### MUNSANG COLLEGE

### 2020 - 2021 Mock Examination

# F.6 Mathematics Compulsory Part

Paper	1
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Class:	_	Name :	Class Number :
*Please circle t	he ini	tial of your subject teacher: CHF / CYL / HYC / MKL / WF	L
Time allowed	:	2 hours 15 minutes	
Full mark	:	105	

### INSTUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the space provided on this cover.

This question-answer book consists of 28 printed pages.

- 2. This paper consists of THREE sections, A(1), A(2) and B. Each section carries 35 marks.
- 3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
- 4. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class and class number on each sheet, and fasten them with string INSIDE this book.
- 5. Unless otherwise specified, all working must be clearly shown.
- 6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
- 7. The diagrams in this paper are not necessarily drawn to scale.

	Marker's Use Only	Examiner's Use Only
	Marker No.	Examiner No.
Question No.	Marks	Marks
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Section A	(1) <i>(</i>	(35	marke)	ì
Section At	11	JJ	шагка	,

1. Make *n* the subject of the formula  $\frac{5m-n}{2} = \frac{n}{3} + 1$ .

5m-n	_	n	ı	
2	_	3	+	)

(3 marks)

Answers written in the margins will not be marked.

(3 marks)

$$3(5m-n) = 2n+6$$

·IM

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{2}\right)$$

IM

$$7m - 6 = 2n + 3n$$

-6

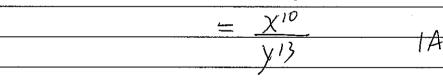
 $= 3m - \frac{6}{5}$ 

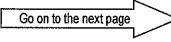

2. Simplify  $\frac{\left(x^5y^{-3}\right)^2}{v^7}$  and express your answer with positive indices.

 $\frac{(x^{5}y^{-3})^{2}}{y^{7}} = \frac{x^{10}y^{-6}}{y^{7}}$   $= x^{10}y^{-6} = 10$   $= x^{10}y^{-6} = 10$ 

 $= x^{10} y^{-6-7}$ 

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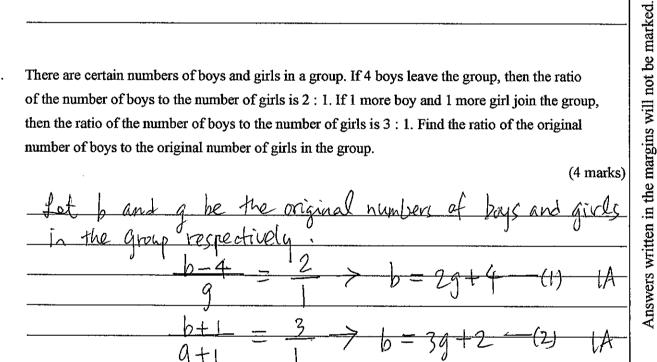




$= 2x [x^{2} - 4(y-z)^{2}]$ $= 2x [x^{2} - (2y-2z)^{2}]$  A	
$= 2x \left[ x^2 - (2y - 2z)^2 \right]$	
= 2x (x+2y-2z)(x-2y+2z)	

4. There are certain numbers of boys and girls in a group. If 4 boys leave the group, then the ratio of the number of boys to the number of girls is 2: 1. If 1 more boy and 1 more girl join the group, then the ratio of the number of boys to the number of girls is 3:1. Find the ratio of the original number of boys to the original number of girls in the group.

