**Maryknoll Convent School**
**(Secondary Section)**
**Form 6 Mock Examination February 2020**
**Mathematics Compulsory Part**
**Paper 1**

Time allowed: $2\frac{1}{4}$ hours

**Name:** _____( )

**Form 6** _____

1. **This paper consists of THREE sections, A(1), A(2) and B.**

2. **Attempt ALL questions in this paper. Write all your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.**

3. **Supplementary answer sheets will be supplied on request.**

4. **Unless otherwise specified, all working must be clearly shown.**

5. **Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.**

6. **The diagrams in this paper are not necessarily drawn to scale.**

**Section A(1) (35 marks)**

1. Simplify $8^{2a} \cdot \frac{4^b}{64^{a-b}}$ where $a$ and $b$ are positive integers and express your answer as a power of 2 with a positive index.

(3 marks)

2. Make $a$ the subject of the formula $\frac{b}{3} + a = \frac{a}{c} - 1$.

(3 marks)



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3. Simplify $\frac{1}{3x-2y}-\frac{3}{3y-2x}$.

(3 marks)

4. Factorize

(a) $8m-12n$,

(b) $12m^2-20mn+3n^2$,

(c) $12m^2-20mn+3n^2-8m+12n$.

(4 marks)



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5. (a) Find the range of values of $x$ which satisfy both $1-\frac{32+x}{3}<-x-1$ and $5x-33\geq 0$.

(b) How many odd integer(s) satisfy both inequalities in (a)?

(4 marks)

6. A toy is sold at a discount of $x\%$ on its marked price. The marked price and the selling price of the toy are \$168 and \$126 respectively.

(a) Find $x$.

(b) If the percentage profit for selling the toy is 20%, find the cost of the toy.

(4 marks)



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7. In Figure 1, $ABCD$ is a circle. $BD$ is a diameter of the circle, $CD = CA$ and $\angle ACD = 80°$.

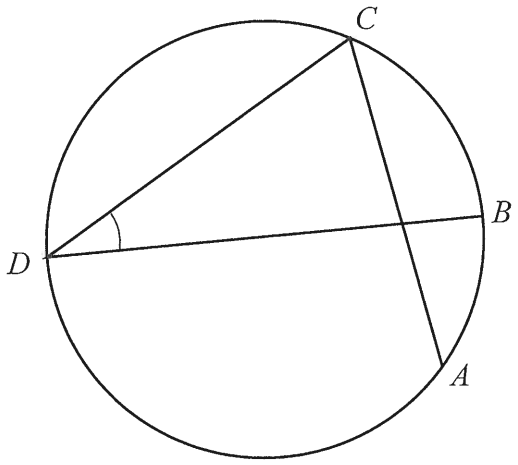


Figure 1

(a) Find $\angle BDC$.

(b) Suppose $\overset{\frown}{BC}$ is longer than $\overset{\frown}{AB}$ by 12 cm. Find the circumference of the circle.

(4 marks)



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8. It is given that $z$ varies directly as $\sqrt{x}$ and inversely as $y^3$. When $x=16$ and $y=2$, $z=-1$.

(a) Express $z$ in terms of $x$ and $y$.

(b) Someone claims that if the values of $x$ and $z$ are increased from 16 to 36 and from $-1$ to $\frac{3}{16}$ respectively, the value of $y$ will also increase. Do you agree? Explain your answer.

(5 marks)



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9. The frequency distribution table and the cumulative frequency distribution table below show the distribution of the lengths of fish in an aquarium.

| *Length (cm)* | *Frequency* |
|---|---|
| 151 – 160 | $a$ |
| 161 – 170 | 11 |
| 171 – 180 | $b$ |
| 181 – 190 | $c$ |
| 191 – 200 | 5 |

| *Length less than (cm)* | *Cumulative frequency* |
|---|---|
| 150.5 | 0 |
| 160.5 | 3 |
| 170.5 | $x$ |
| 180.5 | 38 |
| 190.5 | 45 |
| 200.5 | $y$ |

(a) Find the values of $a$, $b$ and $c$.

(b) If a fish is randomly selected from the aquarium, find the probability that the length of the selected fish is less than 190.5 cm but not less than 170.5 cm.

(5 marks)



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**Section A(2) (35 marks)**

10. A solid metal sphere of radius 18 cm is melted and recast into a solid right circular cone. The cone is now cut into a smaller cone and a frustum by a plane parallel to its base. It is given that the height of the smaller cone and the height of the frustum are the same.

