S6 Mathematics (Compulsory Part) Mock Examination 2022 – 2023
General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Marker should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used.
'A' marks	awarded for the accuracy of the answers.
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified)

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner. e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answer which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Solution	Marks	Remarks
1. $\dfrac{(a^{-1}b)^2}{a^3b^{-2}}$		
$=\dfrac{a^{-2}b^2}{a^3b^{-2}}$	1M	
$=\dfrac{b^{2+2}}{a^{3+2}}$	1M	
$=\dfrac{b^4}{a^5}$	1A	
	(3)	
2. (a) $9h^2-6h+1$		
$=(3h-1)^2$	1A	
(b) $9h^2k^2-6hk^2+k^2$		
$=k^2(3h-1)^2$	1A	
(b) $9h^2k^2-6hk^2+k^2-9h^2+6h-1$		
$=k^2(3h-1)^2-(3h-1)^2$	1M	
$=(3h-1)^2(k^2-1)$		
$=(3h-1)^2(k-1)(k+1)$	1A	
	(4)	
3. (a) 153	1A	
(b) 152.3	1A	
(c) 200	1A	
	(3)	
4. $\dfrac{a}{b}=\dfrac{4}{5}$		
$a:b=4:5=12:15$		
$11b=15c$		
$b:c=15:11$	1M	either a:b or b:c
$a:b:c=12:15:11$		
Let $a=12k, b=15k$ and $c=11k$ where k is a non-zero constant.	1M	
$\dfrac{a+b}{3c}$		
$=\dfrac{12k+15k}{3(11k)}$	1M	
$=\dfrac{9}{11}$	1A	

Solution	Marks	Remarks
	(4)	
5. Let $x be the cost of the concert ticket.		
Then the marked price is $(1+150\%)x = 2.5x$	1M	
And the selling price is $(2.5x)(0.8) = 2x$	1M	
$2x - x = 680$	1M	
$x = 680$		
Thus, the marked price of the concert ticket is $1700	1A	
	(4)	
6. (a) $\frac{2x+8}{-3} < x$		
$2x+8 > -3x$		
$5x > -8$	1M	for putting x on one side
$x > \frac{-8}{5}$		
$\frac{3(x+7)}{5} > 6 - x$		
$3x+21 > 30-5x$		
$8x > 9$		
$x > \frac{9}{8}$	1A	
Therefore, we have $x > \frac{-8}{5}$ or $x > \frac{9}{8}$		
Thus, the solution of (*) is $x > \frac{-8}{5}$	1A	
(b) -1	1A	
	(4)	
7. Note that the probability of drawing a red ball is $\frac{24}{n+36}$	1M	either one
And the probability of drawing a black ball is $\frac{12}{n+36}$		
$\frac{24}{n+36} - \frac{12}{n+36} = \frac{1}{4}$	1M +1A	
$\frac{12}{n+36} = \frac{1}{4}$		
$n = 12$	1A	
	(4)	

Solution	Marks	Remarks
8. $x+\angle EDC=180°$		
$x=60°$	1A	
$\angle BED=y$	1M	
$\frac{\angle BED}{\angle EBC}=\frac{4}{6}$	1M	
$\frac{y}{60°}=\frac{4}{6}$		
$y=40°$	1A	
	(4)	
9. (a) Mean = 1.9	1A	
Median = 1.5	1A	
Standard Deviation ≈ 1.11	1A	
(b) The new median = 2	1M	
The change in median = 2 – 1.5 = +0.5	1A	
The median is increased by 0.5		
	(5)	
10. (a) $b-12=40$	1M	either one
$b=52$	1A	
$43-a=20$		
$a=23$	1A	
	(3)	
(b) When x is maximum, i.e. x = 34, z attains the maximum		
$z=34+18=52$	1M	
When z is minimum, i.e. z = 37, x attains the minimum		
$x=37-18=19$	1M	
When the data are combined, the smallest datum (i.e. 12) and the largest datum (i.e. 52) are the same as those of the data for marketing team A.		
Thus, the claim is agreed.	1f.t.	
	(4)	