(4 marks)



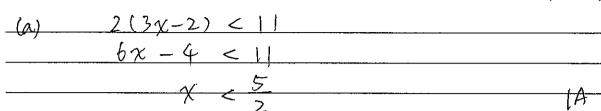
5.	Friderich wants to buy 25 identical gifts from a gift shop. His budget is \$1000 and the marked each gift is \$48. If Friderich can get one gift free of charge for every purchase of five gifts, doe have enough budget to buy all the 25 gifts? Explain your answer.	
	\$1000 = 205 \$48 ir Friderich can buy 20 gifts without any offer	marks)
	If Friderich can get one free gift for every  purchase of 5 gifts, he can get  20 = 4 gifts in addition	/  
	As a total, he can buy 24 gifts by \$1000 under the special offer with \$40 left.	(M
	in Friderich does not have enough budget to buy all 25 gifts.	[A

6.	(a)	Find the range of values of $x$ which satisfy both	2(3x-2)<11	and	$\frac{x-5}{2}$	$\leq \frac{4x}{3}$
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Find the number of integers satisfying both inequalities in (a). (b)

(4 marks)

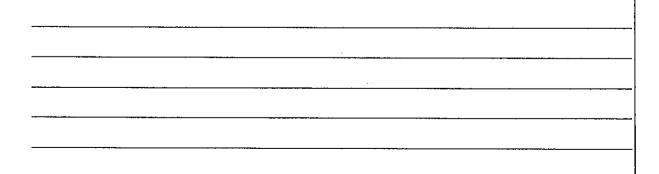
Answers written in the margins will not be marked.



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2 3	
$3x - 15 \leq 8x$	
-15 < 5x	lM
v > -3	

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The	required	range	۱۳۲	-3 <	$\chi < \Sigma$	lA
(,, )	b	J			2	

(b)	Integers So	itisfying the	snegualities are	
,	-2 -2	1, 0	, 1, 2 .	
			6 such integers.	IA
	J			



7.	The coordinates of points $P$ and $Q$	2 are $(-4, -2)$ and $(1, 3)$ respectively. $P$ is rotated anti-	i-clockwise
	about the origin through 90° to F	P'. $Q$ is translated rightwards by 8 units to the point	Q'.

- (a) Write down the coordinates of P' and Q'.
- (b) Prove that PQ is parallel to P'Q'.

(4 marks)

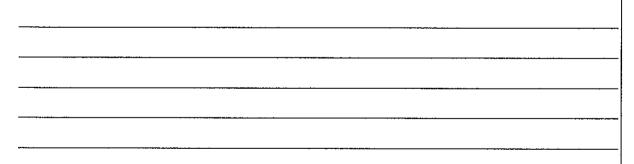
(a) 
$$P' = (2, -4)$$
 [A]
$$Q' = (9, 3)$$
 [A]

(b) Slope of 
$$PQ = \frac{-2-3}{-4-1} = 1$$

Slope of  $P'Q' = \frac{-4-3}{2-9} = 1$ 

)	Slone	of	PQ	=	Slone	nf	PQ
	-	,	,				•

)	PQ	// P'Q'	l f.t
•		· · · · · · · · · · · · · · · · · · ·	,
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8. In Figure 1, BD is a diameter of the circle ABCD. If  $\angle ACD = 54^{\circ}$ , find the ratio of the arc lengths  $\triangle B : \triangle D$ .

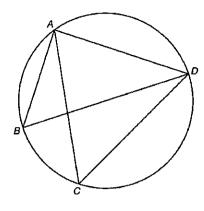


Figure 1

(5 marks)

RD	) 15 a	diane	ter.		
)	L BAD	= 900	1 Lin	semicircle	)
					_

 $\frac{\angle BAD + \angle ABD + \angle BDA = 180^{\circ} (\angle Sum \ of \ \Delta)}{\angle ABD + \angle BDA = 90^{\circ}}$ 

LABD = LACD (LS in the Same Sogment)

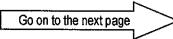
1701 - 760

AB: AD = LBDA: LABD (arcs pup to Ls at Ofe = 360: 540

= 2 : 3

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9. The following table shows the numbers of books read by 40 students in a certain week.

<del></del>				
Number of books read	1	2	3	4
Number of students	x	8	9	у

It is given that x and y are positive integers.

- Find the least possible value and the greatest possible value of the mean of the distribution.
- (b) Leonhard has the following claim. 'If the mode of the distribution is 4, the median of the distribution must not be less than 3.' Is his claim correct? Explain your answer.