(a) Find the volume of the smaller cone in terms of $\pi$.

(2 marks)

(b) Suppose the height of the frustum is 9 cm.
The frustum is now painted on all its faces and the cost of the painting is \$ 370 per $m^2$.
The craftsman claims that the cost of painting the frustum is more than \$ 310. Do you agree? Explain your answer.

(4 marks)



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11. In Figure 2, $ABCD$ is a rectangle. $E$ is a point lying on $AB$ such that $AC$ and $DE$ intersect at $F$.

(a) Prove that $\Delta AFE \sim \Delta CFD$.

(2 marks)

(b) It is given that $AE = 3$ cm, $EB = 7$ cm and the area of $\Delta CFD = 50$ cm$^2$.

(i) Find the length of $AD$.

(ii) Find the area of $\Delta CFE$.

(iii) Is there a point $P$ lying on $EC$ such that the distance between $P$ and $F$ is less than 2 cm? Explain your answer.

(5 marks )

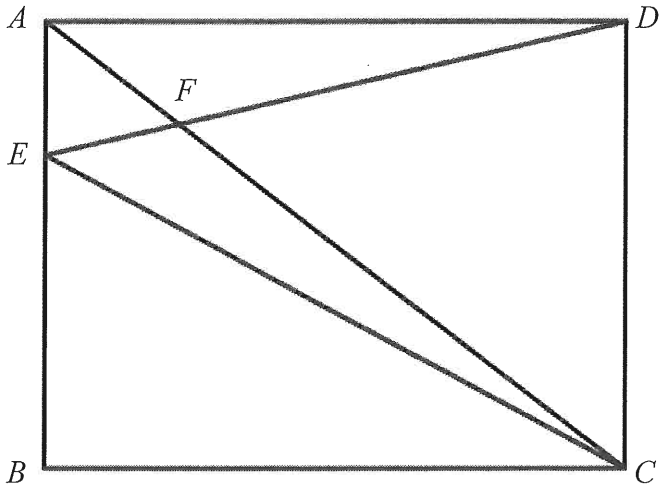


Figure 2



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12. The coordinates of the points $A$ and $B$ are (0,5) and (6,–7) respectively. Let $P$ be a moving point in the rectangular coordinate plane such that $P$ is equidistant from $A$ and $B$. Denote the locus of $P$ by $\Gamma$.

(a) Find the equation of $\Gamma$. (2 marks)

(b) Let $G(h,k)$ be a point in the rectangular coordinate plane. Find, in terms of $k$, the equation of the circle centred at $G$ and passing through $A$ and $B$ in general form. (3 marks)

(c) Let $C = (6,3)$. Using (b), find the equation of the circle passing through $A$, $B$ and $C$. (1 mark)



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13. Let $f(x) = ax^3 + 19x^2 - bx - 3$ where $a$ and $b$ are constants. It is given that $3x - 1$ is a factor of $f(x)$. When $f(x)$ is divided by $x - 2$, the remainder is 125.

(a) Find $a$ and $b$. (3 marks)

(b) Factorize $f(x)$. (3 marks)

(c) Someone claims that the equation $f(x) = 2x + 6$ has at least one unreal root. Do you agree? Explain your answer. (2 marks)



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14. The stem-and-leaf diagram below shows the distribution of the weights (in kg) of a group of 14 women, where $a$, $b$, $x$ and $y$ are non-negative integers.

| ***Stem (10 kg)*** | *Leaf (1 kg)* | | | |
|---|---|---|---|---|
| **4** | $a$ | 3 | 6 | $x$ |
| **5** | 4 | 6 | 6 | 7 |
| **6** | 0 | 3 | $y$ | 7 |
| **7** | 5 | $b$ | | |

It is given that the range of the distribution is 32 kg.

(a) Find the values of $a$ and $b$.

(2 marks)

(b) It is given that the inter-quartile range of the distribution is half of the range of the distribution.

(i) Find the possible values of $x$ and $y$.

(ii) Someone claims that when the standard deviation of the distribution attains its greatest value, the mean of the distribution is also at its greatest. Do you agree? Explain your answer.

(6 marks)



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**Section B (35 marks)**

15. The ages of a group of 15 people are arranged in ascending order as follows.

$m$, $n$, 14, 18, 25, 29, 32, 37, 40, 41, 49, 55, 61, $p$, $q$, where $m$, $n$, $p$ and $q$ are positive integers.

It is given that the mean age of the group is 37.

