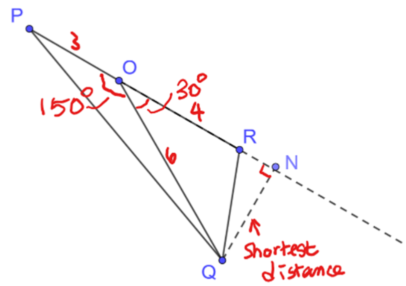
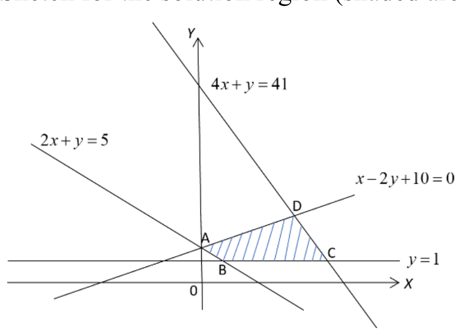
2019-2020 F6 Math (Core) Paper 2 – Detail Solution

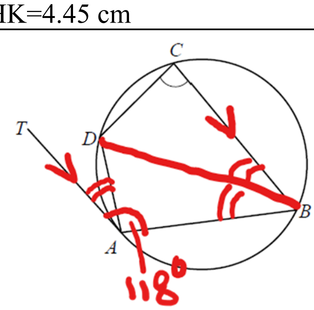
Qn	Ans.	Solution
1	C	0.0259198 = 0.03 (correct to 2 decimal places). or = 0.0259 (correct to 3 significant figures). or = 0.0259 (correct to 4 decimal places). or = 0.025920 (correct to 5 significant figures).
2	B	$\frac{a^{6a}a^6}{8a^3}=\frac{a^{3+6a}}{8}$
3	A	$a_2=a_3-a_1=10-a_1$ ---(1) $a_3=a_4-a_2$ ---(2) Sub (1) into(2), $10=a_4-10+a_1$ $a_4=20-a_1$ ---(3) $a_4=a_5-a_3$ ---(4) Sub (3) into (4), $20-a_1=a_5-10$ $a_5=30-a_1$ ---(5) $a_5=a_6-a_4$ ---(6) Sub (3) & (5) into (6), $30-a_1=44-20+a_1$ $a_1=3$
4	C	$x-\frac{x-1}{2}\le 2-\frac{2x-1}{6}$ or $4x-5>7-2x$ $12x-6x+6\le 24-4x+2$ $4x+2x>7+5$ $x\le 2$ $x>2$ Combined solution: All real numbers
5	C	The graph opens downward. $\therefore a<0$ i.e. I is correct. When $x=0$, $y-\text{int}=-c>0$, $\therefore c<0$ i.e. II is wrong. As the graph with two x-intercepts, $\Delta=(-1)^2-4a(-c)>0$ $ac>-\frac{1}{4}$ i.e. III correct
6	D	$(a+b)^2-4(a-b)^2=((a+b)+2(a-b))((a+b)-2(a-b))$ $=(3a-b)(3b-a)$
7	C	$(x-h)^2+k(x-1)\equiv x(x+h)-k+1$ $LHS=(x-h)^2+k(x-1)$ $RHS=x(x+h)-k+1$ $=x^2+(k-2h)x+h^2-k$ $=x^2+hx-k+1$ Compare both sides, $h^2-k=-k+1$ i.e. $h=1$ or $-1\,(rej)$ When $h=1$, $k-2h=h$ i.e. $k=3h=3(1)=3$
8	C	$\frac{\alpha}{x\beta}+\alpha=\frac{1}{\beta}$ Multiply both sides by $x\beta$, $\alpha+\alpha x\beta=x$ $\alpha x\beta=x-\alpha$ $\beta=\frac{x-\alpha}{\alpha x}$

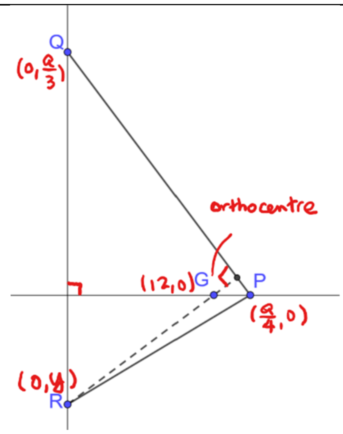
9	D	$\begin{cases}15t+24s=27-(1)\\2t+5s=27-(2)\end{cases}$ $(1)\times 2-(2)\times 15:$ $2(15t+24s)-15(2t+5s)=27\times 2-27\times 15$ $48s-75s=-351$ i.e. $s=13$ $\therefore\ 15t+24(13)=27$ i.e. $t=-19$ $\therefore\ s-t=13-(-19)=32$
10	D	$2x=3y=4z$ i.e. $\frac{x}{y}=\frac{3}{2}$ *and* $\frac{y}{z}=\frac{4}{3}$ Let $x=3k,\ y=2k,\ z=1.5k$ $\therefore (x+y):(y+z)=5k:3.5k=10:7$
11	B	$a=k_1+k_2b^2$ i.e. $\begin{cases}k_1+4k_2=17-(1)\\k_1+9k_2=42-(2)\end{cases}$ $(2)-(1):\ 5k_2=25$ i.e. $k_2=5\ \ \therefore k_1=17-4(5)=-3$ When $b=3,\ a=-3+5(3)^2=42$ When $b=3(1+20\%)=3.6,\ a=-3+5(3.6)^2=61.8$ % change in a = $\frac{61.8-42}{42}\times 100\%=47.1428571\%$
