Paper 2
Full Solutions

Section A

1. B

$x^2 - y^2 - x + y = (x+y)(x-y) - (x-y)$
$= \underline{\underline{(x-y)(x+y-1)}}$

2. D

$(-4)^{444}\left(\frac{1}{2^{222}}\right) = (-1)^{444}(2^2)^{444}\left(\frac{1}{2^{222}}\right)$
$= 2^{888-222}$
$= \underline{\underline{2^{666}}}$

3. B

$\frac{x+6y}{3x} = \frac{y}{x} + 1$
$\frac{x+6y}{3x} = \frac{y+x}{x}$
$x + 6y = 3(y+x)$
$x + 6y = 3y + 3x$
$3y = 2x$
$x = \underline{\underline{\frac{3y}{2}}}$

4. C

$\left(\frac{\pi}{5}\right)^3 \approx 0.248\ 050\ 213$

For option A:
$0.248\ 050\ 213 = 0.25$ (cor. to 2 sig. fig.)
For option B:
$0.248\ 050\ 213 = 0.248$ (cor. to 3 d.p.)
For option C:
$0.248\ 050\ 213 = 0.2481$ (cor. to 4 sig. fig.)
For option D:
$0.248\ 050\ 213 = 0.248\ 05$ (cor. to 5 d.p.)
$\therefore$ The answer is C.

5. B

$4 - x < 2 - 3x$ or $x + 3 > 2x - 5$
$2x < -2$ $x < 8$
$x < -1$
$\therefore$ $x < 8$

6. D

L.H.S. $= n(x-3)^2 - 2x$
$= n(x^2 - 6x + 9) - 2x$
$= nx^2 - 6nx + 9n - 2x$
$= nx^2 - (6n+2)x + 9n$

R.H.S. $= 9x^2 + mx(x+2) + 18$
$= 9x^2 + mx^2 + 2mx + 18$
$= (9+m)x^2 + 2mx + 18$

$\therefore$ $nx^2 - (6n+2)x + 9n \equiv (9+m)x^2 + 2mx + 18$

By comparing the coefficients of x^2 and the constant term, we have

$\begin{cases} n = 9 + m & \ldots\ldots(1) \\ 9n = 18 & \ldots\ldots(2) \end{cases}$

By substituting (1) into (2), we have
$9(9+m) = 18$
$9 + m = 2$
$m = \underline{\underline{-7}}$

7. C

$f(x) = (x-1)^3 - 6(x-1) + 4$

For option A:
$f(-1) = (-1-1)^3 - 6(-1-1) + 4$
$= (-2)^3 - 6(-2) + 4$
$= 8$
$\neq 0$
$\therefore$ $x + 1$ is not a factor of $f(x)$.

For option B:
$f(2) = (2-1)^3 - 6(2-1) + 4$
$= 1^3 - 6(1) + 4$
$= -1$
$\neq 0$
$\therefore$ $x - 2$ is not a factor of $f(x)$.

For option C:
$f(3) = (3-1)^3 - 6(3-1) + 4$
$= 2^3 - 6(2) + 4$
$= 0$
$\therefore$ $x - 3$ is a factor of $f(x)$.
$\therefore$ The answer is C.

8. C

$x^2 + k(x+5) = 16$
$x^2 + kx + 5k - 16 = 0$
$\because$ $x^2 + kx + 5k - 16 = 0$ has equal roots.
$\therefore$ $\Delta = 0$
$k^2 - 4(1)(5k-16) = 0$
$k^2 - 20k + 64 = 0$
$(k-4)(k-16) = 0$
$k = \underline{\underline{4}}$ or $k = \underline{\underline{16}}$

9. B

$y = (px+q)^2 - 3$
$= p^2\left[x - \left(-\frac{q}{p}\right)\right]^2 - 3$

$\because$ $p^2 > 0$
$\therefore$ The graph of $y = (px+q)^2 - 3$ opens upwards.
$\because$ $p < 0$ and $q > 0$
$\therefore$ $-\frac{q}{p} > 0$
i.e. The x-coordinate of the vertex of the graph is positive.
$\therefore$ The answer is B.

