Methodist College

First Mock Examination, 2022 – 2023

F.6 Mathematics

Paper 2

Name: ______________________________ Date: 20 October 2022

Class: ____________ Class No.: ____ Time 11:15 am -12:30 pm (1 ¼ hours)

Solution

INSTRUCTIONS:

1. Read carefully the instructions on the Answer Sheet. After the announcement of the start of the examination, you should first insert the information required in the spaces provided.
2. When told to open this paper, you should check that all the questions are there. Look for the word '**END OF PAPER**' after the last question.
3. All questions carry equal marks.
4. **ANSWER ALL QUESTIONS.** You are advised to use an HB pencil to mark all the answers on the Answer Sheet, so that wrong marks can be completely erased with a clean rubber. You must mark the answers clearly; otherwise you will lose marks if the answers cannot be captured.
5. You should mark only **ONE** answer for each question. If you mark more than one answer, you will receive NO MARKS for that question.
6. No marks will be deducted for wrong answers.
7. The diagrams in this paper are not necessarily drawn to scale.

Section A

1. $\dfrac{(2a^6)^3}{4a^2} =$

   A. $2a^{16}$.

   B. $2a^9$.

   C. $8a^{16}$.

   D. $8a^9$.

Solution

$$\frac{(2a^6)^3}{4a^2} = \frac{2^3 a^{18}}{4a^2} = 2a^{18-2} = 2a^{16}$$

The answer is A.

2. If $\dfrac{a}{x} + \dfrac{b}{y} = 3$, then $x =$

   A. $\dfrac{by}{3y-a}$.

   B. $\dfrac{by}{a-3y}$.

   C. $\dfrac{ay}{b-3y}$.

   D. $\dfrac{ay}{3y-b}$.

Solution

$$\frac{a}{x} + \frac{b}{y} = 3$$

$$\frac{ay+bx}{xy} = 3$$

$$ay = 3xy - bx$$

$$x = \frac{ay}{3y-b}$$

The answer is D.

3. $\dfrac{1}{3x+7}-\dfrac{1}{3x-7}=$

A. $\dfrac{14}{49-9x^2}$.

B. $\dfrac{14}{9x^2-49}$.

C. $\dfrac{6x}{49-9x^2}$.

D. $\dfrac{6x}{9x^2-49}$.

<u>Solution</u>

$$\frac{1}{3x+7}-\frac{1}{3x-7}=\frac{(3x-7)-(3x+7)}{(3x)^2-7^2}=\frac{-14}{9x^2-49}=\frac{14}{49-9x^2}$$

The answer is A.

4. $3m^2-5mn+2n^2+m-n\ =$

A. $(m+n)(3m-2n-1)$.

B. $(m+n)(3m+2n-1)$.

C. $(m-n)(3m-2n+1)$.

D. $(m-n)(3m+2n+1)$.

<u>Solution</u>

$$3m^2-5mn+2n^2+m-n=(m-n)(3m-2n)+(m-n)=(m-n)(3m-2n+1)$$

The answer is C.

5. Let c be a constant. If $f(x) = x^3 + cx^2 + c$, then $f(c) + f(-c) =$

A. 0 .

B. $2c$.

C. $2c^3 + 2c$.

D. $-2c^3 + 2c$.

Solution

$f(c) + f(-c) = c^3 + c(c)^2 + c + (-c)^3 + c(-c)^2 + c = c^3 + c^3 + c - c^3 + c^3 + c = 2c^3 + 2c$

The answer is C.

6. Let $g(x) = ax^3 + 4ax^2 - 24$, where a is a constant. If $x + 2$ is a factor of $g(x)$, then $g(-1) =$

A. -27.

B. -15.

C. 0.

D. 12.

Solution

$$g(-2) = 0$$
$$a(-2)^3 + 4a(-2)^2 - 24 = 0$$
$$-8a + 16a - 24 = 0$$
$$8a = 24$$
$$a = 3$$
$$g(-1) = 3(-1)^3 + 12(-1)^2 - 24 = -15$$

The answer is B.

7. If $a = 9.23$ (correct to 2 decimal places), find the range of value of a.

A. $9.22 < a \le 9.24$

B. $9.22 \le a < 9.24$

C. $9.225 < a \le 9.235$

D. $9.225 \le a < 9.235$

Solution

The answer is D.

8. The monthly salary of Aska is 15% higher than that of Gary while the monthly salary of Gary is 5% lower than that of Ceci. It is given that the monthly salary of Aska is $97 750. The monthly salary of Ceci is

A. $59 500.

B. $85 000.

C. $89 474.

D. $112 413.

Solution

Let the monthly salary of Aska be $$A$.

Let the monthly salary of Ceci be $$C$.

Let the monthly salary of Gary be $$G$.

$$A = 1.15G \quad \& \quad G = 0.95C$$
$$A = 1.15(0.95C)$$
$$A = 1.0925C$$
$$97750 = 1.0925C$$
$$C = 89474$$

The answer is C.