12	B	*Let* $2x^3+5x^2-7x+2=(x^2+3x+k)(2x+h)$ $=2x^3+6x^2+2kx+hx^2+3hx+kh$ $=2x^3+(6+h)x^2+(2k+3h)x+kh$ Compare the coefficient of x^2 on both sides, $6+h=5\ \ \therefore h=-1$ Compare the constant term on both sides, $2=kh=k(-1)\ \ \therefore k=-2$
13	D	Interest = $127000(1.0025^{60}–1) ≈ $20530
14	A	Since the cross section is a kite, the two diagonals are ⊥. Let $x = 5k$ and $y = 16k$ $6.25^2-(5k)^2=13^2-(16k)^2 \Rightarrow k=0.75 \Rightarrow x=3.75$ & $y=12$ Length of prism = 1575÷[(3.75+12)×10×0.5]=20 cm Total surface area = (6.25+13)×2×20+1575÷20×2=927.5 cm^2
15	D	Let K be the centre of the bigger circle. Consider △AKO.AK=KO =3 and OA =1. ∠AKB=sin^{-1}(0.5÷3)×2×2=38.37627291° ∠AOB= $2\times\cos^{-1}(0.5\div 3)$ =160.8118635° Area of shaded region = π3^2×(38.37627291°÷360°)–0.5×3^{2}sin(38.37627291°)+ π1^2×(160.8118635°÷360°)–0.5×1^{2}sin(160.8118635°) ≈1.46 cm^2
16	B	$\Delta ABC\sim\Delta ZYX$,∠ABC =b and ∠ACB = a. Height = ABsin$b$ and AC = ABsin$b$÷(sin$a$)
17	C	AB:HF = AE:EF = 3:2 ⇒ HD:DF:FC = 1:1:2 AG:GD = AB:HD = 3:1 ⇒ AG:AD = 3:4. Together with AE:AF = 3:5, we have Area AGE : Area ADF = 3×3 : 4×5 = 9:20 Area DFEG : Area ADF = 11:20 ⇒ Area ADF = 40 DF:AB = 1:3 ⇒ Area ABF = 40×3 = 120 AE:EF = 3:2 ⇒ Area ABE = $120\times\frac{3}{5}$ = 72

18	C	$0.5\div(1-\cos\theta)-0.5\div(1+\cos\theta)$ $=0.5\times[1+\cos\theta-1+\cos\theta]\div(1-\cos^2\theta)$ $=\cos\theta\div\sin^2\theta=\dfrac{1}{\sin\theta\tan\theta}$
19	C	$\because$ $AB = AC \Rightarrow \angle ABC = \angle ACB = 80°$ (base $\angle$s, isos Δ) $\because$ $CB = CE \Rightarrow \angle CEB = \angle CBE = 80°$ (base $\angle$s, isos Δ) $\angle ECB = 180°-80°-80°=20°$ ($\angle$ sum of Δ) and $\angle ECF = 80°-20°=60°$ $\because CE = EF \Rightarrow \angle EFC = \angle ECF = 60°$ (base $\angle$s, isos Δ) $\angle CEF = 180°-60°-60°=60°$ ($\angle$sum of Δ) Therefore, I is true. $\angle DEF = 180°-80°-60°=40°$ (adj. $\angle$s on st. line) $\because EF=FD \Rightarrow \angle EDF = \angle DEF = 40°$ (base $\angle$s, isos. Δ) $\angle AFD = 40°-20°=20°$ (ext. $\angle$ of Δ) $AD = FD$ (sides opp. eq. $\angle$s) and $FD = BC$ (given) $\Rightarrow AD = BC$. Therefore, II is not true and III is true.