(5 marks)

22(1) + 8(2) + 9(3) + 1(4)

correct

Answers written in the margins will not be marked.

## Section A(2) (35 marks)

10. The stem-and-leaf diagram below shows the test scores of 30 students in a class..

Stem (tens)	<u>Le</u>	af (	<u>unit</u>	<u>s)</u>				
1	a							
2								
3	1	3						
4	5	6	9					
5	0	1	2	3	7	9	9	
6	0	0	3	4	4	6	6	9
7	1	b	3	5	5	5		
8	3	6						
9	2							
			_					

- (a) If the range and the inter-quartile range of these scores are 81 and 22 respectively, find the values of a and b.(3 marks)
- (b) Due to a mistake in recording, the score of a student should be 11 instead of 71.
  - (i) What is the change in the mean of the test scores?
  - (ii) Bernhard claims that for the two statistical measures in (a), correcting the score from 71 to 11 will only affect the value of the inter-quartile range. Do you agree? Explain your answer.

(a) Range = 92 - (10 + a) = 81i. a = 1Interguartile range = (70 + b) - 51 = 22i. b = 3(5 marks)

(b) (i) Let m be the original mean  $\frac{(30 m - 71 + 11) - 30 m}{(20 m - 2)} = \frac{30}{2}$ The mass decreases for  $\frac{3}{2}$ 

The mean decreases by 2

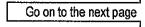
(ii) The corrected datum does not affect the largest and the smallest data, the range remain unchanged.

The original &3 and &, are 73 and 51 while the corrected Q3 and Q, are 73 and 50, the interguartile range is changed.

The claim is correct.

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11. The cost of printing n copies of a brochure for a company is C. It is known that C is partly constant and partly varies as  $\sqrt{n}$ .

When  $n = 10\,000$ ,  $C = 212\,000$  and when  $n = 40\,000$ ,  $C = 224\,000$ .

(a) Find the cost of printing 62 500 copies of that brochure.

(4 marks)

(b) After a careful investigation, the company decides to increase the number of copies of brochure printed from 62 500 to 250 000. The company claims that the extra cost of printing the brochures is less than \$50 000. Do you agree? Explain your answer.

Let  $C = k_1 + k_2 \sqrt{h}$  (2 marks) Where  $k_1$ ,  $k_2$  are non-zero constants

When N = 10000, C = 212000  $212000 = k_1 + k_2 \sqrt{10000}$ 

When N=40000, C=224000 [either]

(2)-(1)  $100k_2 = 12000$ 

Subot into (1) K1 = 200000

in C = 200 000 + 120 \in

When n= 62500,

C= 200000+120 162500 = 230000

The required cost is \$230000

	When $N=250000$ $C = 200000 + 120 \sqrt{250000} = 260000$
	The extra cost = 260000 - 230000
	= 30  on
	< 50 000
	The claim is agreed -
<del></del>	

12. Figure 2(a) shows a right conical vessel of base radius 9 cm. The curved surface area of the vessel is  $135\pi$  cm<sup>2</sup>. The vessel is now fully filled with water.

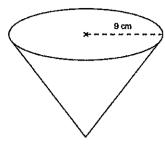


Figure 2(a)

- (a) Find
  - (i) the height of the vessel,
  - (ii) the volume of the vessel in terms of  $\pi$ .

(4 marks)

(b) In Figure 2(b), the water in the vessel is poured into three identical paper cups which are similar in shape to the vessel.

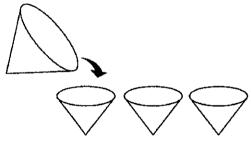


Figure 2(b)

If the water just fills up the three cups without overflow, find the base radius of the paper cup.