9. In the figure, the equations of the straight lines L_1 and L_2 are $ax + y + b = 0$ and $x + cy - d = 0$ respectively. Which of the following are true?

I. $c > 0$

II. $d = c$

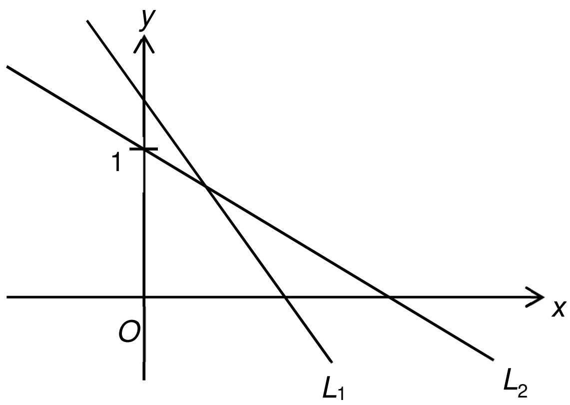
III. $ac > 1$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III



Solution

Statement I :

$$L_2 : y = \frac{-x}{c} + \frac{d}{c}$$

$m_{L2} < 0$

$\therefore \quad c > 0$

Statement II :

$$L_2 : y = \frac{-x}{c} + \frac{d}{c}$$

Substitute (0 ,1) into L_2, $c = d$.

Statement III :

$$L_2 : y = \frac{-x}{c} + \frac{d}{c} \text{ and } L_1 : y = -ax - b$$

$m_{L2} > m_{L1}$

$\therefore \quad -\frac{1}{c} > -a$

$\frac{1}{c} < a$

$ac > 1$

The answer is D.

10. Which of the following statements about the graph of $y = (3 - x)(-x + 2) + 6$ is/are true?

I. The graph opens downwards.

II. The graph passes through the point $(1 , 8)$.

III. The x-intercepts of the graph are irrational.

A. I only

B. II only

C. I and III only

D. II and III only

<u>Solution</u>

$y = (3 - x)(-x + 2) + 6$

$y = x^2 - 5x + 12$

Statement I :

Since $a = 1$, the graph opens upwards.

Statement I is incorrect.

Statement II :

Substitute (1, 8) into $y = x^2 - 5x + 12$, L.H.S. = R.H.S. = 8.

Statement II is correct.

Statement III :

$0 = x^2 - 5x + 12$

$\Delta = (-5)^2 - 4(12) = -23$

$\Delta < 0$

$\therefore$ No real roots.

Statement III is incorrect.

The answer is B.

11. Let a , b and c be non-zero numbers. If $2a = 3b$ and $a : c = 1 : 4$,

then $\dfrac{a+3b}{b+3c} =$

A. $\dfrac{38}{3}$.

B. 1.

C. $\dfrac{2}{3}$.

D. $\dfrac{9}{38}$.

Solution

$2a = 3b$ & $a : c = 1 : 4$

$b = \dfrac{2}{3}a$ $\quad c = 4a$

$$\frac{a + 3b}{b + 3c} = \frac{a + 3(\frac{2}{3}a)}{(\frac{2}{3}a) + 3(4a)} = \frac{9}{38}$$

The answer is D.

12. It is given that w varies as the cube of u and the square root of v. When $u = 3$ and $v = 16$, $w = 216$. When $u = 2$ and $v = 9$, $w =$

A. 48.

B. 108.

C. 288.

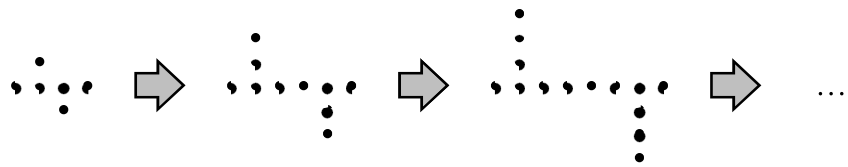
D. 342.

Solution

$$w = ku^3\sqrt{v}$$
$$216 = (3)^3(\sqrt{16})k$$
$$k = 2$$
$$w = 2(2)^3(\sqrt{9})$$
$$w = 48$$

The answer is A.

13. In the figure, the 1st pattern consists of 6 dots. For any positive integer n, the $(n + 1)^{th}$ pattern is formed by adding 4 dots to the n^{th} pattern. Find the number of dots in the 9th pattern.



A. 30

B. 34

C. 38

D. 42

Solution

The number of dots in the 9th pattern = 6 + 4 (8) = 38.

The answer is C.

14. The solution of $2(1-x)-10>-4$ or $\frac{7x+3}{3}\geq -6$ is

A. $x \geq -3$.

B. $x < -2$.

C. $-3 \leq x < -2$.

D. all real solution of x.

Solution

$$\frac{7x+3}{3} \geq -6 \qquad \text{or} \qquad 2(1-x)-10 > -4$$

$$7x \geq -21 \qquad\qquad 2-2x > 6$$

$$x \geq -3 \qquad\qquad -2x > 4$$

$$x < -2$$

The solution is all real solution of x.