20	C	$\angle CAB = \angle DBC$ (given) $\angle ABC = \angle BCD$ (alt.$\angle$s, $AB//CD$) $\Delta ABC \sim \Delta BCD$ $\dfrac{AB}{BC}=\dfrac{BC}{CD}$ (corr. sides, $\sim\Delta$s) $\therefore BC=\sqrt{3\times 12}=6\text{cm}$
21	D	$(n-2)\times 180°=2340°$ $n=15$ Each exterior angle = $\dfrac{360°}{15}=24°$ There is a unique line of symmetry through each vertex. Therefore, I, II and III are true.
22	D	$\angle OBC = 180°-104°-20°=56°$ ($\angle$ sum of Δ) Join OC, $\angle OCB=\angle OBC=56°$ (base $\angle$s, isos. Δ) $\angle OCD = 104°-56°= 48°$ $\angle CDO = \angle OCD = 48°$ (base $\angle$s, isos. Δ)
23	A	Slope / x-intercept / y-intercept L_1: $-\dfrac{5}{a}$, $\dfrac{b}{5}$, $\dfrac{b}{a}$ L_2: $-c$, $\dfrac{d}{c}$, d By comparing the slopes, $-c>-\dfrac{5}{a}>0$ $\therefore c<0, a<0$ and $c<\dfrac{5}{a}\Rightarrow ac>5$ $(\because a<0)$ So, I is not true. By comparing the y-intercepts, $d>\dfrac{b}{a}\Rightarrow ad<b$ $(\because a<0)$ So, II is true. By comparing the x-intercepts, $\dfrac{b}{5}<\dfrac{d}{c}\Rightarrow bc>5d$ $(\because c<0)$ So, III is not true.
24	A	The required locus: $x^2+(y-a)^2=(y+10)^2$ Sub (4,–5), $4^2+(-5-a)^2=5^2$ $a^2+10a+16=0$ $\therefore a=-2$ or -8

25	B	 Let O be the pole. $\angle POQ = 300° - 150° = 150°$ and $\angle QOR = 330° - 300° = 30°$ P, O, R is a straight line and Let N be the foot of perpendicular of Q to PR. Shortest distance $= QN = 6\sin 30° = 3$
26	B	$3x^2 + 3y^2 - 6x + 24y + 32 = 0$ $x^2 + y^2 - 2x + 8y + \frac{32}{3} = 0$ Centre$=(-\frac{-2}{2}, -\frac{8}{2}) = (1, -4)$ Radius $= \frac{1}{2}\sqrt{D^2 + E^2 - 4F} = \frac{1}{2}\sqrt{(-2)^2 + (8)^2 - 4(\frac{32}{3})} = \frac{1}{2}\sqrt{\frac{76}{3}} \neq 11$ Distance of (0,–8) from centre (1,–4) $= \sqrt{(1-0)^2 + (-4+8)^2} = \sqrt{17} >$ radius . Therefore, only II is true.
