Set 1 – Marking Scheme

Paper 1

Solution	Marks	Remarks
1. $\dfrac{(2x^{-2}y^{5})^{-1}}{16x^3y^4}$		
$=\dfrac{2^{-1}x^{2}y^{-5}}{16x^3y^4}$	1M	for $(ab)^m=a^mb^m$ or $(a^m)^n=a^{mn}$
$=\dfrac{1}{32x^{3-2}y^{4+5}}$	1M	for $\dfrac{c^p}{c^q}=c^{p-q}$ or $\dfrac{c^p}{c^q}=\dfrac{1}{c^{q-p}}$
$=\dfrac{1}{32xy^9}$	1A	
	3	
2. $(1-p)(1-q)=p$		
$(1-q)-p(1-q)=p$	1A	
$(1-q)=p(1-q)+p$	1M	for putting p on one side
$(1-q)=p(2-q)$		
$p=\dfrac{q-1}{q-2}$	1A	$p=\dfrac{1-q}{2-q}$
	3	
3. (a) $45p^2-125q^2$		
$=5[(3p)^2-(5q)^2]$		
$=5(3p-5q)(3p+5q)$	1A	or equivalent
(b) $45p^2-125q^2-12p-20q$		
$=5(3p-5q)(3p+5q)-4(3p+5q)$	1M	for using result in (a)
$=(3p+5q)[5(3p-5q)-4]$		
$=(3p+5q)(15p-25q-4)$	1A	or equivalent
	3	
4. (a) Marked price of phone A		
$=3000(1+60\%)$		
$=\$4800$	1A	
Marked price of phone I		
$=2500(1+100\%)$		
$=\$5000$	1A	
(b) Profit of phone A		
$=4800(1-15\%)-3000$	1M	either one
$=\$1080$		
Profit of phone I		
$=5000(1-15\%)-2500$		
$=\$1750$		
Thus, phone I has a higher profit.	1A	f.t.
	4	

Solution	Marks	Remarks
5. Let x m be the walking distance from Town A to Town B and y m be the walking distance from Town B to Town C.		
$\begin{cases}\dfrac{x}{1.2}+\dfrac{y}{0.9}=75\times 60\\ x+y=5000\end{cases}$	1A + 1A	
From the second equation, $y=5000-x$.		
Substituting into the first equation,		
$\dfrac{x}{1.2}+\dfrac{5000-x}{0.9}=4500$	1M	for eliminating one unknown
$0.9x+1.2(5000-x)=4500\times 0.9\times 1.2$		
$-0.3x+6000=4860$		
$x=3800$	1A	
The walking distance from Town A to Town B is 3800 m.		
Let x m be the walking distance from Town A to Town B.		
$\dfrac{x}{1.2}+\dfrac{5000-x}{0.9}=75\times 60$	1M + 1A + 1A	1A for $y=5000-x$ + 1M for $\dfrac{x}{1.2}+\dfrac{y}{0.9}$
$x=3800$	1A	
The walking distance from Town A to Town B is 3800 m.		
	4	
6. (a) $\dfrac{-32-x}{3}<-x-2$		
$-32-x<-3x-6$		
$x<13$	1A	
$4x-24\geq 0$		
$x\geq 6$	1A	
Thus, the required solution is $6\leq x<13$.	1A	
(b) The multiples of 3 which satisfy both the inequalities in (a) are 6 , 9 and 12 .		
Thus, the required number is 3 .	1A	
	4	
7. (a) $\theta=135°$ or $\theta=315°$	1A	for both correct
(b) $AB=\sqrt{OA^2+OB^2}$ (Pyth. theorem)		
$=\sqrt{6^2+8^2}$	1M	
$=10$		
Let r be the perpendicular distance from O to AB.		
$\dfrac{1}{2}(10)(r)=\dfrac{1}{2}(8)(6)$	1M	
$r=4.8$		
The perpendicular distance from O to AB is 4.8 .	1A	
	4	