The answer is D.

15. The coordinates of the points A and B are (4, 7) and (–2, –6) respectively. Let P be a moving point in the rectangular coordinate plane such that $AP = BP$. Find the equation of the locus of P.

A. $12x + 26y - 25 = 0$

B. $12x - 26y + 25 = 0$

C. $26x + 12y - 25 = 0$

D. $26x - 12y + 25 = 0$

Solution

$$AP = BP$$

$$\therefore \qquad \sqrt{(x-4)^2+(y-7)^2} = \sqrt{(x+2)^2+(y+6)^2}$$

$$x^2 - 8x + 16 + y^2 - 14y + 49 = x^2 + 4x + 4 + y^2 + 12y + 36$$

$$12x + 26y - 25 = 0$$

The answer is A.

16. The mean of 90 integers is 135. If the mean of 40 of these 90 integers is 105, then the mean of the remaining 50 integers is

    A. 156.
    B. 159.
    C. 162.
    D. 165.

Solution

The mean of the remaining 50 integers = $\dfrac{90 \times 135 - 40 \times 105}{50} = 159$.

The answer is B.

17. If the volume of a right circular cone of base radius $3a$ cm and height $4b$ cm is 432 cm^3, then the volume of a right circular cylinder of base radius $7a$ cm and height $5b$ cm is

    A. 1620 cm^3.
    B. 2940 cm^3.
    C. 6370 cm^3.
    D. 8820 cm^3.

Solution

$$\frac{(3a)^2 \pi \times 4b}{3} = 432$$

$$a^2 b\pi = 36$$

The volume of a right circular cylinder = $(7a)^2 \pi \times 5b = 245a^2 b\pi = 8820 \text{ cm}^3$.

The answer is D.

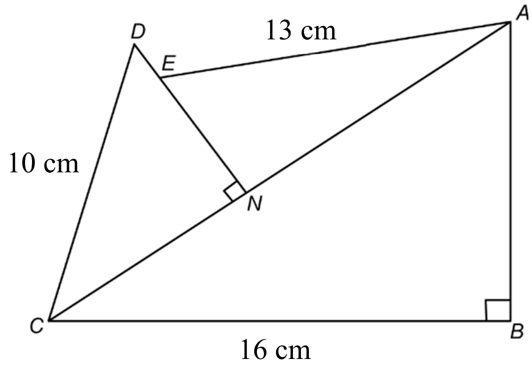
18. In the figure, N is a point lying on AC and E is a point lying on DN. If $DN = 6$ cm and $DE = 1$ cm, then the area of ΔABC is

A. 24 cm^2.

B. 30 cm^2.

C. 96 cm^2.

D. 192 cm^2.



Solution

$CN = \sqrt{10^2 - 6^2} = 8$

$AN = \sqrt{13^2 - 5^2} = 12$

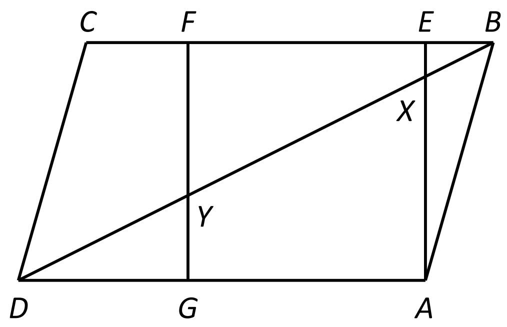
$AB = \sqrt{20^2 - 16^2} = 12$

$\therefore$ The area of $\Delta ABC = \dfrac{12 \times 16}{2} = 96$ cm^2.

The answer is C.

19. In the figure, $ABCD$ is a parallelogram and $AEFG$ is a square. It is given that $BE : EF : FC = 2 : 7 : 3$. BD cuts AE and FG at the points X and Y respectively. If the area of ΔABX is 24 cm^2 , then the area of the quadrilateral $CDYF$ is

A. 54 cm^2.
B. 77 cm^2.
C. 81 cm^2.
D. 87 cm^2.



Solution

$BE : EF : FC = 2 : 7 : 3$

$\therefore BE : DA = 2 : 12 = 1 : 6$

$\Delta BEX \sim \Delta DAX$ (AAA)

$\therefore BX : DX = BE : AD = 1 : 6$

ΔABX & ΔDAX share the same height.

$\therefore$ Area of $\Delta DAX = 24 \times 6 = 144 \text{ cm}^2$

Area of $\Delta BEX = 144 \div 6^2 = 4 \text{ cm}^2$

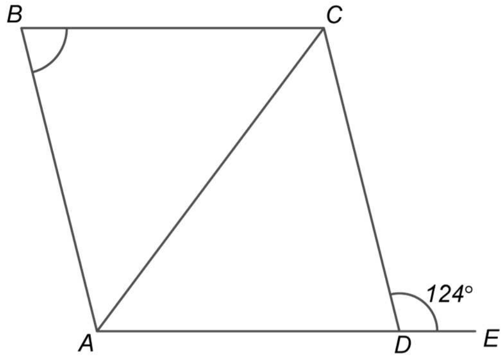
$\Delta BEX \sim \Delta BFY$ (AAA)

$\therefore$ Area of $\Delta BFY = 4 \times (\frac{9}{2})^2 = 81 \text{ cm}^2$

$\therefore$ The area of the quadrilateral $CDYF = 24 + 144 - 81 = 87 \text{ cm}^2$

The answer is D.

