La Salle College

F.6 Mathematics Paper I

2018 – 2019 Mock Examination

Name: ______________________ **Total Mark: 105**

Class: ______________( ) **Time Allowed: 135 min**

Instructions to students:

1. This paper consists of THREE sections, A(1), A(2) and B. Each section carries 35 marks.
2. Unless otherwise specified, all working must be clearly shown.
3. Unless otherwise specified, numerical answers must be exact or correct to 3 significant figures.
4. The diagrams in the paper are not necessarily drawn to scale.

Section A(1) (35 marks)

1. Make b the subject of the formula $\dfrac{6a-b+5c}{2b}=3a+4$. (3 marks)

2. Simplify $\dfrac{(x^{-3}y^2)^4}{x^{-2}y^3}$ and express your answer with positive indices. (3 marks)

3. (a) Round up 123.456 to 2 significant figures.

(b) Round off 123.456 to the nearest integer.

(c) Round down 123.456 to 1 decimal place.

(3 marks)

4. A box contains only black balls and white balls. The probability of drawing a black ball is $\frac{1}{4}$. If there are 10 more white balls than black balls, find the total number of balls in the box.

(3 marks)

5. Factorize

(a) $45x^2 - 125y^2$,

(b) $45x^2 - 125y^2 - 12x - 20y$.

(4 marks)

6. (a) Find the range of values of x which satisfy both $\frac{2x+4}{3} \geq 1$ and $x^2 - 2x - 3 < 0$.

(b) Write down the number of non-negative integer(s) satisfying both inequalities in (a).

(4 marks)

7. The cost of a watch is \$800. The marked price of the watch is 60% higher than the cost. Jeff sold the watch at a discount of x%. If the watch is sold at a profit of 12%, find the value of x.

(4 marks)

8. Jason and Martin both need to travel x km to go to the same office to work. One morning they set off at the same time, Jason walked at 5 km/h for 12 minutes and then got on a bus which travelled at 20 km/h for the rest of the journey. Martin travelled the whole journey in a vehicle at 30 km/h and arrived at the office 45 minutes before Jason. Find the value of x.

(5 marks)

9. In Figure 1, $ADCB$ is a semi-circle and PAB is a straight line. It is given that $CD = CB$ and $\angle BPC = 24°$.

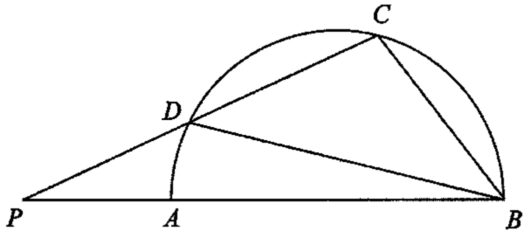

Figure 1

(a) If $\angle CBD = x$, express $\angle DBP$ in terms of x.

(b) Find $\angle CBD$.

(6 marks)

Section A(2) (35 marks)

10. John sells each copy of a weekly magazine at the price of $\$x$ and the total profit of selling the magazines is $\$y$. It is given that one part of y varies directly as x and the other part varies directly as the square of x. When $x = 42$, $y = 79\ 800$; when $x = 50$, $y = 75\ 000$.

    (a) Express y in terms of x. (3 marks)

    (b) Using the method of completing the square, find the greatest total profit of selling the magazines. (3 marks)

11. Let $f(x) = ax^3 + bx^2 - 30x + 16$, where a and b are real constants. It is given that $3x - 2$ is a factor of $f(x)$. When $f(x)$ is divided by $x + 3$, the remainder is $-a - b$.

(a) Find the values of a and b. (3 marks)

(b) Jacky claims that the equation $f(x) = 0$ has at least one irrational root. Do you agree? Explain your answer. (4 marks)

12. The stem-and-leaf diagram below shows the distribution of the ages of the players in a badminton team.

Stem (tens)	Leaf (units)
1	8 9
2	3 3 4 4 5 5 6 7 7 8 8 9 9
3	0 4 6

(a) Find the median and inter-quartile range of the ages of the players. (2 marks)

(b) Ronnie divides the badminton team into two teams, the younger team and the elder team. The younger team consists of the 9 youngest players while the elder team consists of all the other players If Ronnie randomly draws a person from each team to be the captain, find the probability that the difference in the ages of the two captains is less than 10. (2 marks)

(c) The 2 oldest players leave the team while 2 new players join the team. The mean age of the 2 new players is 26. Edward claims that the median of the ages of the new team must be less than that of the original team. Do you agree? Explain your answer. (3 marks)

13. (a) A vessel in the form of a frustum is made by cutting off the lower part of an inverted right circular cone. The vessel has a height of 8 cm with the radii of the top and the base equal to 15 cm and 9 cm respectively. Find the capacity of the vessel in terms of π. (4 marks)

(b) Arthur owns a bigger vessel that is similar in shape to the vessel in (a). The slant edge of this vessel is of length 12 cm. Arthur claims that the capacity of this vessel is less than 6000 cm^3. Do you agree? Explain your answer. (3 marks)

14. P is a moving point in the rectangular coordinate plane such that the perpendicular distance from P to the straight line $L : y = -1$ is equal to the distance between P and $F(0, 1)$. Denote the locus of P by Γ.

(a) Find the equation of Γ. (2 marks)

(b) L_1 is a straight line passing through $A(0, -4)$ with positive slope m. It is given that L_1 cuts the x-axis at point B and L_1 touches Γ at the point T. Find the equation of L_1. (2 marks)

(c) L_2 is a straight line passing through B and perpendicular to L_1. Q is a moving point in the same coordinate plane such that it maintains a fixed distance of $\sqrt{20}$ from L_2.

