

La Salle College
Form Six Mock Examination 2019
Mathematics Compulsory Part Paper 1 (Section A) Marking Scheme

Solution	Remarks / Comments
<p>1. $\frac{6a-b+5c}{2b} = 3a+4$</p> $6a-b+5c = 6ab+8b$ $6ab+9b = 6a+5c$ $3b(2a+3) = 6a+5c$ $b = \frac{6a+5c}{3(2a+3)}$	<p>Very Good. Some candidates were careless having made a as the subject instead.</p>
<p>2. $\frac{(x^{-3}y^2)^4}{x^{-2}y^3}$</p> $= \frac{x^{-12}y^8}{x^{-2}y^3}$ $= x^{-12-(-2)}y^{8-3}$ $= x^{-10}y^5$ $= \frac{y^5}{x^{10}}$	<p>Very Good.</p>
<p>3. (a) 130 (b) 123 (c) 123.4</p>	<p>(a) Fair. Many candidates mistakenly gave 120 as the answer. (b) Very Good. (c) Fair. Many candidates mistakenly gave 123.5 as the answer.</p>
<p>4. Let n be the number of black balls.</p> $\frac{n}{n+(n+10)} = \frac{1}{4}$ $4n = 2n+10$ $2n = 10$ $n = 5$ <p>The total number of balls in the box</p> $= 2 \times 5 + 10$ $= 20$	<p>Good. Some candidates were able to find the number of black balls and white balls respectively. However, they did not compute the total number of balls as required.</p>

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<p>5. (a) $45x^2 - 125y^2$ $= 5(9x^2 - 25y^2)$ $= 5[(3x)^2 - (5y)^2]$ $= 5(3x+5y)(3x-5y)$</p> <p>(b) $45x^2 - 125y^2 - 12x - 20y$ $= 5(3x+5y)(3x-5y) - 4(3x+5y)$ $= (3x+5y)[5(3x-5y) - 4]$ $= (3x+5y)(15x-25y-4)$</p>	<p>(a) Good. Most candidates recognised that the expression could be reduced to the difference of squares after taking out the common factor. However, some were careless that they failed to put down the common factor.</p> <p>(b) Good. Most candidates made use of the results of (a) . However, a number of them finished the factorization by leaving their answer as $(3x+5y)[5(3x-5y)-4]$. Some were also careless factorizing the expression as $5(3x-5y-4)(3x+5y)$.</p>
<p>6. (a) $\frac{2x+4}{3} \geq 1$ $2x+4 \geq 3$ $2x \geq -1$ $x \geq -\frac{1}{2}$ $x^2 - 2x - 3 < 0$ $(x+1)(x-3) < 0$ $-1 < x < 3$</p> <p>Thus, the overall solution is $-\frac{1}{2} \leq x < 3$.</p> <p>(b) 3</p>	<p>(a) Fair. Many candidates were weak in solving the quadratic inequality.</p> <p>(b) Good.</p>
<p>7. The marked price of the watch $= \\$800 \times (1 + 60\%)$ $= \\$1280$</p> <p>The selling price of the watch $= \\$1280 \times (1 - x\%)$ $1280 \times (1 - x\%) = 800 \times (1 + 12\%)$ $1280 \times (1 - x\%) = 896$ $1 - x\% = 0.7$ $x\% = 0.3$ $x = 30$</p>	<p>Good. Some candidates wrongly computed the selling price as $\\$1200 \times x\%$ when a discount of $x\%$ was offered to the marked price.</p>

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<p>8. The distance that Jason travelled on the bus</p> $= \left(x - 5 \times \frac{12}{60} \right) \text{ km}$ $= (x - 1) \text{ km}$ <p>The time that Jason travelled on the bus</p> $= \frac{x - 1}{20} \text{ hours}$ <p>The time that Martin travelled</p> $= \frac{x}{30} \text{ hours}$ $\left(\frac{x - 1}{20} + \frac{12}{60} \right) - \frac{x}{30} = \frac{45}{60}$ $3(x - 1) + 12 - 2x = 45$ $3x - 3 + 12 - 2x = 45$ $x = 36$	<p>Fair. A number of candidates failed to recognise that the unit 'km/h' was used for the speed while 'minutes' was used for the travelling time. Some also missed out the 12-minute walking time when setting up the equation.</p>
<p>9. (a) $\because CD = CB$ (given)</p> <p>$\therefore \angle BDC = x$ (base \angle s, isos. Δ)</p> <p>$\therefore \angle DBP = x - 24^\circ$ (ext. \angle of Δ)</p> <p>(b) Join AD.</p> <p>$\angle ADB = 90^\circ$ (\angle in semi-circle)</p> <p>$(x - 24^\circ) + x + (x + 90^\circ) = 180^\circ$ (opp. \angle s, cyclic quad.)</p> <p>$3x = 114^\circ$</p> <p>$x = 38^\circ$</p> <p>$\angle CBD = 38^\circ$</p>	<p>(a) Good. Most candidates were able to express $\angle DBP$ in terms of x although some of them have missed out the unit.</p> <p>(b) Good. Some candidates wrongly recognised $\angle DCB = 90^\circ$.</p>