10. A

Let \$$x$ be the price of the doll.
Then the price of the toy car $= \$(1+25\%)x = \$1.25x$

$$1.25x + x = 270$$
$$2.25x = 270$$
$$x = 120$$

∴ The prices of the toy car and the doll are \$150 and \$120 respectively.

The difference in price $= \$(150-120)$
$= \underline{\underline{\$30}}$

11. C

$$\frac{1}{2a} = \frac{1}{3b}$$
$$3b = 2a$$
$$a = \frac{3b}{2}$$
$$\frac{1}{3b} = \frac{1}{4c}$$
$$4c = 3b$$
$$c = \frac{3b}{4}$$
$$\frac{a+b}{b+c} = \frac{\frac{3b}{2}+b}{b+\frac{3b}{4}} = \frac{\frac{5b}{2}}{\frac{7b}{4}} = \frac{10}{7}$$

∴ $(a+b) : (b+c) = \underline{\underline{10:7}}$

12. D

Let $z = \frac{ky^2}{\sqrt{x}}$, where k is a non-zero constant.

New value of $x = (1 - 36\%)x = 0.64x$
New value of $y = (1 + 20\%)y = 1.2y$

New value of $z = \frac{k(1.2y)^2}{\sqrt{0.64x}}$
$= \frac{1.44ky^2}{0.8\sqrt{x}}$
$= \frac{1.8ky^2}{\sqrt{x}}$
$= 1.8z$

∴ Percentage change of $z = \frac{1.8z - z}{z} \times 100\%$
$= \frac{0.8z}{z} \times 100\%$
$= +80\%$

∴ z is increased by 80%.

13. C

Let $T(n)$ be the number of dots in the nth pattern.
$T(3) = 9$
$T(4) = 9 + [2(3) + 1] = 16$
$T(5) = 16 + [2(4) + 1] = 25$
$T(6) = 25 + [2(5) + 1] = 36$
$T(7) = 36 + [2(6) + 1] = 49$
$T(8) = 49 + [2(7) + 1] = 64$
$T(9) = 64 + [2(8) + 1] = 81$

∴ The 9th pattern has 81 dots.

Alternative Solution
By inspection, $T(n) = n^2$.
$n^2 = 81$
∴ $n = 9$
∴ The 9th pattern has 81 dots.

14. B

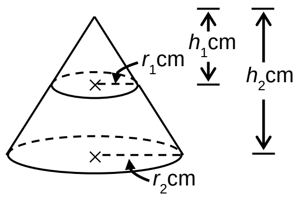
In $\triangle BCD$,
$BD = \sqrt{BC^2 + CD^2}$ (Pyth. theorem)
$= \sqrt{15^2 + 20^2}$ cm
$= 25$ cm

In $\triangle ABD$,
$AB^2 + BD^2 = AD^2$ (Pyth. theorem)
$AB = \sqrt{AD^2 - BD^2}$
$= \sqrt{65^2 - 25^2}$ cm
$= 60$ cm

Area of $ABCD$ = area of $\triangle BCD$ + area of $\triangle ABD$
$= \left(\frac{1}{2}\times 15 \times 20 + \frac{1}{2}\times 60 \times 25\right)$ cm^2
$= (150 + 750)$ cm^2
$= \underline{\underline{900 \text{ cm}^2}}$

15. C

Let r_1 cm and r_2 cm be the base radius of the upper part of the cone and that of the original cone respectively, while h_1 cm and h_2 cm be the height of the upper part of the cone and that of the original cone respectively.



$$h_1 = \frac{1}{2}h_2$$
$$\frac{r_1}{r_2} = \frac{h_1}{h_2}$$
$$\frac{r_1}{6} = \frac{\frac{1}{2}h_2}{h_2}$$
$$\frac{r_1}{6} = \frac{1}{2}$$
$$r_1 = 3$$

∵ The volume of the frustum is 168π cm^3.