20. In the figure, $AB = BC$ and D is a point lying on AE such that $AC = AD$. If $AE \parallel BC$, then $\angle ABC =$



A. 44°.

B. 56°.

C. 62°.

D. 68°.

Solution

$\angle CDA = 56°$ (adj. ∠s on st. line)

$\angle ACD = 56°$ (base ∠s, isos Δ)

$\angle BCD = 124°$ (alt. ∠s, $AE \parallel BC$)

$\angle BCA = 124° - 56°$

$= 68°$

$\angle BCA = 68°$ (base ∠s, isos Δ)

$\angle ABC = 180° - 68° - 68°$ (∠ sum of Δ)

$= 44°$

The answer is A.

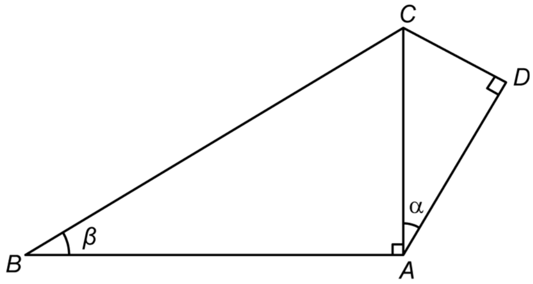
21. In the figure, $\dfrac{CD}{AB} =$

A. $\cos\alpha \tan\beta$.

B. $\sin\alpha \tan\beta$.

C. $\dfrac{\cos\alpha}{\tan\beta}$.

D. $\dfrac{\sin\alpha}{\tan\beta}$.



Solution

$$\tan\beta = \frac{AC}{AB}$$

$$AB = \frac{AC}{\tan\beta}$$

$$\sin\alpha = \frac{CD}{AC}$$

$$CD = AC\sin\alpha$$

$$\frac{CD}{AB} = \frac{AC\sin\alpha}{\frac{AC}{\tan\beta}} = \sin\alpha\tan\beta$$

The answer is B.

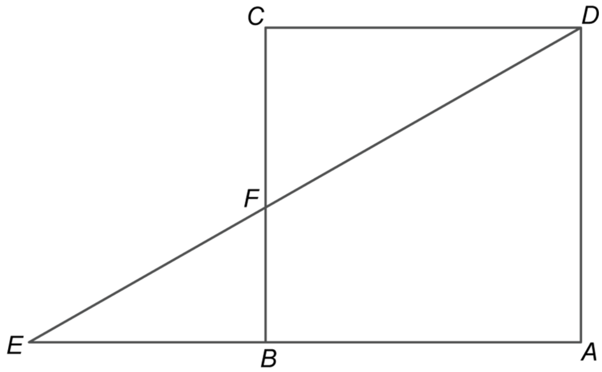
22. In the figure, $ABCD$ is a square. E is a point lying on AB produced such that $BE = 4$ cm. BC and DE intersect at the point F. If $EF = 5$ cm, then $DE =$

A. 12 cm.

B. 15 cm.

C. 16 cm.

D. 20 cm.



Solution

$BF = \sqrt{5^2 - 4^2} = 3$

$\Delta EFB \sim \Delta EDA$ (AAA) , $AB = AD$

$$\therefore \quad \frac{BF}{AD} = \frac{BE}{BE + AD}$$

$$\frac{3}{AD} = \frac{4}{4 + AD}$$

$$12 + 3AD = 4AD$$

$$AD = 12$$

$\therefore DE = \sqrt{16^2 + 12^2} = 20$ cm

The answer is D.

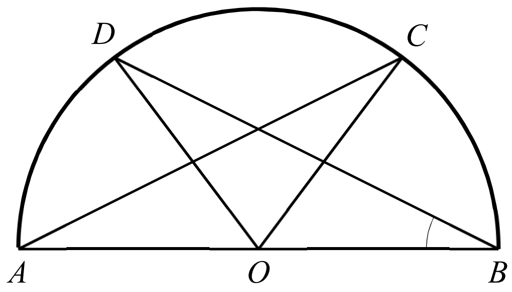
23. In the figure, O is the centre of the semi-circle $ABCD$. If $AC = BD$ and $\angle COD = 56°$, then $\angle ABD =$

A. 29° .

B. 31° .

C. 36° .

D. 48° .



Solution

Let	x	=	$\angle ABD$.	
Since	AC	=	BD ,	
	$\angle CAB$	=	x .	
	$\angle CAD$	=	28°	(∠ at centre twice ∠ at ⊙^{ce})
and	$\angle ADB$	=	90°	(∠ in semi-circle)
∴	$x + 28° + 90° + x$	=	180°	(∠ sum of Δ)
	$\angle ABD = x$	=	31°	