27	B	Use (a, b) to represent an outcome where a is the value of the first chosen coin and b is the value of the second chosen coin. The possible outcomes are: (10, 5), (10, 5), (10, 2), (5, 10), (5, 5), (5, 2), (5, 10), (5, 5), (5, 2), (2, 10), (2, 5), (2, 5), i.e. 12 possible outcomes in total Counting those with total not less than 10, prob. $= \frac{8}{12} = \frac{2}{3}$
28	D	Since the respective total numbers of banknotes John and Mary each have are unknown, both options A and B may not be true. $k = 100 - 35 - 20 - 25 = 20$ $\theta = 360° - 125° - 72° - 90° = 73°$
29	A	Before: range = 163 – 139 = 24 min, inter-quartile range = 158 – 142 = 16 min After: range = 158 – 131 = 27 min, inter-quartile range = 150 – 134 = 16 min Hence, both I and II are true. Upper quartile (after the marathon) = median (before the marathon) > min and lower quartile (before the marathon), hence III may not be true.
30	A	For any value of m (i.e. m = 5, 6, 7 or 8), we have: (1) The mode is 8, i.e. $b = 8$. (2) The median is m, i.e. $c = m$. Case 1: $m = 5$ $a = 6$, $b = 8$ and $c = 5$ Case 2: $m = 6$ $a = 6.076\ 923\ 077\ldots$, $b = 8$ and $c = 6$ Case 3: $m = 7$ $a = 6.153\ 846\ 154\ldots$, $b = 8$ and $c = 7$ ∴ II is not always true. Case 4: $m = 8$ $a = 6.230\ 769\ 231\ldots$, $b = 8$ and $c = 8$ ∴ III is not always true.

31	D	$\dfrac{1-\log x+1}{\log x-1}=\dfrac{4\log x-17}{11\log x+5}$ $(2-\log x)(11\log x+5)=(4\log x-17)(\log x-1)$ $-11(\log x)^2+17\log x+10=4(\log x)^2-21\log x+17$ $15(\log x)^2-38\log x+7=0$ $(3\log x-7)(5\log x-1)=0$ $\log x=\dfrac{7}{3}$ or $\dfrac{1}{5}$ $\therefore \log\dfrac{1}{x^2}=\log x^{-2}=-2\log x=-2\left(\dfrac{7}{3}\right)$ or $-2\left(\dfrac{1}{5}\right)=-\dfrac{14}{3}\ or -\dfrac{2}{5}$
32	C	$2^{10}+(-1+5)\times 2^7+(-2+3)\times 2^2+2$ $=2^{10}+4\times 2^7+2^2+2$ $=2^{10}+2^9+2^2+2^1=11000000110_2$
33	D	Given $a_2=a_1r>0$ and $\dfrac{a_1}{1-r}=-3$ i.e. $a_1=-3+3r$ Combine together result $(-3+3r)r>0$ $\therefore r<0$ or $r>1(rej)$ As S_∞ exist, $-1<r<0$ i.e. I correct Since $a_1=-3+3r$ and $-1<r<0$ $-3<3r<0$ $-6<-3+3r<-3$ i.e. $-6<a_1<-3$ i.e. II correct $S_{2k}=\dfrac{a_1(1-r^{2k})}{1-r}=\dfrac{-3(1-r)(1-r^{2k})}{(1-r)}=-3+3r^{2k}$ $\because 3r^{2k}=3(r^k)^2>0$ $\therefore S_{2k}=-3+3r^{2k}>-3$ i.e. III correct
34	D	$\dfrac{y^{\frac{1}{3}}}{2}+\dfrac{x}{1}=1$ $y^{\frac{1}{3}}+2x=2$ $y^{\frac{1}{3}}=2-2x$ $y=(2-2x)^3=-8x^3+24x^2-24x+8$
35	A	Sketch for the solution region (shaded area):  $A(0,5)$ When $y=1$, $2x+1=5$ i.e. $x=2$ $\therefore B(2,1)$