∴ $\frac{1}{3}\pi r_2^2 h_2 - \frac{1}{3}\pi r_1^2 h_1 = 168\pi$
$$\frac{1}{3}\pi(6)^2 h_2 - \frac{1}{3}\pi(3)^2\left(\frac{1}{2}h_2\right) = 168\pi$$
$$\frac{1}{3}\pi h_2\left(6^2 - 3^2 \times \frac{1}{2}\right) = 168\pi$$
$$h_2 = 16$$

∴ The height of the frustum $= \frac{16}{2}$ cm
$= \underline{\underline{8 \text{ cm}}}$

16. D

$\because$ $AB = BE$

$\therefore$ $\angle BAE = \angle BEA$ (base $\angle$s, isos. $\triangle$)

In $\triangle ABE$,

$\angle BAE + \angle BEA = \angle ABF$ (ext. $\angle$ of $\triangle$)

$2\angle BEA = 132°$

$\angle BEA = 66°$

$\angle DAE = \angle BEA$ (alt. $\angle$s, AD // FC)

$= 66°$

$\because$ $AE = DE$

$\therefore$ $\angle ADE = \angle DAE$ (base $\angle$s, isos. $\triangle$)

$= 66°$

$\angle DEC = \angle ADE$ (alt. $\angle$s, AD // FC)

$= \underline{\underline{66°}}$

17. A

Consider $\triangle ABD$ and $\triangle ADE$.

$\because$ The height of $\triangle ABD$ with base BD = the height of $\triangle ADE$ with base DE

$\therefore$ $\dfrac{\text{Area of } \triangle ABD}{\text{Area of } \triangle ADE} = \dfrac{BD}{DE} = \dfrac{1}{3}$

$$\text{Area of } \triangle ABD = \frac{1}{3} \times \text{area of } \triangle ADE$$
$$= \frac{1}{3} \times 18 \text{ cm}^2$$
$$= 6 \text{ cm}^2$$

In $\triangle ABC$ and $\triangle BEC$,

$\angle ABC = \angle BEC$ (given)

$\angle ACB = \angle BCE$ (common angle)

$\angle BAC = 180° - \angle ABC - \angle ACB$ ($\angle$ sum of $\triangle$)

$= 180° - \angle BEC - \angle BCE$

$= \angle EBC$ ($\angle$ sum of $\triangle$)

$\therefore$ $\triangle ABC \sim \triangle BEC$ (AAA)

Let x cm^2 be the area of $\triangle BEC$.

$\because$ $\dfrac{AB}{BE} = \dfrac{8}{1+3}$

$= \dfrac{2}{1}$

$\therefore$ $\dfrac{\text{Area of } \triangle ABC}{\text{Area of } \triangle BEC} = \left(\dfrac{2}{1}\right)^2$

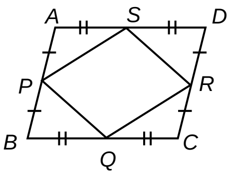
$$\frac{6+18+x}{x} = 4$$
$$24 + x = 4x$$
$$24 = 3x$$
$$x = 8$$

$\therefore$ Area of $\triangle ABC$ = (6 + 18 + 8) cm^2

$= \underline{\underline{32 \text{ cm}^2}}$

18. A

Refer to the figure.



$\because$ $ABCD$ is a parallelogram and P, Q, R and S are the mid-points of AB, BC, CD and AD respectively.

$\therefore$ $AP = PB = DR = CR$ and $BQ = QC = AS = SD$ (opp. sides of // gram)

For I:

$AP = CR$

$\therefore$ I must be true.

For II:

$AP = CR$ and $AS = QC$

$\angle PAS = \angle RCQ$ (opp. $\angle$s of // gram)

$\therefore$ $\triangle PAS \cong \triangle RCQ$ (SAS)

$\therefore$ $PS = RQ$ (corr. sides, $\cong$ $\triangle$s)

Similarly,

$\triangle PBQ \cong \triangle RDS$ (SAS)

$\therefore$ $PQ = RS$ (corr. sides, $\cong$ $\triangle$s)

$\therefore$ $PQRS$ is a parallelogram. (opp. sides equal)

$\therefore$ $\angle QPS = \angle SRQ$ (opp. $\angle$s of // gram)

$\therefore$ II must be true.