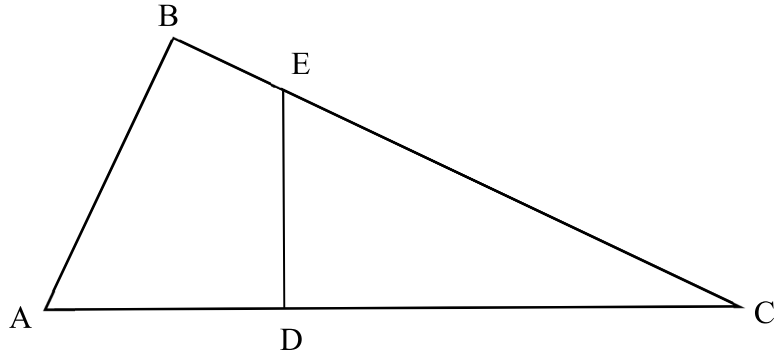
24. In the figure, ABC is a right-angled triangle with $\angle ABC = 90°$. Let D and E be points lying on AC and BC respectively such that $ABED$ is a cyclic quadrilateral. If $AB = 660$ cm, $AD = 572$ cm and $BE = 275$ cm, then $CD =$

A. 429 cm.

B. 625 cm.

C. 715 cm.

D. 728 cm.



Solution

$\angle EDC = \angle ABC = 90°$ (ext. ∠, cyclic quad.)

$AE = \sqrt{660^2 + 275^2} = 715$

$ED = \sqrt{715^2 - 572^2} = 429$

$\angle BAD = \tan^{-1}(\frac{275}{660}) + \tan^{-1}(\frac{429}{572}) = 59.4898°$

$\angle CED = \angle BAD = 59.4898°$ (ext. ∠, cyclic quad.)

$CD = 429\tan(59.4898°) = 728$ cm

The answer is D.

25. The coordinates of the point P are $(3\sqrt{3}, 9)$. P is translated downwards by 6 units to the point Q. Q is then rotated anti-clockwise about the origin through 90° to the point R. Find the polar coordinates of R.

A. (6, 30°)

B. (6, 120°)

C. $(6\sqrt{3}, 30°)$

D. $(6\sqrt{3}, 120°)$

Solution

$P(3\sqrt{3}, 9) \rightarrow Q(3\sqrt{3}, 3) \rightarrow R(-3, 3\sqrt{3})$

$$r = \sqrt{(3)^2 + (3\sqrt{3})^2} = 6$$

$$\theta = 180° - \tan^{-1}(\frac{3\sqrt{3}}{3}) = 120°$$

Therefore, the polar coordinates of R is (6, 120°).

The answer is B.

26. The equation of the straight line L is $8x - 3y + 12 = 0$. If P is a moving point in the rectangular coordinate plane such that the perpendicular distance from P to L is equal to 6, then the locus of P is

A. a sector.

B. a square.

C. a parabola.

D. a pair of straight lines.

Solution

The answer is D.

27. The coordinates of the points G and H are (1, 4) and (5, −8) respectively. A point P is moving on a rectangular coordinate plane such that $PG = GH$. The equation of the locus of P is

A. $3x + y - 7 = 0$.

B. $x - 3y - 9 = 0$.

C. $x^2 + y^2 - 2x - 8y - 143 = 0$.

D. $x^2 + y^2 - 10x + 16y - 71 = 0$.

Solution

$$PG = GH$$

$$\therefore \quad \sqrt{(x-1)^2 + (y-4)^2} = \sqrt{(5-1)^2 + (-8-4)^2}$$

$$x^2 + y^2 - 2x - 8y - 143 = 0$$

The answer is C.

28. The equations of the circles C_1 and C_2 are $x^2 + y^2 + 8x - 4y - 5 = 0$ and $2x^2 + 2y^2 + 8x - 4y - 5 = 0$ respectively. Let G_1 and G_2 be the centres of C_1 and C_2 respectively. Denote the origin by O. Which of the following is/are true?

I. G_1 , G_2 and O are collinear.

II. The radii of C_1 and C_2 are equal.

III. O is equidistant from G_1 and G_2.

A. I only

B. II only

C. I and III only

D. II and III only

Solution

Statement I:

G_1 (4, −2) , G_2 (2, −1), O (0, 0)

Slope of $G_1 O$ = Slope of $G_2 O$ = −0.5

Statement I is correct.

Statement II:

The radii of C_1 = $\sqrt{(4)^2 + (-2)^2 - (-5)} = 5$

The radii of C_2 = $\sqrt{(2)^2 + (-1)^2 - (-\frac{5}{2})} = \sqrt{7.5}$

Statement II is incorrect.