Solution	Remarks / Comments
<p>10. (a) Let $y = hx + kx^2$, where h and k are non-zero constants. So, we have $42h + 42^2 k = 79800$ and $50h + 50^2 k = 75000$. Solving, we have $h = 4000$ and $k = -50$. Thus, we have $y = 4000x - 50x^2$</p> <p>(b) $y = 4000x - 50x^2$ $= -50(x^2 - 80x)$ $= -50 \left[x^2 - 80x + \left(\frac{80}{2}\right)^2 - \left(\frac{80}{2}\right)^2 \right]$ $= -50(x - 40)^2 + 80000$ Thus, the greatest total profit is \$ 80000 .</p>	<p>(a) Very Good. A small number of candidates poorly presented the partial variation as $y \propto x + x^2$.</p> <p>(b) Good. Most candidates managed to make use of the method of completing the square to find the greatest total profit. However, some of them wrongly substituted $y = 0$ before using the method of completing square.</p>
<p>11. (a) $\begin{cases} f\left(\frac{2}{3}\right) = 0 \\ f(-3) = -a - b \end{cases}$</p> <p>$\begin{cases} a\left(\frac{2}{3}\right)^3 + b\left(\frac{2}{3}\right)^2 - 30\left(\frac{2}{3}\right) + 16 = 0 \\ a(-3)^3 + b(-3)^2 - 30(-3) + 16 = -a - b \end{cases}$</p> <p>$\begin{cases} 2a + 3b = 27 \quad \dots (1) \\ 13a - 5b = 53 \quad \dots (2) \end{cases}$</p> <p>By solving (1) and (2) , we have $a = 6$ and $b = 5$.</p> <p>(b) $f(x) = 0$ $(3x - 2)(2x^2 + 3x - 8) = 0$ $x = \frac{2}{3}$ or $x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-8)}}{2(2)} = \frac{-3 \pm \sqrt{73}}{4}$</p> <p>Hence, there are two distinct irrational roots. Thus, the claim is agreed.</p>	<p>(a) Very Good.</p> <p>(b) Fair. Most candidates were able to factorise the cubic function into two factors. However, some only claimed that the equation $2x^2 + 3x - 8 = 0$ has irrational roots without any explanation. Some merely used their calculators to find the approximate values, in decimals (which are rational), of the roots. Some also claimed that there are irrational roots by using $\Delta > 0$ only. In addition, candidates should note that it is not acceptable to draw the conclusion by simply putting down ‘\therefore Yes’ or ‘\therefore Yes, the claim is correct’ at the end of their argument. They were expected to clearly indicate whether they agree or disagree with the claim.</p>

Solution	Remarks / Comments
<p>12. (a) The median</p> $= \frac{26+27}{2}$ $= 26.5$ <p>The inter-quartile range</p> $= 29 - 24$ $= 5$ <p>(b) The required probability</p> $= \frac{2+4+7+7+7+7+8+8+8}{9 \times 9}$ $= \frac{58}{81}$ <p>(c) Let x and y be the ages of the new players. Assume that $x \leq y$.</p> <p>Note that $\frac{x+y}{2} = 26$, i.e. $x+y = 52$.</p> <p>Case 1 : When $x = y = 26$,</p> $\text{the new median} = \frac{26+26}{2} = 26 < 26.5 .$ <p>Case 2 : When $x \leq 25$, $y = 52 - x$,</p> <p>then we have $y \geq 52 - 25 = 27$.</p> $\text{The new median} = \frac{25+26}{2} = 25.5 < 26.5$ <p>Thus, the claim is agreed.</p>	<p>(a) Good. Some candidates could not identify the upper and lower quartiles correctly.</p> <p>(b) Good. Some candidates were careless in counting the number of favourable outcomes.</p> <p>(c) Fair. Many candidates were unable to discuss the situation by separating it into two cases. Some used particular numbers to argue that the new median is smaller than the original one in the case that one player is of at most 25 years of age while the other is of at least of 27 years of age. In addition, candidates should note that they were expected to address to the situation ‘the median of the ages of the new team must be less than that of the original team’ directly instead of the other way round, i.e. they should not argue by saying ‘if the new median age is of at least 26.5’.</p>