For III:

$\because$ $\triangle QCR \cong \triangle SDR$ only when $\angle QCR = \angle SDR$.

$\therefore$ III may not be true.

$\therefore$ The answer is A.

19. A

$\angle BDC = \angle BAC = 34°$ ($\angle$s in the same segment)

$\dfrac{\angle ACB}{\angle BAC} = \dfrac{\overset{\frown}{AB}}{\overset{\frown}{BC}}$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$$\angle ACB = \frac{1}{2} \times 34°$$
$$= 17°$$

$\dfrac{\angle DBC}{\angle BAC} = \dfrac{\overset{\frown}{CD}}{\overset{\frown}{BC}}$ (arcs prop. to $\angle$s at $\odot^{ce}$)

$$\angle DBC = \frac{3}{2} \times 34°$$
$$= 51°$$

In $\triangle BCD$,

$\angle BDC + \angle BCD + \angle DBC = 180°$ ($\angle$ sum of $\triangle$)

$$34° + (17° + \angle ACD) + 51° = 180°$$
$$\angle ACD = \underline{\underline{78°}}$$

20. C

Let r cm be the radius of the sector OAB.

In $\triangle OAC$,

$\cos 60° = \dfrac{OC}{OA}$ and $\sin 60° = \dfrac{AC}{OA}$

$OC = \dfrac{r}{2}$ $AC = \dfrac{\sqrt{3}r}{2}$

$$\text{Area of } \triangle OAC = \frac{1}{2} \times OC \times AC$$
$$= \frac{1}{2} \times \frac{r}{2} \times \frac{\sqrt{3}r}{2} \text{ cm}^2$$
$$= \frac{\sqrt{3}r^2}{8} \text{ cm}^2$$

$\because$ Area of sector OAB – area of $\triangle OAC$ = area of the shaded region

$\therefore$ $\dfrac{60°}{360°} \times \pi r^2 - \dfrac{\sqrt{3}r^2}{8} = 32$

$$r^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right) = 32$$
$$r^2 \approx 104.2032$$
$$r = 10.2 \quad \text{(cor. to the nearest 0.1)}$$

$\therefore$ The radius of the sector is 10.2 cm.

21. B

In $\triangle ABE$,

$$\cos\alpha = \frac{AB}{BE}$$

$$BE = \frac{AB}{\cos\alpha}$$

$\angle EBC = 90° - \alpha$

In $\triangle CBE$,

$$\tan(90° - \alpha) = \frac{CE}{BE}$$

$$CE = BE \times \frac{1}{\tan\alpha} = \frac{AB}{\cos\alpha} \times \frac{\cos\alpha}{\sin\alpha} = \underline{\underline{\frac{AB}{\sin\alpha}}}$$

22. C

$$(n-2)\times 180° = 12 \times \frac{360°}{n}$$

$$n - 2 = \frac{24}{n}$$

$$n^2 - 2n - 24 = 0$$

$$n = 6 \quad \text{or} \quad n = -4 \text{ (rejected)}$$

∴ The polygon is a regular hexagon.

For option A:

The value of n is 6.

∴ Option A is not true.

For option B:

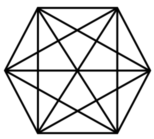
Size of each interior angle of a regular hexagon

$$= \frac{(6-2)\times 180°}{6}$$

$$= 120°$$

∴ Option B is not true.

For option C:



Number of diagonals of a regular hexagon = 9

∴ Option C is true.

For option D:

The number of folds of rotational symmetry of a regular hexagon is 6.

∴ Option D is not true.

∴ The answer is C.

23. A

For I:

x-intercept of $L_1 = \frac{1}{a}$

∵ From the graph, the x-intercept of L_1 is negative.

∴ $\frac{1}{a} < 0$

$a < 0$

∴ I is true.

For II:

y-intercept of $L_1 = \frac{1}{b}$

y-intercept of $L_2 = \frac{1}{3}$

∵ From the graph, the y-intercept of L_1 > the y-intercept of L_2

∴ $\frac{1}{b} > \frac{1}{3}$

$0 < b < 3$

∴ II is true.