Statement III:

$G_1 O = \sqrt{(4)^2 + (2)^2} = \sqrt{20}$

$G_2 O = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$

Statement III is incorrect.

The answer is A.

29. Two numbers are randomly drawn at the same time from eight balls numbered 2 , 3 , 4 , 5 , 6 , 7 , 8 and 9 respectively. Find the probability that the two numbers drawn are consecutive integers.

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{2}{9}$

D. $\frac{7}{9}$

Solution

The required probability = $\frac{7}{8C2}=\frac{1}{4}$

The answer is B.

30. Consider the following positive integers:

$$4 \quad 7 \quad 8 \quad 8 \quad a \quad b \quad c \quad d$$

If the mode and the mean of the above positive integers are 3 and 6 respectively, which of the following must be true?

I. The median of the above positive integers is 5.5.

II. The range of the above positive integers is 4.

III. The inter-quarter range of the above positive integers is 5.

A. I only

B. II only

C. I and III only

D. II and III only

Solution

Since mode is 3, 3 of *a, b, c* and *d* must be 3. Assume *a, b, c* = 3.

$$\text{The mean} = \frac{4+7+8+8+3+3+3+d}{8} = 6$$

$$d = 12$$

Consider the following positive integers:

$$3 \quad 3 \quad 3 \quad 4 \quad 7 \quad 8 \quad 8 \quad 12$$

Statement I:

$$\text{The median} = \frac{4+7}{2} = 5.5$$

Statement I is correct.

Statement II:

Range = 12 − 3 = 9

Statement II is incorrect.

Statement III:

IQR = 8 − 3 = 5

Statement III is correct.

The answer is C.

Section B

31. $15\times16^{13}+17\times16^{10}+16^{3}+18=$

    A. $E0110000000112_{16}$.

    B. $E0110000001012_{16}$.

    C. $F0110000000112_{16}$.

    D. $F0110000001012_{16}$.

Solution

$15\times16^{13}+17\times16^{10}+16^{3}+18=15\times16^{13}+1\times16^{11}+1\times16^{10}+1\times16^{3}+1\times16^{1}+2\times16^{0}$

$=F0110000001012_{16}$

The answer is D.

32. If the roots of the equation $4(\log_{\pi}x)^2-3\log_{\pi}x+1=\log_{\pi}x$ are α and β, then $\alpha\beta=$

    A. $\pi^{\frac{1}{4}}$.

    B. π.

    C. $\log_{\pi}1$.

    D. $\log_{\pi}10$.

Solution

$4(\log_{\pi}x)^2-4(\log_{\pi}x)+1=0$

$\log_{\pi}x=0.5$ (repeated)

$\alpha\beta=(\pi^{0.5})(\pi^{0.5})$

$\alpha\beta=\pi$

The answer is B.

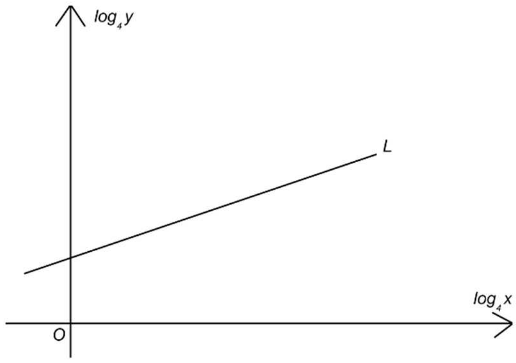
33. In the figure, the straight line L shows the relation between $\log_4 x$ and $\log_4 y$. It is given that L passes through the points (5, 5) and (3, 2). If $y = kx^a$, then $k =$

A. $-\frac{5}{2}$.

B. $\frac{1}{32}$.

C. $\frac{3}{2}$.

D. 32.



Solution

$$\frac{(\log_4 y) - 2}{(\log_4 x) - 3} = \frac{5-2}{5-3}$$

$$2\log_4 y - 4 = 3\log_4 x - 9$$

$$\log_4 y = \frac{3}{2}\log_4 x - \frac{5}{2}$$

$$\log_4 k = -\frac{5}{2}$$

$$k = \frac{1}{32}$$

The answer is B.

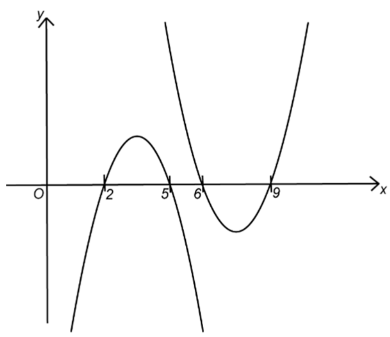
34. Let f(x) be a quadratic function. The figure below may represent the graph of $y = \mathrm{f}(x)$ and

A. the graph of $y = -\mathrm{f}(2x)$.

B. the graph of $y = \mathrm{f}(-x) + 4$.

C. the graph of $y = -\mathrm{f}(x + 4)$.

D. the graph of $y = -\mathrm{f}(2x) + 4$.



Solution

The answer is C.

