-\frac{b}{a} > 0$

$b > 0$

II is false

y-intercept < 1, we have

$-\frac{b}{a} < 1$

$-b > a$

$a + b < 0$

III is false

27. The equations of the straight line L and the circle C are $kx - 5y + k = 0$ and $2x^2 + 2y^2 - 8x - 12y + 15 = 0$ respectively, where k is a constant. If L divides C into two equal parts, find the y-intercept of L.

A. 5

B. $\frac{6}{5}$

C. 1

D. –1

ANS: C

$2x^2 + 2y^2 - 8x - 12y + 15 = 0$

$x^2 + y^2 - 4x - 6y + \frac{15}{2} = 0$

Centre of C = $\left(-\frac{-4}{2}, -\frac{-6}{2}\right)$ = (2 , 3)

Sub (2 , 3) into $kx - 5y + k = 0$,

$2k - 5(3) + k = 0$

$k = 5$

equation of the straight line:

$5x - 5y + 5 = 0$

$y = x + 1$

y-intercept is 1

28. A box contains 4 blue balls and 3 red balls. If two balls are randomly drawn from the box one by one with replacement, then the probability of drawing one red ball and one blue ball is

A. $\frac{2}{7}$.

B. $\frac{12}{49}$.

C. $\frac{4}{7}$.

D. $\frac{24}{49}$.

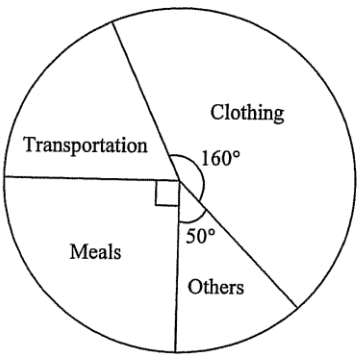
ANS: D

		2nd ball						
		B	B	B	B	R	R	R
1st ball	B	BB	BB	BB	BB	BR	BR	BR
	B	BB	BB	BB	BB	BR	BR	BR
	B	BB	BB	BB	BB	BR	BR	BR
	B	BB	BB	BB	BB	BR	BR	BR
	R	RB	RB	RB	RB	RR	RR	RR
	R	RB	RB	RB	RB	RR	RR	RR
	R	RB	RB	RB	RB	RR	RR	RR

From the table,

The required probability = $\frac{24}{49}$.

29. The pie chart below shows the expenditure of Ian in a certain month. Ian spends \$900 on transportation that month. Find his expenditure on clothing that month.



A. \$1350

B. \$1800

C. \$2400

D. \$5400

Ans: C

Let the total expenditure on that month be $$x$.

$$x \times \frac{360° - 160° - 50° - 90°}{360°} = 900$$

$$x = 5400$$

Ian's expenditure on clothing that month

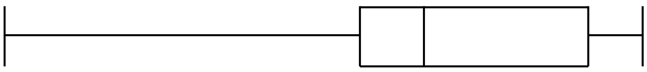
$$= 5400 \times \frac{160°}{360°}$$

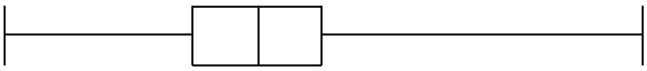
= $2400

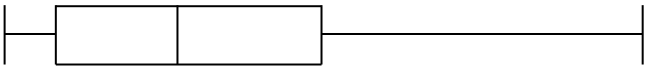
30. The stem-and-leaf diagram below shows the distribution of the daily salaries (in $100) of some employees of a company.

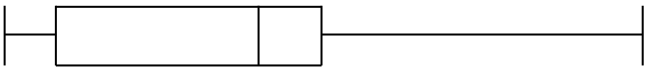
Stem (tens)	Leaf (units)
1	8 9 9
2	1 1 7 9
3	0 0 2 2 2 2 5 6 7
4	9
5	1

Which of the following box-and-whisker diagrams may represent the distribution of their daily salaries?

A. 

B. 

C. 

D. 