Solution	Marks	Remarks
8. (a) $AD = CD$ and $BD = CD$ (given) $AD = BD$ $\angle DAB = \angle DBA$ (base $\angle$s, isos. Δ)	1M	for both
$\angle DBC = \angle DCB$ (base $\angle$s, isos. Δ)		
$\angle DAB + \angle DBA + \angle DBC + \angle DCB = 180^\circ$ ($\angle$ sum of Δ)	1M	
$2\angle DBA + 2\angle DBC = 180^\circ$ $\angle DBA + \angle DBC = 90^\circ$ $\angle ABC = 90^\circ$ Thus, the claim is agreed.	1A	f.t.
$AD = CD$ and $BD = CD$ (given) $AD = BD = CD$ AC is the diameter of the circle passing through A, B and C.	1M	
$\angle ABC = 90^\circ$ ($\angle$ in semi-circle)	1M	
Thus, the claim is agreed.	1A	f.t.
(b) $\angle CAB = 180^\circ - 90^\circ - 30^\circ$ ($\angle$ sum of Δ) $= 60^\circ$ $\angle DBA = \angle CAB$ (base $\angle$s, isos. Δ) $= 60^\circ$ $\angle ADB = 180^\circ - 60^\circ - 60^\circ$ ($\angle$ sum of Δ) $= 60^\circ$ ΔABD is equilateral. $AB = AD = DC$	1M	
$\sin\angle ACB$ $= \frac{AB}{AC}$ $= \frac{AD}{2AD}$ $= \frac{1}{2}$	1A	f.t.
	5	
9. (a) The median = 26 marks	1A	
The range = 30 − 14 = 16 marks	1A	
The inter-quartile range = 30 − 15 = 15 marks	1A	
(b) The third quartile is the average of the 25th highest score and the 26th highest score. Since it is equal to the full score, 30 marks, both of them should be 30 marks. The top 26 students, who originally have 30 marks, face a mark deduction and the scores of other students remain unchanged.	1M	
Thus, it is impossible for all the students to pass the test after the rechecking.	1A	f.t.
	5	

Solution	Marks	Remarks
10. (a) By remainder theorem, $f(k) = k$ $2k^3 - 5k^2 - k + 4k = k$	1M	
$2k^3 - 5k^2 + 2k = 0$ $k(2k-1)(k-2) = 0$ $k = 0$ or $k = \frac{1}{2}$ or $k = 2$	1A	for all correct $k = 0$ or $k = 0.5$ or $k = 2$
(b) Take $k = 2$. By definition, $x - 2$ is a factor of $f(x) - 2$. By long division, $f(x) - 2 = 2x^3 - 5x^2 - x + 6$ $= (x-2)(2x-3)(x+1)$	1M	
$f(x) = 2$ $f(x) - 2 = 0$ $(x-2)(2x-3)(x+1) = 0$ $x = 2$ or $x = \frac{3}{2}$ or $x = -1$	1A	for all correct $x = 2$ or $x = 1.5$ or $x = -1$
Take $k = 2$. $f(x) - 2 = 2x^3 - 5x^2 - x + 6$ $f(2) - 2 = 2(2)^3 - 5(2)^2 - 2 + 6 = 0$	1M	for substitution
So $x - 2$ is a factor of $f(x) - 2$. $f(x) = 2$ $f(x) - 2 = 0$ $(x-2)(2x-3)(x+1) = 0$ $x = 2$ or $x = \frac{3}{2}$ or $x = -1$	1A	for all correct $x = 2$ or $x = 1.5$ or $x = -1$
(c) Take $k = \frac{1}{2}$. By long division, $g(x) = 2x^2 - 4x - 3$	1A	
Let $2x^2 - 4x - 3 = 4$. Then $2x^2 - 4x - 7 = 0$. $\Delta = (-4)^2 - 4(2)(-7) = 72 > 0$ The graph of $y = g(x)$ intersects the line $y = 4$ at two distinct points. Thus, the claim is agreed.	1A	f.t.
	6	