Solution	Marks	Remarks
11. (a) Let $f(x) = ax + bx^3$, where a and b are non-zero constants.	1A	
So, we have $2a + 8b = 18$ and $5a + 125b = 360$	1M	For substitution
Solving, we have $a = -3$ and $b = 3$		
Thus, we have $f(x) = -3x + 3x^3$	1A	
	(3)	
(b) $f(x) = 0$		
$-3x + 3x^3 = 0$	1M	
$3x(x-1)(x+1) = 0$		
$x = 0$, $x = 1, x = -1$		
Thus, the x-intercepts of y = f(x) are 0, 1 and –1	1A	
	(2)	
12. (a) The ratio of the volume of two cones = $\left(\sqrt{1}\right)^3 : \left(\sqrt{9}\right)^3$ = 1 : 27.	1M	
The volume of smaller circular cone		
$= \frac{4}{3}\pi(84)^3 \times \frac{1}{1+27}$	1M	
$= 28224\pi \text{ cm}^3$	1A	
	(3)	
(b) Let r cm be the radius of the smaller cone		
$\frac{1}{3}\pi r^2(48) = 28224\pi$	1M	
$r = 42$		
The total surface of the smaller cone		
$= \pi(42)^2 + \pi(42)\sqrt{42^2 + 48^2}$	1M	
$= (42\sqrt{4068} + 1764)\pi \text{ cm}^2$		
The total surface area of two cones		
$= (42\sqrt{4068} + 1764)\pi \times (1+9) \text{ cm}^2$	1M	
$\approx 13.95745761 \text{ m}^2$		
$> 13\text{m}^2$		
Thus, the claim is agreed.	1f.t.	

Solution	Marks	Remarks
13. (a) Let $f(x)=(x^2-6x+8)Q(x)+mx-6$ where Q(x) is a polynomial.	1M	
Since $f(2)=0$, we have $(0)Q(2)+mx-6=0$	1M	
Thus, we have $m=3$	1A	
	(3)	
(b) $f(x)=(x^2-6x+8)(ax+b)+3x-6$ where a and b are constants.	1M	
Since we have $f(3)=0$ and $f(0)=-30$,	1M	
we have $3a+b=3$ and $8b-6=-30$	1M	
solving we have $a=2$ and $b=-3$	1A	
Hence, we have $f(x)=(x^2-6x+8)(2x-3)+3x-6$		
$f(x)=(x-2)(x-4)(2x-3)+3(x-2)$		
$f(x)=(x-2)(x-3)(2x-5)$		
The roots of the equation $f(x)=0$ are 2, 3, and $\frac{5}{2}$		
Note $\frac{5}{2}$ is not an integer. The claim is disagreed.	1f.t.	
	(5)	
14. (a) Let $G(h,k)$ $GA=GB$		
$(h-8)^2+(k-8)^2=(h-12)^2+(k-2)^2$ ------(1)	1M	
G lies on L_1		
$\therefore h-8k+4=0$ $h=8k-4$ ------(2) Sub (2) into (1)		
$(8k-4-8)^2+(k-8)^2=(8k-4-12)^2+(k-2)^2$		
$(8k-12)^2+(k-8)^2=(8k-16)^2+(k-2)^2$		
We get $k=1$ and $h=4$ $\therefore G(4,1)$ The radius of C		
$=\sqrt{(8-4)^2+(8-1)^2}$	1M	