For III:

x-intercept of $L_2 = \frac{1}{c}$

∵ From the graph, the x-intercept of L_2 is negative.

∴ $\frac{1}{c} < 0$

$c < 0$

∴ III is not true.

∴ The answer is A.

24. A

Slope of $L_1 = -\frac{4}{-3} = \frac{4}{3}$

∵ L_2 is perpendicular to L_1.

∴ Slope of L_2 × slope of $L_1 = -1$

$$\text{Slope of } L_2 \times \frac{4}{3} = -1$$

$$\text{Slope of } L_2 = -\frac{3}{4}$$

y-intercept of $L_1 = -\frac{6}{-3} = 2$

∵ L_2 has the same y-intercept as L_1.

∴ The equation of L_2 is

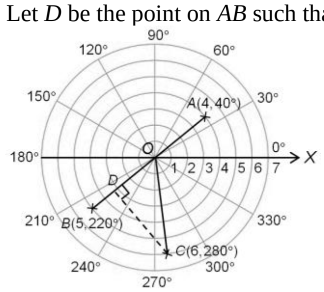
$$y = -\frac{3}{4}x + 2$$

$$4y = -3x + 8$$

$$3x + 4y - 8 = 0$$

25. C

Let D be the point on AB such that $CD \perp AB$.



$\angle COD = 280° - 220° = 60°$

In $\triangle COD$,

$$\sin\angle COD = \frac{CD}{OC}$$

$$CD = 6 \times \sin 60° = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$$

∴ The perpendicular distance from C to AB is $3\sqrt{3}$.

26. B

The equation of the circle C is:

$$2x^2 + 2y^2 + 20x - 12y + 15 = 0$$

$$x^2 + y^2 + 10x - 6y + \frac{15}{2} = 0$$

For I:

Centre of $C = \left(-\frac{10}{2}, -\frac{(-6)}{2}\right) = (-5, 3)$

$\therefore$ I is true.

For II:

Radius of $C = \sqrt{\left(\frac{10}{2}\right)^2 + \left(\frac{-6}{2}\right)^2 - \frac{15}{2}}$ units

$= \sqrt{\frac{53}{2}}$ units

$\therefore$ II is not true.

For III:

Distance between the point (2, 0) and the centre of C

$= \sqrt{(-5-2)^2 + (3-0)^2}$ units

$= \sqrt{(-7)^2 + 3^2}$ units

$= \sqrt{58}$ units

$>$ the radius of C

i.e. The point (2, 0) lies outside C.

$\therefore$ III is true.

$\therefore$ The answer is B.

27. C

$\because$ $AP^2 + BP^2 = AB^2$

$\therefore$ $\angle APB = 90°$ (converse of Pyth. theorem)

$\therefore$ The locus of P is a circle with diameter AB. (converse of $\angle$ in semi-circle)

i.e. The centre of the locus of P lies on the straight line $4x - 7y + k = 0$.

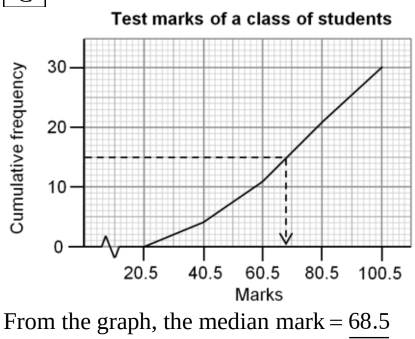
The centre of the locus of $P = \left(-\frac{(-8)}{2}, -\frac{(-6)}{2}\right) = (4, 3)$

By substituting (4, 3) into the equation $4x - 7y + k = 0$, we have

$$4(4) - 7(3) + k = 0$$

$$k = \underline{\underline{5}}$$

28. C



From the graph, the median mark $= \underline{\underline{68.5}}$

29. A

The possible outcomes:

1st ball \ 2nd ball	2	4	5	7	9
2		(2, 4)	(2, 5)	(2, 7)	(2, 9)
4	(4, 2)		(4, 5)	(4, 7)	(4, 9)
5	(5, 2)	(5, 4)		(5, 7)	(5, 9)
7	(7, 2)	(7, 4)	(7, 5)		(7, 9)
9	(9, 2)	(9, 4)	(9, 5)	(9, 7)	

$P(\text{the sum is odd}) = \frac{12}{20} = \frac{3}{5}$

$P(\text{the sum is even}) = \frac{8}{20} = \frac{2}{5}$

Expected number of tokens obtained $= 15 \times \frac{3}{5} + 25 \times \frac{2}{5}$

$= \underline{\underline{19}}$

30. C

$\because$ The mean of the ten integers is 6.