Solution	Marks	Remarks
11. (a) Let $C = k_1 n + k_2\sqrt{n}$, where k_1 and k_2 are non-zero constants.	1A	
$\begin{cases} 34.5 = k_1(25) + k_2\sqrt{25} \\ 144 = k_1(100) + k_2\sqrt{100} \end{cases}$	1M	for substitution
Solving, we have $k_1 = 1.5$ and $k_2 = -0.6$.	1A	for both correct
The printing cost of a 50-page document $= 1.5(50) - 0.6\sqrt{50}$ $\approx \$70.75735931$ $\approx \$70.8$	1A	r.t. $70.8
(b) $1.45n = 1.5n - 0.6\sqrt{n}$		
$1.45 = 1.5 - \frac{0.6}{\sqrt{n}}$	1M	
$\sqrt{n} = 12$ $n = 144$ Thus, the number of pages in the document is 144 .	1A	
	6	
12. (a) $\begin{cases} (80+c)-(40+a)=39 \\ (40+a)+(40+b)+(80+c)+1076 = 20\times 62.5 \end{cases}$	1M	for setting simultaneous equations
$\begin{cases} a-1=c \\ a+b+c=14 \end{cases}$ $2a+b=15$ ...... (1) From the stem-and-leaf diagram, $a \le b \le 5$ ...... (2)	1M	
From (1) and (2), $3b \ge 15$ $b \ge 5$ The only possible value for b is 5 . Thus, the only possible solution for a , b and c is $a = 5$, $b = 5$ and $c = 4$.	1A	for all correct
(b) The median = 61 words per minute	1A	
The mode = 45 words per minute	1A	
(c) The median.	1A	
It is because the mode is at the lowest end of the distribution, while the median is at the centre of the distribution. OR The mode is far away from the centre of the distribution, while the median is close to the centre of the distribution. OR Any other suitable reason.	1A	either one
	7	

JointUs

Solution	Marks	Remarks
13. (a) Let h cm be the height of the circular frustum. By similar triangles,		
$\frac{x-h}{x} = \frac{\frac{8}{2}}{\frac{16}{2}}$	1M	
$h = \frac{x}{2}$	1A	or equivalent
Thus, the height is $\frac{x}{2}$ cm .		
(b) Volume of the cylinder $= \pi\left(\frac{8}{2}\right)^2\left(20 - \frac{x}{2}\right)$	1M	
$= (320\pi - 8\pi x)$ cm^3	1A	or equivalent
(c) Capacity of the container $= \frac{1}{3}\pi\left(\frac{16}{2}\right)^2(x) - \frac{1}{3}\pi\left(\frac{8}{2}\right)^2\left(x - \frac{x}{2}\right) + (320\pi - 8\pi x)$	1M	
$= \left(\frac{32}{3}\pi x + 320\pi\right)$ cm^3	1A	or equivalent
$\frac{32}{3}\pi x + 320\pi \ge 2000$ $x \ge \frac{375}{2\pi} - 30$	1A	
And $\frac{x}{2} < 20$ $x < 40$ $\frac{375}{2\pi} - 30 \le x < 40$	1A	
Capacity of the container $= \frac{1}{3}\pi\left(\frac{16}{2}\right)^2(x)\left[1 - \left(\frac{8}{16}\right)^3\right] + (320\pi - 8\pi x)$	1M	
$= \left(\frac{32}{3}\pi x + 320\pi\right)$ cm^3	1A	or equivalent
$\frac{32}{3}\pi x + 320\pi \ge 2000$ $x \ge \frac{375}{2\pi} - 30$	1A	
And $\frac{x}{2} < 20$ $x < 40$ $\frac{375}{2\pi} - 30 \le x < 40$	1A	
	8	

JointUs

Solution	Marks	Remarks
14. (a) (i) The coordinates of B are $(0, -5)$.	1A	
(ii) The coordinates of M are $\left(\frac{p}{2}, \frac{q-5}{2}\right)$.	1A	
(b) The slope of $AM = \frac{5-\frac{q-5}{2}}{0-\frac{p}{2}} = \frac{q-15}{p}$	1A	
The slope of $OC = \frac{q}{p}$	1A	
(c) Slope of $AM \times$ Slope of $OC = -1$		
$\frac{q-15}{p} \times \frac{q}{p} = -1$	1M	
$q^2 - 15q = -p^2$		
$p^2 + q^2 - 15q = 0$		
The equation of the locus of C is		
$x^2 + y^2 - 15y = 0$	1A	
$x^2 + \left(y - \frac{15}{2}\right)^2 = \left(\frac{15}{2}\right)^2$		
Thus, the locus of C is a circle with centre $\left(0, \frac{15}{2}\right)$		$(0, 7.5)$, 7.5
and radius $\frac{15}{2}$, excluding the points $(0, 0)$ and $(0, 15)$.	1A + 1A	1A for correct centre and radius
		1A for excluding the two points
	8	