Solution	Remarks / Comments
<p>13. (a) Let h cm be the height of the removed cone.</p> $\frac{h}{9} = \frac{h+8}{15}$ $15h = 9h + 72$ $6h = 72$ $h = 12$ <p>The capacity of the vessel</p> $= \left[\frac{1}{3} \pi \times 15^2 \times (12+8) - \frac{1}{3} \pi \times 9^2 \times 12 \right] \text{cm}^3$ $= 1176 \pi \text{ cm}^3$ <p>(b) The slant edge of the smaller vessel</p> $= \left[\sqrt{(12+8)^2 + 15^2} - \sqrt{9^2 + 12^2} \right] \text{cm}$ $= 10 \text{ cm}$ <p>Let $V \text{ cm}^3$ be the capacity of the larger vessel.</p> $\frac{V}{1176 \pi} = \left(\frac{12}{10} \right)^3$ $V = 2032.128 \pi$ ≈ 6384.118396 > 6000 <p>Thus, the claim is disagreed.</p>	<p>(a) Good. Most candidates were able to find the capacity of the frustum. However, some wrongly gave $\frac{h}{8} = \frac{9}{15}$ when attempting to find the height of the removed cone.</p> <p>(b) Good. Most candidates were able to use the properties of similar solids to finish their arguments.</p>

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<p>14. (a) Let (x, y) be the coordinates of P.</p> $\sqrt{(x-0)^2 + (y-1)^2} = y - (-1)$ $x^2 + y^2 - 2y + 1 = y^2 + 2y + 1$ $y = \frac{1}{4}x^2$ <p>(b) The equation of L_1 is $y = mx - 4$.</p> <p>Sub $y = mx - 4$ into $y = \frac{1}{4}x^2$:</p> $\frac{1}{4}x^2 = mx - 4$ $x^2 = 4mx - 16$ $x^2 - 4mx + 16 = 0 \quad \dots (*)$ <p>$\therefore L_1$ is a tangent to Γ at T</p> <p>$\therefore \Delta = 0$</p> $(-4m)^2 - 4(1)(16) = 0$ $16m^2 - 64 = 0$ $m^2 = 4$ $m = 2 \text{ or } m = -2 \text{ (rejected)}$ <p>Thus, the equation of L_1 is $y = 2x - 4$.</p> <p>(c) (i) The locus of Q is a pair of straight lines that are parallel to L_2 which have a perpendicular distance of $\sqrt{20}$ from L_2.</p> <p>(ii) Let θ be the inclination of L_1.</p> $\tan \theta = 2$ $\theta = \tan^{-1} 2$ <p>The slope of $L_2 = -1 \div 2 = -0.5$</p> <p>Let c be the difference between the y-intercept of L_2 and that of the locus of Q.</p> $\cos(180^\circ - \tan^{-1} 2 - 90^\circ) = \frac{\sqrt{20}}{c}$ $c = 5$ <p>Thus, the equations of the locus of Q are $y = -0.5x + 6$ and $y = -0.5x - 4$.</p>	<p>(a) Fair. A number of candidates could not even set up the equation by using the distance formula. Some also wrongly gave $\sqrt{(x-0)^2 + (y+1)^2}$ as the distance between the moving point and the line L.</p> <p>(b) Fair. A number of candidates managed to set up the equation as $y = mx - 4$. However, they could not proceed any further.</p> <p>(c) (i) Fair. Most candidates pointed out that the locus is a pair of parallel lines. However, a number of them did not point out that the pair of straight lines is parallel to L_2. Some also did not mention the perpendicular distance between the locus and L_2.</p> <p>(ii) Poor. Most candidates could not distinguish 'perpendicular distance' from 'vertical distance'.</p>