$\therefore$ $\frac{4+5+7+8+9+10+12+a+b+c}{10} = 6$

$$55 + a + b + c = 60$$

$$a + b + c = 5$$

Suppose $a \le b \le c$. There are 2 cases

Case 1: $a = 1$, $b = 1$ and $c = 3$

Case 2: $a = 1$, $b = 2$ and $c = 2$

For I:

In both cases, arrange the ten integers in ascending order:

a b c 4 5 7 8 9 10 12

Median of the ten integers $= \frac{5+7}{2} = 6$

$\therefore$ I must be true.

For II:

Case 1: $a = 1$, $b = 1$ and $c = 3$
Mode of the ten integers = 1

Case 2: $a = 1$, $b = 2$ and $c = 2$
Mode of the ten integers = 2

$\therefore$ II may not be true.

For III:

In both cases, $a = 1$

Range of the ten integers = 12 – 1 = 11

$\therefore$ III must be true.

$\therefore$ The answer is C.

Section B

31. A

$4a^3b^3 = 2^2 \bullet a^3 \bullet b^3$

$8a^4 = 2^3 \bullet a^4$

$12ab^2 = 2^2 \bullet 3 \bullet a \bullet b^2$

$\therefore$ The H.C.F. of $4a^3b^3$, $8a^4$, $12ab^2 = 2^2 \bullet a$

$= \underline{\underline{4a}}$

32. A

$\because$ The graph of $y = f(x)$ is obtained by reducing the graph of $y = g(x)$ along the x-axis to $\frac{1}{2}$ times the original.

$\therefore$ $f(x) = g(2x)$

33. D

$$\begin{aligned}4^{16}+8^{16}&=(2^2)^{16}+(2^3)^{16}\\&=2^{32}+2^{48}\\&=(2^4)^8+(2^4)^{12}\\&=1\times16^8+1\times16^{12}\\&=\underline{\underline{1000100000\,000_{16}}}\end{aligned}$$

34. D

The slope of the straight line $=\dfrac{4-0}{0-(-4)}=\dfrac{4}{4}=1$

∴ The equation of the straight line is $\log_3 y=x+4$.

∵ $\log_3 y=x+4$

∴ $$\begin{aligned}y&=3^{x+4}\\&=3^x(3^4)\\&=81(3^x)\end{aligned}$$

35. A

$$\begin{aligned}\frac{k}{2+i}+4+i&=\frac{k}{2+i}\times\frac{2-i}{2-i}+4+i\\&=\frac{k(2-i)}{2^2-i^2}+4+i\\&=\frac{2k-ki}{5}+4+i\\&=\left(\frac{2k}{5}+4\right)+\left(1-\frac{k}{5}\right)i\end{aligned}$$

∵ The real part and the imaginary part are equal.

∴ $$\begin{aligned}\frac{2k}{5}+4&=1-\frac{k}{5}\\\frac{2k+k}{5}&=-3\\3k&=-15\\k&=\underline{\underline{-5}}\end{aligned}$$

36. C

For I:

∵ $\dfrac{(\log x)^2}{\log x}=\log x$

$\dfrac{(\log x)^3}{(\log x)^2}=\log x$

∴ $\log x$, $(\log x)^2$, $(\log x)^3$ is a geometric sequence.

∴ I is true.

For II:

∵ $\dfrac{\log x^2}{\log x}=\dfrac{2\log x}{\log x}=2$

$\dfrac{\log x^3}{\log x^2}=\dfrac{3\log x}{2\log x}=\dfrac{3}{2}\neq 2$

∴ $\log x$, $\log x^2$, $\log x^3$ is not a geometric sequence.