ANS: D

Minimum datum = 18

Q_1 = 21

$Q_2 = \frac{30+32}{2} = 31$

Q_3 =35

Maximum datum = 51

Section B

31. $300ACE0_{16} =$

A. $3 \times 16^6 + 10 \times 16^3 + 12 \times 16^2 + 14 \times 16$.

B. $3 \times 16^6 + 11 \times 16^3 + 13 \times 16^2 + 15 \times 16$.

C. $3 \times 16^7 + 10 \times 16^4 + 12 \times 16^3 + 14 \times 16^2$.

D. $3 \times 16^7 + 11 \times 16^4 + 13 \times 16^3 + 14 \times 16^2$.

ANS: A

32. Let k be a constant. Find the values of k such that $x^2 + (k-1)x + 9 > 0$ for any real number x.

A. $-5 < k < 7$

B. $-7 < k < 5$

C. $k < -5$ or $k > 7$

D. $k < -7$ or $k > 5$

ANS: A

$\Delta < 0$

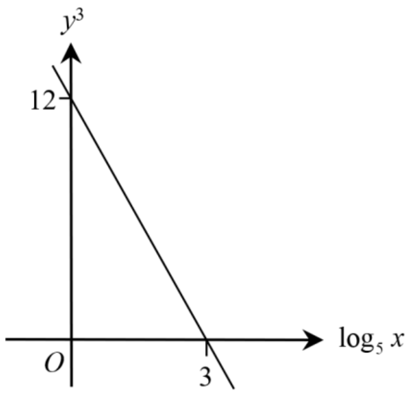
$(k-1)^2 - 4(1)(9) < 0$

$(k-1+6)(k-1-6) < 0$

$(k+5)(k-7) < 0$

$-5 < k < 7$

33. The graph in the figure shows the linear relation between $\log_5 x$ and y^3. If $x = 5$, then $y =$



A. 2.

B. 8.

C. –2.

D. –8.

ANS: A

$y^3 = \frac{12-0}{0-3}\log_5 x + 12$

$y^3 = -4\log_5 x + 12$

When $x = 5$

$y^3 = -4\log_5 5 + 12$

$y^3 = 8$

$y = 2$

34. If a is a real number and n is an integer, then the real part of $\dfrac{ai^{4n+1}-2i^{4n+2}}{ai^{4n+3}+2i^{4n}}$ is

A. 1.

B. $\dfrac{4a}{4+a^2}$.

C. $\dfrac{4-a^2}{4+a^2}$.

D. $\dfrac{4+a^2}{4-a^2}$.

ANS: C

$$\frac{ai^{4n+1}-2i^{4n+2}}{ai^{4n+3}+2i^{4n}}$$

$$=\frac{ai+2}{-ai+2}$$

$$=\frac{ai+2}{-ai+2}\times\frac{ai+2}{ai+2}$$

$$=\frac{(ai+2)^2}{4-a^2i^2}$$

$$=\frac{4+4ai+a^2i^2}{4+a^2}$$

$$=\frac{4+4ai-a^2}{4+a^2}$$

$$=\frac{4-a^2}{4+a^2}+\frac{4a}{4+a^2}i$$

Real part is $\dfrac{4-a^2}{4+a^2}$

35. Consider the following system of inequalities:

$$\begin{cases} x-3y+18\geq 0 \\ 2x+y+1\leq 0 \\ -1\leq y\leq 3 \end{cases}$$

Let R be the region which represents the solution of the above system of inequalities. Find the constant k such that the minimum value of $2x + 3y + k$ is 7, where (x, y) is a point lying in R.

A. 2

B. 10

C. 16

D. 52

ANS: D

The vertex of the region R are (–9 , 3), (–2 , 3), (–21 , –1), (0 , –1).

Let $V = 2x + 3y + k$

At (–9 , 3), $V = -9 + k$

At (–2 , 3), $V = 5 + k$

At (–21 , –1), $V = -45 + k$

At (0 , –1), $V = -3 + k$

Minimum value will be obtained at (–21 , –1).