Now, two more people with the same age join the group and the median age of the group remains unchanged.

(a) Does the standard deviation of the ages of the new group increase, decrease or remain unchanged as compared with the original one? Explain your answer.

(3 marks)

(b) 5 people are selected at random from the new group consisting of 17 people. Find the probability that at least half of the selected people are older than 45.

(2 marks)



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16. The coordinates of the centre $G$ of circle $C$ are (12,3). The straight line $L$ passes through the points $P(0,24)$ and $Q(18,0)$. $L$ touches $C$ at $N$.

(a) Find the coordinates of $N$ and hence the equation of $C$.

(4 marks)

(b) Let $R$ be a point such that $C$ is the inscribed circle of $\Delta PQR$. Is $\Delta PQR$ an obtuse-angled triangle? Explain your answer.

(2 marks)



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17. Let $f(x) = 2x^2 - 4kx + 5k^2 + 3$, where $k$ is a non-zero real constant.

(a) Using the method of completing the square, express the coordinates of the vertex of the graph of $y = f(x)$ in terms of $k$. (2 marks)

(b) The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ vertically. It is given that the graph of $y = g(x)$ touches the $x$-axis.

(i) Find $g(x)$ in terms of $k$.

(ii) Under two transformations, $g(x)$ is changed to $h(x) = -2x^2 + 4kx - 2k^2 + 3$. Describe the geometric meaning of the transformations.

(5 marks)



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18. City A and City B both have rapidly growing populations. At the end of year 2000, the population of City A is three times that of City B. In each subsequent year, the population of City A and City B each increases by a constant number of people respectively. The increase in the population of City B is 3 times that of City A. It is given that at the end of 2008, the population of City B is 382.8 thousand; and at the end of 2012, the population of City B is greater than that of City A by 25%.

Let $a$ thousand be the population of City B at the end of year 2000 and $d$ thousand be the increase in the population of City A in each year.

(a) Find $a$ and $d$. (2 marks)

(b) City A has a cash disbursement policy. Each person in the population receives \$5000 in cash each year from the government starting from the end of year 2008. The policy shall be reviewed immediately when the total cash disbursed by the government since the end of 2008 first exceeds 25 billion dollars. At the end of which year should the policy be reviewed?

(4 marks)



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19. Figure 3 shows a pyramid VABCD with height VO. The base ABCD is a parallelogram where AC and BD intersects at O. It is given that AC = 20 cm, BD = VO = 24 cm and $\angle BOC = 60°$.

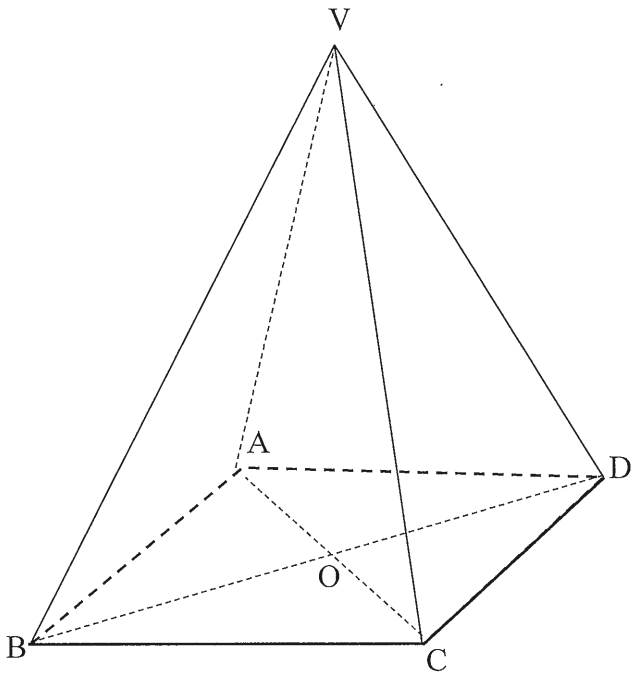


Figure 3

(a) Find BC, VC and $\cos \angle VCB$. (Leave your answers in $\sqrt{\ }$ form if necessary.)

(5 marks)

(b) Find the area of $\triangle VCD$. (1 mark)

(c) P is a moving point on VC. Let PC = $x$ cm.

(i) Find BP in terms of $x$.

Hence, find the value of $x$ when BP attains its minimum length.

(ii) When BP attains its minimum length, P is at the point F on VC. Is $\angle BFD$ the angle between VBC and VCD? Explain your answer.

(5 marks)

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END OF PAPER

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