∴ II is not true.

For III:

∵ $\dfrac{\log_4 x}{\log_2 x}=\dfrac{\log x}{\log 4}\times\dfrac{\log 2}{\log x}=\dfrac{1}{2}$

$\dfrac{\log_{16} x}{\log_4 x}=\dfrac{\log x}{\log 16}\times\dfrac{\log 4}{\log x}=\dfrac{1}{2}$

∴ $\log_2 x$, $\log_4 x$, $\log_{16} x$ is a geometric sequence.

∴ III is true.

∴ The answer is C.

37. C

By substituting $x = 0$ into $y = x - 4$, we have

$y = 0 - 4 = -4$

i.e. $y = x - 4$ intersects the y-axis at (0, –4).

By substituting $x = 0$ into $y = 4 - x$, we have

$y = 4 - 0 = 4$

i.e. $y = 4 - x$ intersects the y-axis at (0, 4).

By substituting $x = 2$ into $y = x - 4$, we have

$y = 2 - 4 = -2$

i.e. $x = 2$ and $y = x - 4$ intersect at (2, –2).

By substituting $x = 2$ into $y = 4 - x$, we have

$y = 4 - 2 = 2$

i.e. $x = 2$ and $y = 4 - x$ intersect at (2, 2).

At (0, –4), $4x+3y=4(0)+3(-4)=-12$

At (0, 4), $4x+3y=4(0)+3(4)=12$

At (2, –2), $4x+3y=4(2)+3(-2)=2$

At (2, 2), $4x+3y=4(2)+3(2)=14$

∴ The greatest value of $4x + 3y=\underline{\underline{14}}$

38. C

$$\begin{aligned}\cos^2 x&=2\sin x+1\\1-\sin^2 x&=2\sin x+1\\\sin^2 x+2\sin x&=0\\\sin x(\sin x+2)&=0\end{aligned}$$

$\sin x=0$ or $\sin x+2=0$

$\sin x=-2$ (rejected)

When $\sin x = 0$, $x = 0°$ or $180°$ or $360°$.

∴ The equation $\cos^2 x=2\sin x+1$ has 3 roots.

39. D

Let $AB = AD = x$ cm.

In △ABD,

$BD^2=AB^2+AD^2$ (Pyth. theorem)

$$\begin{aligned}(5\sqrt{2})^2&=x^2+x^2\\50&=2x^2\\x^2&=25\end{aligned}$$

$x=5$ or $x=-5$ (rejected)

∴ $AB = 5$ cm

$\angle AEB+\angle AED=180°$ (adj. ∠s on st. line)

$\angle AEB+60°=180°$

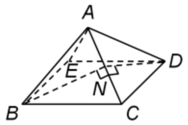
$\angle AEB=120°$

In △ABE, by the sine formula,

$$\begin{aligned}\frac{AE}{\sin 45°}&=\frac{AB}{\sin 120°}\\AE&=\frac{5}{\frac{\sqrt{3}}{2}}\times\frac{\sqrt{2}}{2}\text{ cm}\\&=\underline{\underline{\frac{5\sqrt{6}}{3}\text{ cm}}}\end{aligned}$$

40. C

With the notations in the figure,



Let N be a point on AC such that $BN \perp AC$ and $DN \perp AC$.
The angle between the planes ABC and ACD is $\angle BND$.

∵ $ABCDE$ is a right pyramid with square base $BCDE$ and $AB = BC$.

∴ $AB = AC = BC = CD$

Let $AB = AC = BC = CD = x$ cm.