$-45 + k = 7$

$k = 52$

36. If a, b, c, d is an arithmetic sequence, which of the following must be true?

I. $ad = bc$

II. $a + d = b + c$

III. $a < b < c < d$

A. I only

B. II only

C. I and III only

D. II and III only

ANS: B

Let the common difference be x.

$b = a + x$

$c = a + 2x$

$d = a + 3x$

ad

$= a(a + 3x)$

$= a^2 + 3ax$

bc

$= (a + x)(a + 2x)$

$= a^2 + 3ax + 2x^2$

$= ad + 2x^2$

$\neq ad$ when $x \neq 0$

I is false

$= a + a + 3x$

$= a + x + a + 2x$

$= b + c$

II is true

Common difference can be positive or negative.

III is false

37. For $0° \le x < 360°$, how many roots does the equation $7\cos x\sin x = 8\sin x$ have?

A. 0

B. 2

C. 4

D. 5

ANS: B

$7\cos x\sin x = 8\sin x$

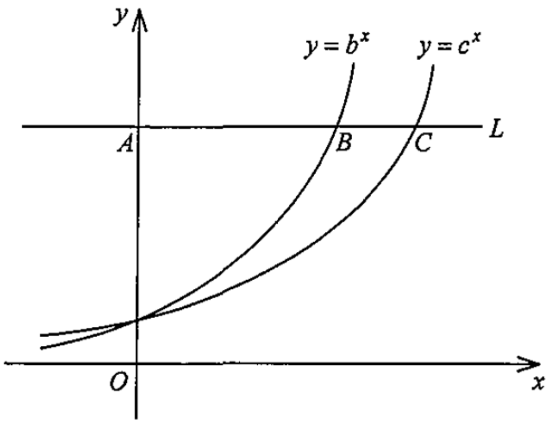
$7\cos x\sin x - 8\sin x = 0$

$\sin x(7\cos x - 8) = 0$

$\sin x = 0$ *or* $\cos x = \frac{8}{7}$ (*rejected*)

For $0° \le x < 360°$, $\sin x = 0$ has 2 roots.

38. The figure shows the graph of $y = b^x$ and the graph of $y = c^x$ on the same rectangular coordinate system, where b and c are positive constants. If a horizontal line L cuts the y-axis, the graph of $y = b^x$ and the graph of $y = c^x$ at A , B and C respectively, which of the following must be true?



I. $b > c$

II. $\dfrac{AC}{AB} = \log_b c$

III. $0 < \dfrac{1}{bc} < 1$

A. I only

B. I and II only

C. I and III only

D. II and III only

ANS: C

I is true

$$c^{AC} = b^{AB}$$
$$AC \log c = AB \log b$$
$$\frac{AC}{AB} = \frac{\log b}{\log c}$$
$$= \log_c b$$
$$\neq \log_b c$$

II is false

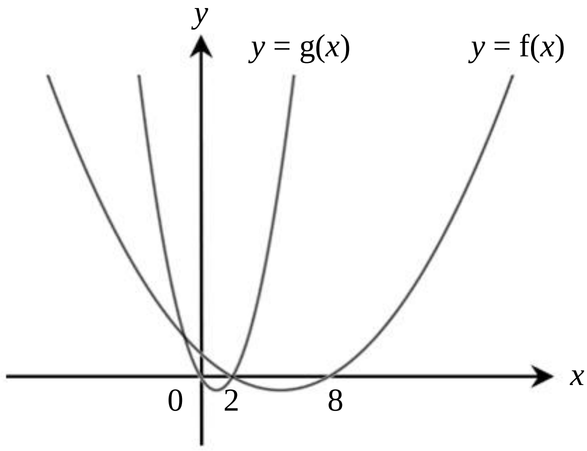
$b > 1$ and $c > 1$

$bc > 1$

$0 < \dfrac{1}{bc} < 1$

III is true

39. If the figure below shows the graphs of y = f(x) and y = g(x) on the same coordinate system, then



A. $g(x) = f(3x+2)$.

B. $g(x) = f\left(\frac{x}{3}+2\right)$.

C. $g(x) = f(3x+6)$.

D. $g(x) = \frac{1}{3}f(x-2)$.

ANS: A

Method I

In the figure, g(0) = f(2) = 0 and g(2) = f(8) = 0

So we put x = 0 and x = 2

In option A, g(0) = f(2) = 0 and g(2) = f(8) = 0

In option B, g(0) = f(2) = 0 and g(2) = f($\frac{8}{3}$)