2 (marks)

(a) (i) Let h cm be the height of the ve	ssel
$\pi \cdot 9 \cdot \sqrt{9^2 + h^2} = 135\pi$	19
$\frac{81+1^2}{} = 225$	
h2 = 144	
h = 12	
i. The height of the vessel is 1=	2cm. LA
	· · · · · · · · · · · · · · · · · · ·
(ii) The Volume = = = Tr (9)2(12)	IM.
= 324 TV CM	LA

(b) Let rem b  3  r3  The base ra	= 243			
	• • •	(cor to	3 sig fro	7) 24 cm .
The base ra	= 6.24 dius of th	e paper (	2 5, 9 + 50	<del>])</del> 24 cm +
				1
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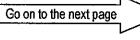
13.	The cubic poly	ynomial f(x	) is divisible by	x-2. WI	hen $f(x)$	is divided by	$x^2-4$ , the
	remainder is	4x+k, when	e k is a constant.				

Find the value of k. (a)

(3 marks)

Answers written in the margins will not be marked

It is given that f(x) is also divisible by x+4. When f(x) is divided by x, (b) the remainder is 40. Georg claims that all the roots of the equation f(x) = 0 are integers. Do you agree? Explain your answer. (4 marks)



14. In Figure 3, the straight line L: 3x - 5y + 15 = 0 cuts the x-axis and the y-axis at A and B respectively.

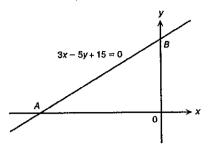


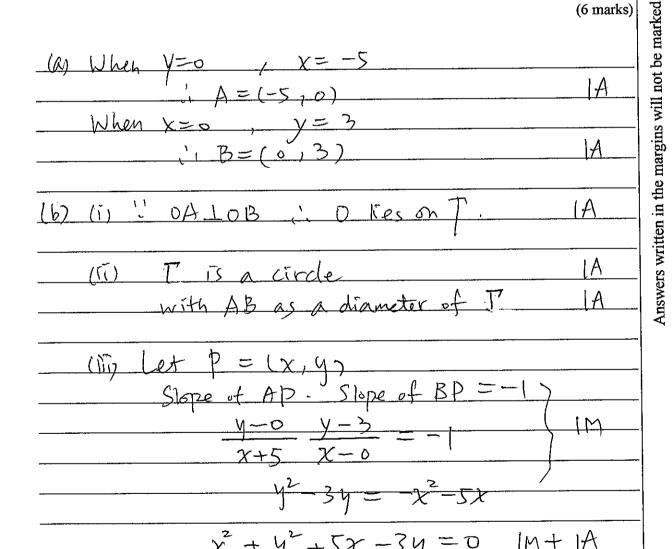
Figure 3

Find the coordinates of A and B. (a)

(2 marks)

- (b) P is a moving point on the coordinate plane such that  $AP \perp BP$ . Denote the locus of P by  $\Gamma$ .
  - Does the origin O lie on  $\Gamma$ ? Explain your answer.
  - Describe the geometric relation between the line segment AB and  $\Gamma$ .
  - (iii) Find the equation of  $\Gamma$ .

(6 marks)



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Section	R	(35	marke	ì
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- There are 8 kittens and 4 puppies in a garden. A photo is taken with 5 of the cubs being randomly 15. selected in the garden. Find the probabilities that
  - 3 kittens and 2 puppies are selected,

(2 marks)

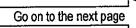
3 kittens and 2 puppies are selected provided that there is at least 1 puppy selected.

(4 marks)

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1A for numerator

Answers written in the margins will not be marked.



- The yearly profit of Évariste & Co. in 2012 was \$P. The yearly profit increases at a constant rate of r% per year. Let T(n) denote the yearly profit of Évariste & Co. in the nth year after 2012.
  - (a) It is given that the yearly profits in 2014 and 2016 were \$1 028 500 and \$1 244 485 respectively. (4 marks) Find the values of P and r.
  - (b) Find the total yearly profit of Évariste & Co. from 2012 to 2019.

