MATHEMATICS Compulsory Part

PAPER 1

Question-Answer book

Time allowed : 2 hours 15 minutes

This paper must be answered in English

Candidate Number						
Class					
Class Number					

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number, Class and Class Number in the spaces provided.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.

SECTION A(1) (35 marks)

1. Simplify $\dfrac{(m^3 n^{-2})^{-4}}{m^{-3} n^{10}}$ and express your answer with positive indices. (3 marks)

2. Make z the subject of the formula $x = \dfrac{y + 2z}{3 + z}$. (3 marks)

Page total

3. Factorize

(a) $a^2 + 3ab + 2b^2$,

(b) $a^2 + 3ab + 2b^2 - 4a^2b - 8ab^2$.

(3 marks)

4. The cost of a wallet is $120. The percentage profit is 25% if the wallet is sold at its marked price.

(a) Find the marked price of the wallet.

(b) If the wallet is sold at a discount of 10% on its marked price, what is the new percentage profit?

(4 marks)

© Pearson Education Asia Limited 2017

Page total

5. Consider the compound inequality

$$-6x \geq 12 \quad \text{and} \quad \frac{2x+5}{3} > 2(x+1) \qquad \ldots\ldots(*)$$

(a) Solve (*).

(b) Write down the greatest negative integer satisfying (*).

(4 marks)

6. Cathy and John bought some stamps. If Cathy gives 3 stamps to John, they will have the same number of stamps. If John gives 3 stamps to Cathy, the number of stamps that Cathy has will be 4 times that of John. Find the total number of stamps Cathy and John have. (4 marks)

Page total

7. The bar chart below shows the distribution of the ratings of a restaurant voted by a group of customers. It is known that the mean rating is 2.7.

Distribution of the ratings of a restaurant

(a) Find the value of k.

(b) If a customer is randomly picked from the group, find the probability that his rating is higher than the median of the ratings voted by the group of customers.

(4 marks)

© Pearson Education Asia Limited 2017

Page total

8. It is given that $f(x)$ is the sum of two parts, one part varies inversely as $\sqrt{x}$ and the other part is a constant. Suppose that $f(4) = 15$ and $f(25) = 9$.

(a) Find $f(x)$.

(b) Solve the equation $f(x) = 10$.

(5 marks)

9. A piece of ribbon is measured as 2.6 m, correct to the nearest 0.1 m.

   (a) Find the range of the actual length of the ribbon.

   (b) Penny claims that if the ribbon is evenly divided into 180 smaller pieces of ribbon, the length of each piece of ribbon can be measured as 1.50 cm, correct to 3 significant figures. Do you agree? Explain your answer.

   (5 marks)

Page total

SECTION A(2) (35 marks)

10. The coordinates of the points A and B are $(-4, 3)$ and $(6, 5)$ respectively. Let L be the perpendicular bisector of the line segment AB.

    (a) Find the equation of L.

    (3 marks)

    (b) If AB is a chord of the circle $x^2 + y^2 - 2ax + 4ay - 85 = 0$, where a is a constant, find the value of a.

    (3 marks)

Page total

11. Let $f(x)$ be a polynomial. When $f(x)$ is divided by $x - 1$, the quotient is $x^2 + (h - 1)x - h$ and the remainder is k, where h and k are constants. When $f(x)$ is divided by $x + h$, the remainder is 5. It is given that $f(x)$ is divisible by $x - 2$.

   (a) Find the values of h and k. (4 marks)

   (b) Someone claims that all the roots of the equation $f(x) = 0$ are rational numbers. Do you agree? Explain your answer. (3 marks)

Page total

12. In Figure 1, *ABCD* is a trapezium with *AD* // *BC*. *E* is a point lying on *CD* such that *AE* and *BE* are the angle bisectors of $\angle BAD$ and $\angle ABC$ respectively.

Figure 1

(a) Is $\triangle ABE$ a right-angled triangle? Explain your answer. (2 marks)

(b) Let *P* be a point on *AB* such that $\triangle ADE \cong \triangle APE$.

    (i) Name a triangle which is congruent to $\triangle BCE$. Give the proof.

    (ii) If *AE* = 12 cm and *BE* = 5 cm, find the length of *AD* + *BC*.

(5 marks)

© Pearson Education Asia Limited 2017 | Page total

© Pearson Education Asia Limited 2017

Page total

13. A factory introduces a new machine for manufacturing water pipes. At first, 20 trial pipes are produced. The factory manager records the lengths of the 20 trial pipes in the following stem-and-leaf diagram.

Stem (10 cm)	Leaf (1 cm)
12	a 7 8 8 8 8 9
13	0 5 6 6 7 8
14	0 1 b 4
15	3 8 9

It is given that the range and the inter-quartile range of the lengths of the 20 trial pipes are 35 cm and 13 cm respectively.