In option C, g(0) = f(6) and g(2) = f(12)

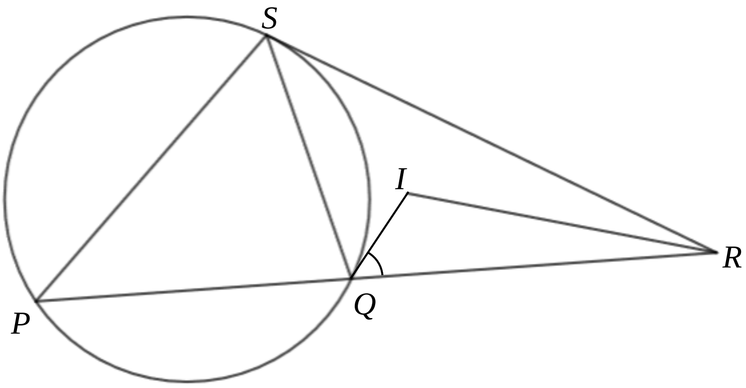
In option D, g(0) = $\frac{1}{3}$ f(–2) and g(2) = $\frac{1}{3}$ f(0)

Only option A gives the same result from the graph.

Method II

The graph of y = f(x) is first translated to the left by 2 units and then reduced to $\frac{1}{3}$ of the original along the x-axis to obtain the graph of y = g(x) .

40. In the figure, PQS is a circle. PQ is produced to R such that RS is the tangent to the circle at S. I is the in-centre of ΔQRS. If $3\overset{\frown}{PQ} = 4\overset{\frown}{QS}$ and $\angle SPQ = 3\angle IRQ$, then $\angle IQR =$



A. 15°.

B. 45°.

C. 52.5°.

D. 60°.

ANS: C

Let $\angle IRQ$ be x.

$\angle IRS = \angle IRQ = x$

$\angle SPQ = 3x$

$\overset{\frown}{PQ} : \overset{\frown}{QS} = 4 : 3$

$\angle PSQ = 4x$

$\angle QSR = \angle SPQ = 3x$

In ΔPSR,

$3x + (4x + 3x) + (x + x) = 180°$

$x = 15°$

In ΔSQR,

$\angle SQR = 180° - 45° - 30° = 105°$

$\angle IQR = \dfrac{1}{2}\angle SQR = 52.5°$

41. If the straight line $2x - y + k = 0$ and the circle $x^2 + y^2 - 4x + 2y - 1 = 0$ intersect at A and B, then the x-coordinate of the mid-point of AB is