(i) Describe the locus of Q.

(ii) Find the equation of the locus of Q.

(4 marks)

Section B (35 marks)

15. Five boys and three girls stand in a line for taking photos. Find the number of the arrangements that can be made if

 (a) three girls stand together, (1 mark)

 (b) no girls stand next to each other. (2 marks)

16. It is known that the radii of circles in a sequence form an arithmetic sequence. Let r_n and C_n be the radius and the circumference of the nth circle respectively. Suppose $r_3 = 12$ and $r_8 = 22$.

 (a) Express C_n in terms of n. (2 marks)

 (b) Determine whether the circumferences of the circles form an arithmetic sequence or not. Justify your answer. (2 marks)

17. The radii of the metal balls produced by a machine are normally distributed with mean 10 cm and standard deviation 1 cm. For data in normal distribution, assume that 68%, 95% and 99.7% of data lie between one, two and three standard deviations from the mean respectively.

(a) Find the percentage of the metal balls with volumes between 972π cm^3 and 2304π cm^3.

(3 marks)

(b) It is known that the cost $\$C$ of producing a metal ball is partly constant and partly varies directly as the radius r cm of the ball. When the radius is 8 cm, the cost is \$368. Also, it is known that the costs of 16% of the balls are higher than \$386.

(i) Express C in terms of r.

(ii) Find the mean and the standard deviation of the cost of a metal ball.

(5 marks)

18. A triangular plate ABC is placed such that BC lies on the horizontal ground along the east-west direction and $\angle ACF = 40°$, where P is the projection of A on the ground, as shown in Figure 1. Given that $BC = 5$ cm, $BP = 8$ cm and $CP = 6$ cm.

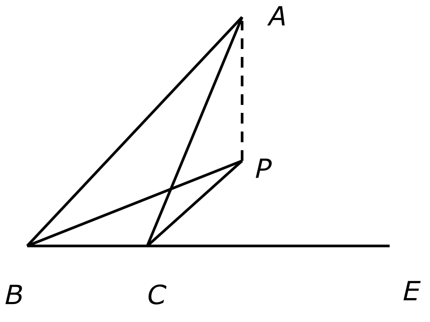

Figure 1

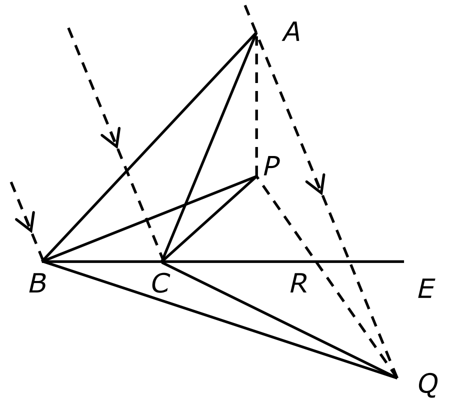

Figure 2

(a) Find $\angle PBC$. (2 marks)

(b) When the sun shines from the bearing N35°W with the angle of elevation 30°, the shadow of the plate on the ground is BCQ as shown in Figure 2.

(i) Let R be the point of intersection of the lines PQ and BC. Find RQ.

(ii) Find the area of the shadow BCQ.

(iii) [Modified] Suppose the bearing of the sun is unchanged and the angle of elevation is decreasing. Is it possible to obtain the area of the shadow twice the area found in (b)(ii)? Justify your answer.

(7 marks)

19. Let ΔABC be an inscribed triangle in a circle, where the angle bisectors of $\angle A$, $\angle B$, $\angle C$ meet the circle at P, Q, R. I is the incentre of ΔABC and M is the point of intersection of AP and QR as shown in Figure (a).

(a) Prove that AP is perpendicular to QR. (3 marks)

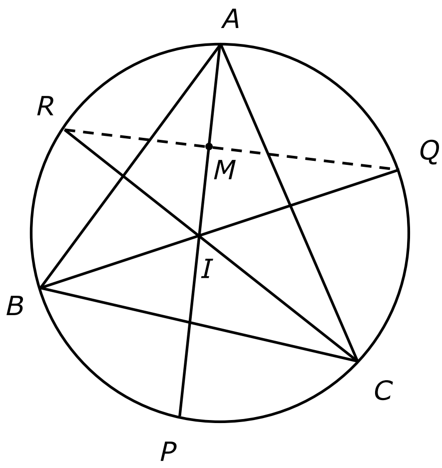

Figure (a)

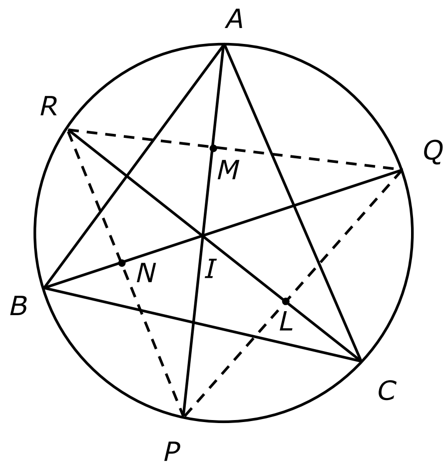

Figure (b)

(b) Let N, L be the points of intersection of BQ and PR, CR and PQ respectively as shown in Figure (b). Suppose the coordinates of I and Q are $(1, -2)$ and $(4, -1)$ respectively and the slope of QR is $\frac{-1}{2}$.

(i) Find the equation of line QR.

(ii) Given that $IN = IL = \sqrt{2}$. A circle S is constructed with centre I and radius $\sqrt{2}$. David claims that QR is a tangent to the circle S. Do you agree? Explain your answer.

(4 marks)

END OF PAPER