Solution	Marks	Remarks
$=\sqrt{65}$		
The equation of circle is		
$(x-4)^2+(y-1)^2=65$	1A	
	(3)	
(b) (i) Γ is a pair of straight lines parallel to L_2, one passes through A and the other passes through B.	1M+1A	for//or passes through A or B
(ii) Note that $L_2 \perp AB$		
Slope of $AB=-\frac{3}{2}$, then the slope of $L_2=\frac{2}{3}$		
The equation of Γ passes through A		
$\frac{y-8}{x-8}=\frac{2}{3}$	1M	for either one
$2x-3y+8=0$		
The equation of Γ passes through B		
$\frac{y-2}{x-12}=\frac{2}{3}$		
$2x-3y-18=0$	1A	
(iii) Note that L_1 is not parallel to L_2 and B lies on L_1.		
$\therefore D$ lies on the part of Γ which passes through A.	1M	
Thus, BGD is a diameter of C and $\angle BAD=90°$		
And the claim is agreed	1Af.t.	
Note that D, G and B lie on L_1 and B, D are on circle C.	1M	
$\therefore BGD$ is a diameter of C		
Thus $\angle BAD=90°$ ($\angle$ in semi-circle)		
And the claim is agreed	1A	
	(6)	

Solution	Marks	Remarks
15. (a) The required probability		
$=\frac{10}{23}\times\frac{9}{22}\times\frac{8}{21}\times\frac{7}{20}+\frac{8}{23}\times\frac{7}{22}\times\frac{6}{21}\times\frac{5}{20}$	1M+1M	1M for P(4green) or P(4 red) 1M for sum of probability
$=\frac{8}{253}$	1A	
	(3)	
(b) The required probability		
$=1-\frac{8}{253}$	1M	1 – part (a)
$=\frac{245}{253}$	1A	
	(2)	
16. (a) The slope of L_1		
$=\frac{4-0}{0-12}$		
$=-\frac{1}{3}$		
The equation of L_1 is		
$y-4=-\frac{1}{3}x$	1M	
$x+3y-12=0$	1A	
The equation of L_2 is		
$y-0=3(x-12)$		
$3x-y-36=0$		
Thus, the system of inequalities is $\begin{cases} x+3y-12\le 0 \\ 3x-y-36\le 0 \\ x\ge 0 \end{cases}$	1A	
	(3)	
(b) Note that the vertices of *R* are the points $(0,4)$, $(12,0)$ and $(0,-36)$		
When $x=0$ and $y=4$, we have $4x-y=-4$.		
When $x=12$ and $y=0$, we have $4x-y=48$.		
When $x=0$ and $y=-36$, we have $4x-y=36$.	1M	for any one
Thus, the greatest value of $4x-y$ is 48.	1A	
	(2)	

Solution	Marks	Remarks
17. (a) Let the first term be a and the common ratio be r		
$a+ar=17496$	1M	
$a(1+r)=17496$ ------(1)		
$\frac{a}{1-r}=19683$	1M	
$a=19683(1-r)$ ------(2)		
Sub (2) into (1)		
$19683(1-r)(1+r)=17496$		
$1-r^2=\frac{8}{9}$		
$r=\frac{1}{3}$ or $-\frac{1}{3}$		
When $r=\frac{1}{3}$ $a=13122$, when $r=-\frac{1}{3}$ $a=26244$	1A	
	(3)	
(b) $G(n)=13122\left(\frac{1}{3}\right)^{n-1}$		
$P(n)=\frac{G(2n+1)}{2}$		
$=6561\left(\frac{1}{3}\right)^{2n}$		
$=3^{8-2n}$	1M	
$\log_3[P(1)P(2)...P(m)]<-50$		
$\log_3\left(3^6\cdot 3^4\cdot 3^2...3^{8-2m}\right)<-50$		
$6+4+2+...+(8-2m)<-50$	1M	for log properties
$(6+8-2m)(\frac{m}{2})<-50$	1M	sum of AS
$-2m^2+14m+100<0$		
$m<\frac{-14-\sqrt{996}}{-4}$ or $m>\frac{-14+\sqrt{996}}{-4}$	1M	for solving quadratic inequality
$m<-4.39$ or $m>11.39$		
Thus the least value of m is 12	1f.t.	
	(5)	