(a) Find the values of a and b. (3 marks)

(b) Two more trial pipes are added to the above sample. It is found that the mean is decreased by 1 cm and the range is increased by 1 cm. Find the lengths of each of the trial pipes. (4 marks)

© Pearson Education Asia Limited 2017

Page total

© Pearson Education Asia Limited 2017

Page total

Kelvin has 4 identical solid metal cubes of side 12 cm. From each of the cubes, Kelvin cuts off the largest square pyramid of same shape. All the remaining metals are melted and recast into 27 smaller identical solid square pyramids which are similar to the larger square pyramids cut off from the cubes.

(a) Find the height of a smaller square pyramid. (3 marks)

(b) Find the ratio of the total surface area of a smaller square pyramid to that of a larger square pyramid. (2 marks)

(c) Kelvin claims that the ratio in (b) will be different if the sides of the metal cubes used at the beginning are not 12 cm. Do you agree? Explain your answer. (3 marks)

© Pearson Education Asia Limited 2017

Page total

Section B (35 marks)

15. The graph in Figure 2 shows the linear relation between x and $\log_{16} y$. The intercepts on both axes of the graph are -4 and 1 respectively. Express the relation between x and y in the form $y = k^{ax+b}$, where a, b and k are integers. (3 marks)

Figure 2

16. 4 boys and 8 girls are randomly arranged to sit in a row.

    (a) Find the probability that all the boys are sitting together. (2 marks)

    (b) Find the probability that none of the boys are sitting next to each other.

                                                                  (2 marks)

© Pearson Education Asia Limited 2017

Page total

17. In a Mathematics competition, Ray and Cindy score 68 marks and 50 marks respectively. The standard scores of Ray and Cindy are 0 and -1.5 respectively.

   (a) Find the mean and the standard deviation of the scores in the Mathematics competition. (3 marks)

   (b) Later, Ray is found cheating in the competition and he is disqualified. Cindy claims that her standard score will increase after excluding Ray's score. Is Cindy's claim correct? Explain your answer. (2 marks)

© Pearson Education Asia Limited 2017

Page total

18. For any positive integer n, let $a_n = 19n - 57$ and $b_n = \dfrac{3125}{m^n}$, where $m > 1$. It is given that the sum to infinity of the series $b_1 + b_2 + b_3 + \ldots$ is $12\,500$.

    (a) Find the value of m. (2 marks)

    (b) Define $z_k = (a_1 + a_2 + \ldots + a_k) + (b_1 + b_2 + \ldots + b_k)i$, where $i = \sqrt{-1}$. If z_k is purely imaginary, find z_k. (3 marks)

19. Figure 3 shows a regular tetrahedron *VABC* of side *a* cm. *M* and *N* are the mid-points of *VB* and *VC* respectively.
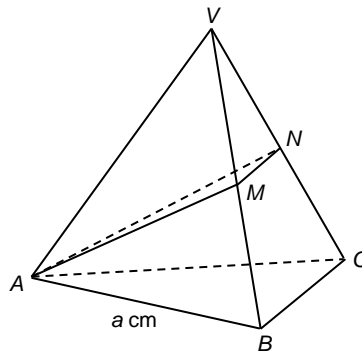

Figure 3

(a) Find $\angle MAN$. (4 marks)

(b) It is given that *X* and *Y* are the points on *VB* and *VC* respectively such that $VX = VY$, $VX < \dfrac{1}{2}VB$ and $VY < \dfrac{1}{2}VC$. Tommy claims that $\angle XAY$ is smaller than $\angle MAN$. Do you agree? Explain your answer. (4 marks)

© Pearson Education Asia Limited 2017

Page total

Page total

20. **(a)** *ABCD* is a quadrilateral with $\angle ABC + \angle ADC = 180°$. Describe the relationship between the two circumcentres of $\triangle ABC$ and $\triangle ABD$.

(2 marks)

(b) In a rectangular coordinate plane, the coordinates of *X* are (7, 7). Point *X* is translated 8 units to the left to a point *Y* and then translated downwards by *h* units to a point *Z*. It is given that *OX* and *OZ* are perpendicular to each other, where *O* is the origin.

(i) Find the value of *h*.

(ii) Isaac claims that the circumcentre of $\triangle XYZ$ is further away from the *y*-axis than that of $\triangle OXY$. Do you agree? Explain your answer.

(iii) Let *P* be a moving point such that the circumcentre of $\triangle XYP$ is also the circumcentre of $\triangle XYZ$. Denote the locus of *P* by Γ. Find the equation of Γ.

(8 marks)

© Pearson Education Asia Limited 2017

Page total

END OF PAPER

Page total