A. $\dfrac{4k}{5}$.

B. $-\dfrac{4k}{5}$.

C. $\dfrac{2k}{5}$.

D. $-\dfrac{2k}{5}$.

ANS: D

$2x - y + k = 0$

$y = 2x + k$

sub $y = 2x + k$ into $x^2 + y^2 - 4x + 2y - 1 = 0$,

$x^2 + (2x + k)^2 - 4x + 2(2x + k) - 1 = 0$

$5x^2 + 4kx + k^2 + 2k - 1 = 0$

x-coordinates of the mid-point of AB

$= -\dfrac{4k}{2(5)}$

$= -\dfrac{2k}{5}$

42. If ΔABC is an obtuse-angled triangle, which of the following must lie inside ΔABC?

I. The circumcenter of ΔABC

II. The in-centre of ΔABC

III. The centroid of ΔABC

A. II only

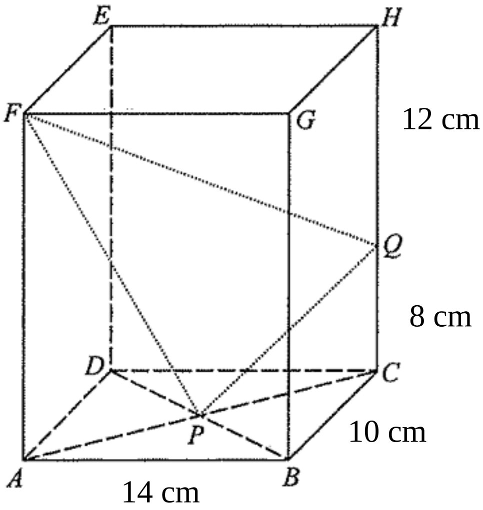
B. I and II only

C. I and III only

D. II and III only

ANS: D

43. In the figure, *ABCDEFGH* is a rectangular block. *AC* and *BD* intersect at *P*. *Q* is a point lying on *CH* such that *CQ* = 8 cm and *QH* = 12 cm . Find $\angle FPQ$ correct to the nearest 0.1°.



A. 31.8°

B. 70.4°

C. 77.8°

D. 88.2°

ANS: B

$$PC = \frac{1}{2}\sqrt{14^2 + 10^2}$$
$$= \sqrt{74}\ cm$$
$$PQ = \sqrt{8^2 + \left(\sqrt{74}\right)^2}$$
$$= \sqrt{138}\ cm$$
$$PF = \sqrt{(12+8)^2 + \left(\sqrt{74}\right)^2}$$
$$= \sqrt{474}\ cm$$
$$QF = \sqrt{12^2 + \left(\sqrt{296}\right)^2}$$
$$= \sqrt{440}\ cm$$
$$\cos\angle FPQ = \frac{\left(\sqrt{138}\right)^2 + \left(\sqrt{474}\right)^2 - \left(\sqrt{440}\right)^2}{2\left(\sqrt{138}\right)\left(\sqrt{474}\right)}$$
$$\angle FPQ \approx 70.35108162°$$
$$\approx 70.4°$$

44. Peter selected 3 different numbers from 1 to 1000 inclusive. Find the probability that the selected numbers can form an arithmetic sequence.

A. $\frac{1}{3}$

B. $\frac{1}{666}$

C. $\frac{1}{999}$

D. $\frac{1}{3996}$

ANS: B

Number of AS with common difference 499 = 2

{1, 500, 999}, {2, 501, 1000}

Number of AS with common difference 498 = 4

{1, 499, 997}, {2, 500, 998}, {3, 501, 999}, {4, 502, 1000}

Number of AS with common difference 497 = 6

{1, 498, 995}, {2, 499, 996}, {3, 500, 997}, {4, 501, 998}, {5, 502, 999}, {6, 503, 1000}

…

Number of AS with common difference 1 = 998

The required probability

$$= \frac{\frac{1}{2} \times (2 + 998) \times 499}{C_3^{1000}}$$

$$= \frac{1}{666}$$

45. Let m_1, r_1 and v_1 be the mean, the range and the variance of the group of numbers $\{a_1, a_2, a_3, \ldots, a_{100}\}$ respectively while m_2, r_2 and v_2 be the mean, the range and the variance of the group of numbers $\{3a_1 + 2, 3a_2 + 2, 3a_3 + 2, \ldots, 3a_{100} + 2\}$. Which of the following must be true?

I. $m_2 = 3m_1 + 2$

II. $r_2 = 3r_1$

III. $v_2 = 3v_1$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

ANS: A

END OF PAPER

1	B	16	A	31	A
2	D	17	B	32	A
3	A	18	D	33	A
4	A	19	D	34	C
5	A	20	B	35	D
6	D	21	B	36	B
7	D	22	C	37	B
8	B	23	C	38	C
9	A	24	C	39	A
10	C	25	A	40	C
11	D	26	A	41	D
12	B	27	C	42	D
13	C	28	D	43	B
14	B	29	C	44	B
15	C	30	D	45	A

A	12
B	11
C	11
D	11