Solution	Marks	Remarks
18. (a) $\angle SPT = \angle RPQ$ (common angle)		
$\angle STP = \angle RQP$ (ext. ∠, cyclic quad.)		
$\Delta PST = \Delta PRQ$ (AA)		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
	(2)	
(i) $\dfrac{PT}{65+187} = \dfrac{65}{PT+197}$	1M	
$PT^2 + 197PT - 16380 = 0$		
$PT = 63\text{cm}$ or -260cm (rejected)	1A	
(ii) Note that $ST \perp PR$ and therefore SR is a diameter of the circle $QRTS$	1M	
$ST = \sqrt{65^2 - 63^2}$		
$= 20\text{cm}$		
$SR = \sqrt{16^2 + 197^2}$	1M	either one diameter
$= \sqrt{39065}\text{cm}$		
The difference in area		
$= \left(\dfrac{\sqrt{39065}}{2}\right)^2 \pi - \left(\dfrac{16}{2}\right)^2 \pi$		
$\approx 30480.51732\text{cm}$		
$\approx 3.048051732\text{cm}$		
$> 3\text{m}^2$		
$\therefore$ The the claim is agreed.	1Af.t.	
	(5)	

Solution	Marks	Remarks
Method II		
Note that $ST \perp PR$ and therefore SR is a diameter		
of the circle $QRTS$	1M	
The difference in area		
$=\left(\frac{SR}{2}\right)^2\pi-\left(\frac{ST}{2}\right)^2\pi$		
$=\left(\frac{SR^2-ST^2}{4}\right)\pi$		
$=\left(\frac{TR^2}{4}\right)\pi$	1M	
$\approx 30480.51732\text{cm}$		
$\approx 3.048051732\text{cm}$		
$>3\text{m}^2$		
$\therefore$ The the claim is agreed.	1Af.t.	
	(5)	
19. (a) $\frac{AC}{\sin\angle ABC}=\frac{AB}{\sin\angle ACB}$	1M	
$\frac{AC}{\sin 10°}=\frac{80}{\sin 50°}$		
$AC=\frac{80\sin 10°}{\sin 50°}$		
$AC=18.1\text{cm}$ (corr. to 3sig.fig.)	1A	
$\because \angle CAB=180°-\angle ABC-\angle ACB$		
$=180°-10°-50°$		
$=120°$		
$\therefore \angle CAD=\angle BAD-\angle CAB$		
$=160°-120°$		
$=40°$		
$CD=\sqrt{AC^2+AD^2-2(AC)(AD)\cos\angle CAD}$	1M	
$=\sqrt{18.13452775^2+50^2-2(18.13452775)(50)\cos 40°}$		
≈ 37.94305833		
$=37.9\text{cm}$ (corr. to 3 sig.fig.)	1A	
	(4)	

Solution	Marks	Remarks
(b) (i) Let H be a point on BC such that $AH \perp BC$ and *A*' be the projection of A on the horizontal ground.		
$AH = AB \sin \angle ABH$	1M	for finding AA'
$= 80 \sin 10°$		
≈ 13.8918524		
The angle between the paper card and the horizontal ground is $\angle AHA'$.	1M	
The shortest distance AA'		
$= AH \sin \angle AHA'$		
$\approx 13.8918524 \sin 20°$		
$\approx 4.751293969\text{cm}$		
$= 4.75\text{cm}$ (corr. to 3sig.fig.)	1A	
(ii) Let *D'* be the projection of *D* on the horizontal ground.		
T be the point where *AD* produced and *BC* produced meet.		
Note that $\Delta DTD' \sim \Delta ATA'$ and $AT = AB$.		
$\frac{DD'}{AA'} = \frac{DT}{AT}$	1M	for finding *DD*'
$DD' \approx \frac{30}{80}(4.751293969)$		
$\approx 1.781735238\text{cm}$		
The angle between CD and the horizontal ground is $\angle DCD'$		
$\sin \angle DCD' = \frac{DD'}{CD}$	1M	for finding $\angle DCD'$
$\approx \frac{1.781735238}{37.94305833}$		
$\angle DCD' \approx 2.69°$		
$< 3°$		
$\therefore$ The claim is disagreed.	1f.t.	
	(6)	