35. Let $u = \dfrac{7}{a+i}$ and $v = \dfrac{7}{a-i}$, where a is a real number. Which of the following must be true?

I. $u + v$ is a rational number.

II. The real part of u is equal to the real part of v.

III. The imaginary part of $\dfrac{1}{u}$ is equal to the imaginary part of $\dfrac{1}{v}$.

A. I only

B. II only

C. I and III only

D. II and III only

Solution

Statement I:

$$u + v = \frac{7(a-i)+7(a+i)}{a^2+1} = \frac{14a}{a^2+1}$$

$\dfrac{14a}{a^2+1}$ is unnecessarily a rational number.

Statement II:

$$u = \frac{7(a-i)}{a^2+1} = \frac{7a}{a^2+1} - \frac{7}{a^2+1}i \quad , \quad v = \frac{7(a+i)}{a^2+1} = \frac{7a}{a^2+1} + \frac{7}{a^2+1}i$$

The real part of u = the real part of v = $\dfrac{7a}{a^2+1}$.

Statement III:

$$\frac{1}{u} = \frac{a}{7} + \frac{1}{7}i \quad , \quad \frac{1}{v} = \frac{a}{7} - \frac{1}{7}i$$

$$\frac{1}{7} \neq -\frac{1}{7}$$

The answer is B.

36. If the sum of the first n terms of a sequence is $6n^2 - n$, which of the following is/are true?

I. 17 is a term of the sequence.

II. The 1st term of the sequence is 5.

III. The sequence is a geometric sequence.

A. I only

B. I and II only

C. I and III only

D. II and III only

<u>Solution</u>

Statement I:

$$T(n) = (6n^2 - n) - [6(n-1)^2 - (n-1)] = 17$$
$$12n = 24$$
$$n = 2$$

Statement I is correct.

Statement II:

$$T(1) = 6(1)^2 - 1 = 5$$

Statement II is correct.

Statement III:

$$T(n) = (6n^2 - n) - [6(n-1)^2 - (n-1)]$$
$$= 12n - 7$$
$$T(3) = 29$$
$$\frac{T(3)}{T(2)} = \frac{29}{17}, \quad \frac{T(2)}{T(1)} = \frac{17}{5}$$
$$\frac{T(3)}{T(2)} \neq \frac{T(2)}{T(1)}$$

Statement III is incorrect.

The answer is B.

37. For $0° \le x < 360°$, how many roots does the equation $2\sin^2 x - 7\cos x = -3$ have?

A. 2

B. 3

C. 4

D. 5

Solution

$$2\sin^2 x - 7\cos x = -3$$
$$2(1-\cos^2 x) - 7\cos x + 3 = 0$$
$$-2\cos^2 x - 7\cos x + 5 = 0$$

$\cos x = 0.61$ or $\cos x = -4.11$ (rejected)

$x = 52.5°$

or $x = 360° - 52.5°$

$= 307°$

The answer is A.

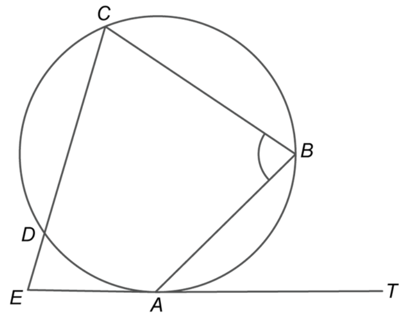
38. In the figure, TA is the tangent to the circle $ABCD$ at the point A. CD produced and TA produced meet at the point E. It is given that $AB = CD$, $\angle BAT = 32°$ and $\angle AED = 66°$. Find $\angle ABC$.

A. 41°

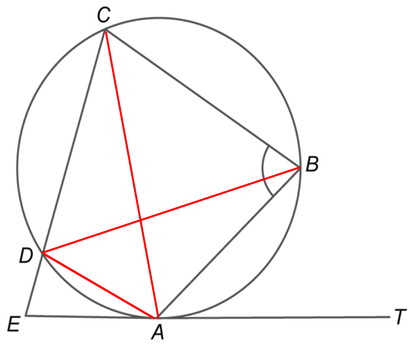
B. 66°

C. 73°

D. 107°



Solution



$\angle ACB = 32°$ ($\angle$ in alt. segment)

Since $AB = CD$, $\angle ACB = \angle CAD = 32°$. (equal chords, equal $\angle$s)

$\angle CAE = \angle ABC$ ($\angle$ in alt. segment)

$\angle DAE = \angle ABC - 32°$

$\angle ADE = \angle ABC$ (ext. $\angle$, cyclic quad.)

$(\angle ABC - 32°) + \angle ABC + 66° = 180°$ ($\angle$ sum of Δ)

$$\angle ABC = 73°$$

The answer is C.

39. Find the constant k such that the circle $x^2 + y^2 + 8x - 4y + k = 0$ and the straight line $2x + y - 9 = 0$ intersect at only one point.