Solution	Marks	Remarks
15. $\log_a y - 3 = 3(\log_{a^2} x + 1)$	1M	for setting up equation
$\log_a y = 3\log_{a^2} x + 6$		
$\log_a y = 3\frac{\log_a x}{\log_a a^2} + 6$	1M	for changing base
$\log_a y = 3\frac{\log_a x}{2} + 6$		
$\log_a y = \log_a x^{\frac{3}{2}} + \log_a a^6$		
$\log_a y = \log_a a^6 x^{\frac{3}{2}}$	1M	
$y = a^6 x^{\frac{3}{2}}$	1A	
	4	
16. (a) The required probability		
$= \frac{C_2^4 C_2^8}{C_4^{12}}$	1M	
$= \frac{56}{165}$	1A	r.t. 0.339
The required probability		
$= \frac{4}{12} \times \frac{3}{11} \times \frac{8}{10} \times \frac{7}{9} \times C_2^4$	1M	
$= \frac{56}{165}$	1A	r.t. 0.339
(b) The required probability		
$= \frac{\frac{C_2^4 C_0^4 C_2^4}{C_4^{12}}}{\frac{56}{165}}$	1M	for numerator
$= \frac{3}{14}$	1A	r.t. 0.214
The required probability		
$= \frac{\frac{4}{12} \times \frac{3}{11} \times \frac{4}{10} \times \frac{3}{9} \times C_2^4}{\frac{56}{165}}$	1M	for numerator
$= \frac{3}{14}$	1A	r.t. 0.214
The required probability		
$= \frac{4}{8} \times \frac{3}{7}$	1M	
$= \frac{3}{14}$	1A	r.t. 0.214
	4	

Solution	Marks	Remarks
17. (a) $\Delta=(-8)^2-4(25)$ $=-36$ <0 Thus, (*) has no real roots.	1A	f.t.
(b) (i) $x=\dfrac{-(-8)\pm\sqrt{(-8)^2-4(25)}}{2}$ $x=4+3i$ or $x=4-3i$	1A	
$\dfrac{\alpha}{\alpha-\beta}$ $=\dfrac{4+3i}{(4+3i)-(4-3i)}$ $=\dfrac{4+3i}{6i}$ $=\dfrac{1}{2}+\dfrac{2}{3i}$ $=\dfrac{1}{2}-\dfrac{2}{3}i$ $\dfrac{\beta}{\beta-\alpha}$ $=\dfrac{4-3i}{(4-3i)-(4+3i)}$ $=\dfrac{4-3i}{-6i}$ $=\dfrac{1}{2}-\dfrac{2}{3i}$ $=\dfrac{1}{2}+\dfrac{2}{3}i$	1A	for both correct
(ii) Sum of roots $=\left(\dfrac{1}{2}+\dfrac{2}{3}i\right)+\left(\dfrac{1}{2}-\dfrac{2}{3}i\right)$ $=1$	1M	
Product of roots $=\left(\dfrac{1}{2}-\dfrac{2}{3}i\right)\left(\dfrac{1}{2}+\dfrac{2}{3}i\right)$ $=\left(\dfrac{1}{2}\right)^2-\left(\dfrac{2}{3}i\right)^2$ $=\dfrac{25}{36}$	1M	
The required quadratic equation is $x^2-x+\dfrac{25}{36}=0$.	1A	or equivalent
	6	

Solution	Marks	Remarks
18. (a) $AC=\sqrt{AB^2+BC^2}$ $=50$ cm $\sin\angle BAC=\dfrac{3}{5}$ and $\cos\angle BAC=\dfrac{4}{5}$ $AM=\dfrac{50}{2}$ $=25$ cm $\dfrac{AM}{AN}=\cos\angle BAC$ $AN=\dfrac{125}{4}$ cm $BN=40-\dfrac{125}{4}$ $=\dfrac{35}{4}$ cm	1A	8.75 cm
$\dfrac{BR}{AB}=\sin\angle BAC$ $BR=24$ cm	1A	
(b) (i) $CM=CM$ (common side) $AC=BC$ (given) $AM=BM$ (prop. of rectangle) $\Delta ACM\cong\Delta BCM$ (SSS) $\angle ACR=\angle BCR$ (corr. ∠s, ≅ Δs)	1M	
$CR=CR$ (common side) $AC=BC$ (given) $\Delta ACR\cong\Delta BCR$ (SAS)	1M	
So $AR=BR=24$ cm (corr. sides, ≅ Δs) . Since ΔAMC is a vertical plane, $AR\perp RB$. $AB=\sqrt{24^2+24^2}$ $=24\sqrt{2}$ cm By cosine formula, $\cos\angle ANB=\dfrac{AN^2+BN^2-AB^2}{2\times AN\times BN}$ $\cos\angle ANB=\dfrac{\left(\frac{125}{4}\right)^2+\left(\frac{35}{4}\right)^2-(24\sqrt{2})^2}{2\times\frac{125}{4}\times\frac{35}{4}}$	1M	
$\angle ANB=100.4163610°\approx100°$	1A	r.t. 100°
(ii) $AR^2=AM^2+MR^2-2(AM)(MR)\cos\angle AMC$	1M	
When $\angle AMC$ decreases from 90° to 0° , $\cos\angle AMC$ increases and AR decreases. $\tan\angle ABR=\dfrac{AR}{BR}$ $\tan\angle ABR$ decreases when AR decreases. Thus, $\angle ABR$ decreases when $\angle AMC$ decreases.	1A	f.t.
	8	