		When $y=1,\ 4x+1=41$ i.e. $x=10\ \therefore \mathrm{C}(10,1)$ $\begin{cases}4x+y=41\\ x-2y+10=0\end{cases}$ i.e. $\mathrm{D}(8,9)$ For $\mathrm{A}(0,5)$, $75-4(0)-3(5)=60$ For $\mathrm{B}(2,1)$, $75-4(2)-3(1)=64$ For $\mathrm{C}(10,1)$, $75-4(10)-3(1)=32$ For $\mathrm{D}(8,9)$, $75-4(8)-3(9)=16$ i.e. Least value is 16
36	A	a,b are roots of the equation $3x^2-6x-2c=0$ $a+b=\frac{-(-6)}{3}=2,\quad ab=\frac{-2c}{3}$ $a^3+b^3=(a+b)(a^2-ab+b^2)$ $=(a+b)((a+b)^2-3ab)$ $=2(2^2-3(\frac{-2c}{3}))=2(4+2c)=8+4c$
37	C	$\frac{i^{23}+i^{24}}{i+k}-ki^{22}=\frac{-i+1}{i+k}-k(-1)=\frac{1-i}{k+i}(\frac{k-i}{k-i})+k=\frac{(1-i)(k-i)}{k^2+1}+k$ $=\frac{(k-1)-(k+1)i}{k^2+1}+k$ $\therefore$ Real part $=\frac{k-1}{k^2+1}+k=\frac{k-1+k(k^2+1)}{k^2+1}=\frac{k^3+2k-1}{k^2+1}$
38	D	Distance = VB. Let VB = x . AB = x and BC = $x\div\sqrt{3}$ $1\div3\times x\times x\times(x\div\sqrt{3})\times\sin135°\times0.5=48\sqrt{6}\Rightarrow x^3=1728\Rightarrow x=12$
39	B	tan∠GFD=10÷(10-5)⇒∠GFD=63.43494882° and ∠FHD=180°−∠*GFD*−30° =86.56505118° EH = 10 −8÷sin86.56505118°×sin63.43494882°=2.831704393 ∠*KGC* = 180°− 63.43494882°= 116.5650512° ∠*GKC* = 180°−∠*KGC*−30°= 33.43494882° EK = 10 −3÷sin33.43494882°×sin116.5650512°=5.130070966 $HK^2=2.831704393^2+5.130070966^2-2(2.831704393)(5.130070966)\cos60°$ HK=4.45 cm
40	C	 *AD* = *CD* => ∠*CBD* = ∠*ABD*=*x* (eq. chords, eq. ∠s) *TA* is a tangent =>∠*TAD* = ∠*ABD* = *x* (∠ in alt. segment) 118°+2*x* = 180° (int. ∠s, *TA*//*CB*) *x* = 31° ∠*DAB* = 118° −31° = 87° ∠ *BCD* = 180°−87° = 93° (opp ∠s, cyclic quad.)

41	B	 Let G be the orthocentre. The coordinates of $P = (\frac{a}{4},0)$ and $Q = (0,\frac{a}{3})$ $m_{PQ} = \frac{\frac{a}{3}-0}{0-\frac{a}{4}} = -\frac{4}{3}$ and $m_{RG} = \frac{y-0}{0-12} = -\frac{y}{12}$ $m_{PQ} \times m_{RG} = -1$ $-\frac{4}{3}\times(-\frac{y}{12}) = -1 \quad \therefore y = -9$
42	C	No. of ways = $P_2^6 \times 5!$ or $7! - 6! \times 2 = 3600$
43	D	Let p be the probability that Mary can solve the problem. $1 - (1 - p)(0.2)(0.3) = 0.976 \;\therefore p = 0.6$ required prob. = $(0.8)(0.3)(0.4) + (0.2)(0.7)(0.4) + (0.2)(0.3)(0.6) = 0.188$
44	A	Let s be the standard deviation. $\frac{78-57}{s} = 3 \quad \therefore s = 7$ Hence, I is true. Standard score of Chris = $\frac{45-57}{7} = -1\frac{5}{7} > -2$ Hence, II is true. If a student's score is x and $50 \le x \le 56$, then the student passes the test but $x <$ mean = 57 which implies that the standard score is negative. Hence, III may not be true.
45	B	$a_{91} = a_1 + 90d = a_1 + 180$, $a_{92} = a_1 + 91d = a_2 + 90d = a_2 + 180$, Similarly, $a_{93} = a_3 + 180, \ldots\ldots, a_{100} = a_{10} + 180$. $\therefore$ Variance of $2a_{91}, 2a_{92}, 2a_{93}, \ldots, 2a_{100}$ $= 2^2 \times$ variance of $a_1, a_2, a_3, \ldots, a_{10} = 4(33) = 132$