A. -25

B. -15

C. 15

D. 25

Solution

$$\begin{cases} x^2 + y^2 + 8x - 4y + k = 0 \\ y = 9 - 2x \end{cases}$$

$$x^2 + (9 - 2x)^2 + 8x - 4(9 - 2x) + k = 0$$

$$x^2 + 81 - 36x + 4x^2 + 8x - 36 + 8x + k = 0$$

$$5x^2 - 20x + 45 + k = 0$$

$$\Delta = (-20)^2 - 4(5)(45 + k) = 0$$

$$400 - 900 - 20k = 0$$

$$k = -25$$

The answer is A.

40. The equation of the three sides of a triangle are $4x+3y=24$, $4x-3y=24$ and $x = a$, where a is a constant. If the x-coordinate of the centroid of the triangle is 30, then a =

A. 24.

B. 30.

C. 42.

D. 45.

Solution

$$\begin{cases} 4x+3y=24 \\ 4x-3y=24 \end{cases}$$

The intersection point is $(6,0)$.

$x = a$ is a vertical line and centroid divides the median by ratio 2:1.

$$\therefore \frac{30-6}{a-30}=\frac{2}{1}$$

$$a=42$$

The answer is C.

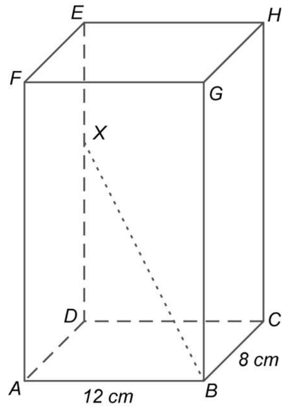
41. In the figure, $ABCDEFGH$ is a rectangular block. Let X be a point lying on DE such that $DX = 9$ cm. Denote the angle between BX and the plane $ABGF$ by θ. Find $\sin\theta$.

A. $\frac{3}{5}$

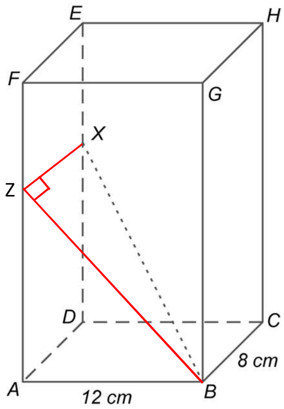
B. $\frac{4}{5}$

C. $\frac{8}{17}$

D. $\frac{15}{17}$



Solution



$\angle XBZ = \theta$

$BD = \sqrt{(8)^2 + (12)^2} = \sqrt{208}$

$BX = \sqrt{(208) + (9)^2} = 17$

$\sin\theta = \frac{8}{17}$

The answer is C.

42. A queue is formed by 6 boys and 2 girls. If girls must be next to each other, how many different queues can be formed?

A. 10080

B. 26040

C. 30240

D. 35280

Solution

The required number = $7! \times 2! = 10080$

The answer is A.

43. A box contains 3 white balls, 4 yellow balls and 3 red balls. A boy and a girl take turns to draw one ball randomly from the box with replacement until one of them draws a white ball or a yellow ball. The boy draws a ball first. Find the probability that the girl draws a white ball.

A. $\frac{9}{91}$

B. $\frac{3}{28}$

C. $\frac{10}{17}$

D. $\frac{17}{20}$

Solution

P (girl draws a white ball in her 1st round) = $\frac{3}{10} \times \frac{3}{10}$

P (girl draws a white ball in her 2nd round) = $\frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}$

P (girl draws a white ball in her 3rd round) = $\frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}$

$\therefore$ The probability that the girl draws a white ball = $\dfrac{(\frac{3}{10} \times \frac{3}{10})}{1 - (\frac{3}{10} \times \frac{3}{10})} = \frac{9}{91}$

The answer is A.

44. In a test, the difference of the test scores and the difference of the standard scores of two students are 40 and 4 respectively. In the test, the standard deviation of the test scores is

A. 10 marks.

B. 20 marks.

C. 36 marks.

D. 44 marks.

Solution

The difference of the test scores $= x_1 - x_2 = 40$

The difference of the standard scores $= z_1 - z_2 = 4$

$$z_1 = \frac{x_1 - \bar{x}}{\sigma},\ z_2 = \frac{x_2 - \bar{x}}{\sigma}$$

$$z_1 - z_2 = \frac{x_1 - \bar{x}}{\sigma} - \frac{x_2 - \bar{x}}{\sigma} = 4$$

$$\frac{x_1 - x_2}{\sigma} = 4$$

$$\sigma = 10$$

The answer is A.

45. There are 55 terms in an arithmetic sequence. If the variance of the first 5 terms of the sequence is 12, then the variance of the last 5 terms of the sequence is

A. 12.

B. 24.

C. 132.

D. 1452.

Solution

The first 5 terms of the sequence have the same common difference as the last 5 terms of the sequence. That means the dispersion of the first 5 terms is the same as the last 5 terms. Therefore, the variance of the last 5 terms of the sequence is 12.

The answer is A.

End of the paper