(2 marks)

- In which year will the total yearly profit of Évariste & Co. first exceed  $5 \times 10^7$ ?
- (3 marks)

Sulest r=10 into (1)

(1-112 = 1028500

Answers written in the margins will not be marked.

Total yearly profit from 2012 to 2019

n 2032, the total yearly profit first exceed

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17.	Let $f(x) = \frac{1}{k} [x^2 + (2k-6)x - 5k + 9]$ , where k is a constant with	$\frac{1}{2} \le k \le \frac{3}{2}$	and
	the point $(3, 1)$ be $A$ .		

(a) Prove that A lies on the graph of y = f(x).

(1 mark)

(b) The graph of y = g(x) is obtained by reflecting the graph of y = f(x) with respect to the y-axis and then translating the resulting graph downwards by 2 units.

Let M be the vertex of the graph of y = g(x). Denote the point (1, -9) by N.

- (i) By the method of completing the square, express the coordinates of M in terms of k.
- (ii) Find k, in surd form, such that the circumcentre of  $\triangle ANM$  lies on AN.
- (iii) It is known that he graph of y = g(x) passes through the same point P for all positive constant k. Let Q be the vertex of the graph of y = g(x) such that the circumcentre C of  $\triangle ANQ$  lies on AN. Henri claims that P, Q and C are collinear.

Do you agree? Explain your answer.

(10 marks)

Answers written in the margins will not be marked.

(a) 
$$f(3) = \frac{1}{k} \left[ 3^2 + (2k-6)(3) - 5k + 9 \right] = 1$$
  
? The graph of  $y = f(x)$  passes through  $A(3, 1)$ . IA

$$(b)(i) g(x) = f(-x) - 2$$

$$= \frac{1}{k} [(-x)^{2} + (2k - 6)(-x) - 5k + 9] - 2$$

$$= \frac{1}{k} [x^{2} + (2k - 6)x + (k - 3)^{2} - 5k + 9] - 2 \text{ IM}$$

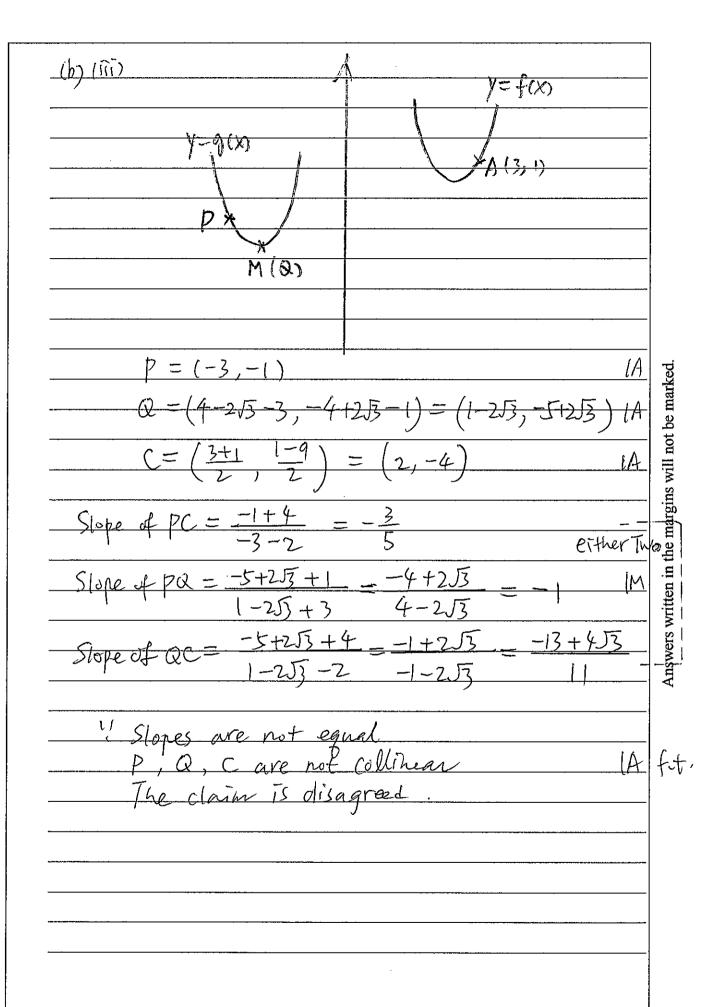
$$= \frac{1}{k} \left[ \left( \frac{(k-1)^{2} - k^{2} + k^{2} - 2}{2} \right) \right]^{2} - k^{2} + k^{2} - 2$$