Solution	Marks	Remarks
19. (a) Let n be the number of cubes. The height of the decoration $= 50+50\times\frac{3}{4}+50\times\left(\frac{3}{4}\right)^2+\ldots+50\times\left(\frac{3}{4}\right)^{n-1}$	1M	
$< 50+50\times\frac{3}{4}+50\times\left(\frac{3}{4}\right)^2+\ldots$ $=\frac{50}{1-\frac{3}{4}}$	1M	
$=200\text{ cm}=2\text{ m}$ Thus, it is impossible that the height of the decoration exceeds 2 m .	1A	f.t.
(b) The area of square faces of the cubes forms a geometric sequence with common ratio $\left(\frac{3}{4}\right)^2=\frac{9}{16}$.	1M	can be absorbed
Let n be the number of cubes in the decoration. $6\times50^2\times\left[1+\left(\frac{9}{16}\right)+\ldots+\left(\frac{9}{16}\right)^{n-1}\right]-2\times50^2\times\left[\left(\frac{9}{16}\right)+\left(\frac{9}{16}\right)^2+\ldots+\left(\frac{9}{16}\right)^{n-1}\right]\le27500$	1M	
$6\times50^2+4\times50^2\times\left[\left(\frac{9}{16}\right)+\left(\frac{9}{16}\right)^2+\ldots+\left(\frac{9}{16}\right)^{n-1}\right]\le27500$ $15000+10000\left(\frac{9}{16}\right)\left[\frac{1-\left(\frac{9}{16}\right)^{n-1}}{1-\frac{9}{16}}\right]\le27500$	1M	for sum of geometric sequence
$1-\left(\frac{9}{16}\right)^{n-1}\le\frac{35}{36}$ $\left(\frac{9}{16}\right)^{n-1}\ge\frac{1}{36}$ $(n-1)\log\frac{9}{16}\ge\log\frac{1}{36}$	1M	for using logarithm
Solving, we have $n\le7.228262519$. The greatest possible value of n is 7 . Thus, the maximum number of plastic cubes in the decoration is 7 .	1A	

Solution	Marks	Remarks
(c) (i) One can imagine that there is another larger cube under the cube with 50 cm side length. The length of the square base of the pyramid is the same as the side length of such larger cube, which extends the geometric sequence by one term before the first one. The length of the square base of the pyramid $=50\div\frac{3}{4}$ $=\frac{200}{3}\text{ cm}$	1A	r.t. 66.7 cm
One can imagine that smaller cubes are placed on top of the decoration one by one. When the number of cubes increases, the volume of the part of the pyramid above the top face of the smallest cube approaches zero. The height of the pyramid is the height of the decoration when there is infinitely many cubes, which is 200 cm .	1A	
(ii) The volumes of the cubes forms a geometric sequence with common ratio $\left(\frac{3}{4}\right)^3=\frac{27}{64}$. The volume of the decoration $=50^3+50^3\times\frac{27}{64}+50^3\times\left(\frac{27}{64}\right)^2+\ldots+50^3\times\left(\frac{27}{64}\right)^6$	1M	
$=\left[50^3\times\frac{1-\left(\frac{27}{64}\right)^7}{1-\frac{27}{64}}\right]\text{cm}^3$	1M	for sum of geometric sequence
The volume of the space in the pyramid $=\frac{1}{3}\left(\frac{200}{3}\right)^2(200)-50^3\times\frac{1-\left(\frac{27}{64}\right)^7}{1-\frac{27}{64}}$ $=80594.33066\text{ cm}^3$ $>80000\text{ cm}^3$ Thus, the epoxy resin will not overflow.	1A	f.t.
	13	