In △BCD,

$$BD^2 = BC^2 + CD^2 \quad \text{(Pyth. theorem)}$$

$$BD = \sqrt{x^2 + x^2} = \sqrt{2}x$$

In △BCN,

$$\sin \angle BCN = \frac{BN}{BC}$$

$$\sin 60° = \frac{BN}{x}$$

$$BN = \frac{\sqrt{3}}{2}x$$

Similarly, $DN = BN = \frac{\sqrt{3}}{2}x$

In △BND, by the cosine formula,

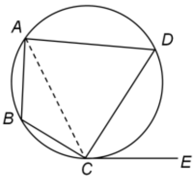
$$\cos \angle BND = \frac{BN^2 + DN^2 - BD^2}{2(BN)(DN)} = \frac{\left(\frac{\sqrt{3}}{2}x\right)^2 + \left(\frac{\sqrt{3}}{2}x\right)^2 - (\sqrt{2}x)^2}{2\left(\frac{\sqrt{3}}{2}x\right)\left(\frac{\sqrt{3}}{2}x\right)} = -\frac{1}{3}$$

$$\angle BND = 109° \quad \text{(cor. to 3 sig. fig.)}$$

∴ The angle between the planes ABC and ACD is 109°.

41. C

Join AC.



$$\angle ABC + \angle ADC = 180° \quad \text{(opp. ∠s, cyclic quad.)}$$

$$\angle ABC + 46° = 180°$$

$$\angle ABC = 134°$$

In △ABC,

∵ $AB = BC$

∴ $\angle BAC = \angle BCA$ (base ∠s, isos. △)

$$\angle ABC + \angle BAC + \angle BCA = 180° \quad \text{(∠ sum of △)}$$

$$134° + 2\angle BCA = 180°$$

$$\angle BCA = 23°$$

$$\angle ACE = \angle ABC \quad \text{(∠ in alt. segment)}$$

$$= 134°$$

$$\angle BCE = \angle BCA + \angle ACE = 23° + 134° = \underline{\underline{157°}}$$

42. B

Let $M(x_M, y_M)$ be the mid-point of OX.
By the mid-point formula, we have

$$x_M = \frac{0+6}{2} = 3 \quad \text{and} \quad y_M = \frac{0+(-6)}{2} = -3$$

∴ The coordinates of M are (3, –3).

Slope of $OX = \frac{-6-0}{6-0} = \frac{-6}{6} = -1$

Slope of $CM = \frac{-2-(-3)}{a-3} = \frac{1}{a-3}$

∵ C is the circumcentre of △OXY.

∴ CM is the perpendicular bisector of OX.

∴ Slope of OX × slope of CM = –1

$$-1 \times \frac{1}{a-3} = -1$$

$$a - 3 = 1$$

$$a = 4$$

Let $N(x_N, y_N)$ be the mid-point of XY.
Similarly, CN is the perpendicular bisector of XY.

∵ x-coordinate of X = x-coordinate of Y

∴ XY is a vertical line.

i.e. CN is a horizontal line.

∴ y-coordinate of N = y-coordinate of C = –2

By the mid-point formula, we have

$$-2 = \frac{-6+b}{2}$$

$$-4 = -6 + b$$

$$b = 2$$

∴ $a + b = 4 + 2 = \underline{\underline{6}}$

43. A

Number of different groups formed without restriction
$= C_5^{15+10}$
$= 53\,130$

Number of different groups formed consists of boys only
$= C_5^{15}$
$= 3003$

Number of different groups formed consists of girls only
$= C_5^{10}$
$= 252$

Number of different groups formed consists of at least one boy and at least one girl
= 53 130 – 3003 – 252
= $\underline{\underline{49\,875}}$

44. C

$$P(\text{at most 2 red bowls}) = 1 - P(\text{3 red bowls}) = 1 - \left(\frac{4}{10}\right)^3 = \underline{\underline{\frac{117}{125}}}$$

45. C

Add 5 to each datum of $\{a-5, b-5, c-5, d-5, e-5, f-5\}$, we get another data set $\{a, b, c, d, e, f\}$ and its median, range and variance are m_1+5, r_1 and v_1 respectively.

Multiply each datum of $\{a, b, c, d, e, f\}$ by 2, we get $\{2a, 2b, 2c, 2d, 2e, 2f\}$ and its median, range and variance are $2(m_1+5)$, $2r_1$ and 2^2v_1 respectively.

$\therefore$ $m_2 = 2(m_1+5) \neq 2m_1+5$

$r_2 = 2r_1$

$v_2 = 2^2v_1 = 4v_1$

$\therefore$ II and III are true.

$\therefore$ The answer is C.