$$21 M = (k-3, -k-1)$$
 1A

$$\frac{1+(k+1)}{3-(k-3)} \cdot \frac{-9+(k+1)}{1-(k-3)} = -1$$

$$k^{2}-8k+4=0$$
  
 $k=4+\sqrt{12}$  (rej.) or  $4-\sqrt{12}$ 

 $k = 4 - 2\sqrt{3}$ 



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18. In Figure 4(a), PTQSR is a paper card in the shape of a concave pentagon. It is given that PT=10 cm, TQ=SQ=8 cm,  $\angle TPR=96^{\circ}$  and  $\angle PTQ=38^{\circ}$ . PS and TR are straight lines intersecting at Q.

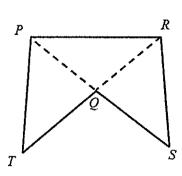


Figure 4(a)

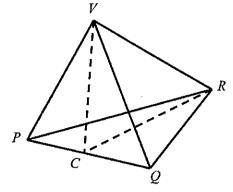


Figure 4(b)

- (a) (i) Find the lengths of PR and QR.
  - (ii) Find  $\angle QPR$ .

(5 marks)

- (b) The paper card in Figure 4(a) is folded along PQ and QR such that T and S meet at a point V as shown n Figure 4(b). Let C be a point lying on PQ satisfying that VC is perpendicular to PQ.
  - (i) Find the length of CR.
  - (ii) David claims that  $\angle VCR$  is the angle between the face VPQ and the face PQR. Do you agree? Explain your answer.

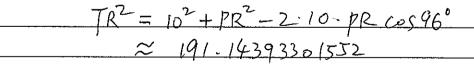
(4 marks)

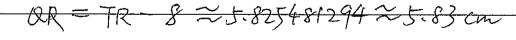
(A) Consider APTR.

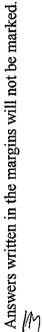
PR = 10SFn38° Sin ( $180^{\circ}$ - $96^{\circ}$ - $38^{\circ}$ )

PR  $\approx 8.558701674$   $\approx 8.556$  cm

(4 marks)







IA

LA

(a) (ii) Consider APTO	P 8.56 R
PQ2 = 102+82-2(10)(8) (0138)	. 46
≈ 37.91827942292	5.83
	10/0
Comider DPQR	389/8
•	T )
SIZEDPR SIN 46°	
5 Fin LAPR 2 0.68052109	228
LRPR≈ 42.884376516	
	178 1- 12c) IA
(h (i) In APRR	pc = ?
8.54	
W2.9° !	
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In AVPC	96-429 0/
PC~10 Cox (96°-42.8843	3765/0380)
€ 6,002021441	AND MA
	PCQ
In APQR	
$ (R^2 = PR^2 + Pc^2 - 2PR). $	
<i>≈</i> 33.9958271888	· ·
- CR ≈ 5.8305940682	6 ≈ 5-83 cm 1A
(5)	
$\frac{(11)}{(12)} \frac{(R^2 + CP^2 \approx 70.0)}{(12)}$	10
$\frac{PR^2 \approx 73.25}{CR^2 + CP^2 \cdot PR^2}$	- IM
	· · · · · · · · · · · · · · · · · · ·
- CLPCR is not a r	ight angle.
- 2, LVCR is not the a	agie it ween the taces.
The claim is not agr.	reed. [A fit

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