# Marymount Secondary School

## Mock Examination 2021 – 2022

## Mathematics (Compulsory Part) Paper 1

Secondary 6

| | |
|---|---|
| Date | : 24 January 2022 |
| Time Allowed | : 2 hours 15 minutes |
| Total Marks | : 105 |

**INSTRUCTIONS**

1. This paper consists of THREE sections, A(1), A(2) and B. Each section carries 35 marks.

2. Attempt ALL questions in this paper. Write your answer in the single-lined papers provided.

3. Graph paper and supplementary answer sheets will be supplied on request.

4. Unless otherwise specified, all working must be clearly shown and numerical answers should be either exact or correct to 3 significant figures.

5. The diagrams in this paper are not necessarily drawn to scale.

## SECTION A(1) (35 marks)

1. Simplify $\frac{(2x^3y^{-2})^3}{x^2y}$ and express your answer with positive indices.

(3 marks)

2. Make $b$ the subject of the formula $\frac{3-2b}{a}=4$.

(3 marks)

3. Factorize
   (a) $8x^2-26xy-15y^2$,
   (b) $4xy-3y^2-8x^2+26xy+15y^2$.

(3 marks)

4. (a) Find the range of values of $x$ which satisfy both $\frac{2(x-2)}{3}-11<4(x+1)$ and $-2x+4\geq 0$.
   (b) How many integers satisfy both inequalities in (a)?

(4 marks)

5. Mary has 20 coins in her coin purse. They are either \$10 coins or \$1 coins. The total value of coins cannot exceed one hundred and twenty dollars. Find the maximum number of \$10 coins in her coin purse.

(4 marks)

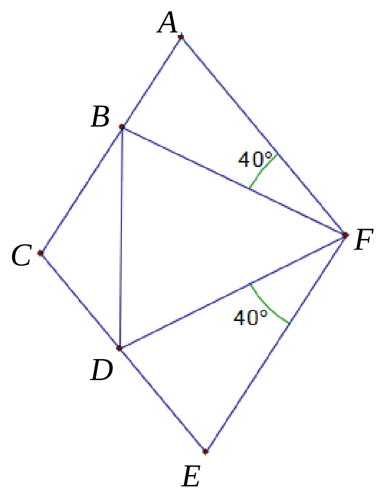
6. The cost of a dress is \$100. If Peter sells it at a discount of 20%, he still makes a 10% profit. Find the marked price of the dress.

(4 marks)

7. In a polar coordinate system, $O$ is the pole. The polar coordinates of the points $P$ and $Q$ are ( $r$ , 315° ) and ( $r$ , 45° ) respectively.
   (a) Describe the geometric relationship between horizontal polar axis and $\angle POQ$.
   (b) If the area of $\Delta POQ$ is 32 square units, find $r$.

(4 marks)

8. In the figure, $ACEF$ is a rhombus, $\Delta BDF$ is an equilateral triangle and $\angle AFB = \angle EFD = 40°$.




(a) Prove $\Delta ABF \cong \Delta EDF$.

(b) Hence, find $\angle CBD$.

(5 marks)

9. The stem-and-leaf diagram below shows the distribution of the scores of 32 S6A students in a mathematics test. Three numbers in the stem-and-leaf diagram are accidentally covered in ink.

| *Stem* (*tens*) | *Leaf* (*units*) |
|---|---|
| 1 | 5 |
| 2 | 4 6 7 8 |
| 3 | 0 1 1 2 3 4 5 |
| 4 | 3 3 [illegible] 7 9 9 |
| 5 | 0 0 2 3 4 |
| 6 | 1 2 3 3 |
| 7 | 3 5 |
| 8 | 1 |

It is known that the three numbers covered in ink are all different. (Note that the following two parts are independent.)

(a) A students is chosen at random. If the probability of choosing a student whose score is at least 45 is $\frac{9}{16}$, find the three numbers covered in ink.

(b) Find the greatest possible value and the least possible value of the median of the distribution.

(5 marks)

## SECTION A(2) (35 marks)

10. (a) Show that the equation of the circle which has points $A\left(\frac{3}{\sqrt{2}}, 2+\frac{3}{\sqrt{2}}\right)$ and $B\left(-\frac{3}{\sqrt{2}}, 2-\frac{3}{\sqrt{2}}\right)$ as ends of a diameter is $x^2+y^2-4y-5=0$.

(3 marks)

(b) Show that the circle in (a) and the circle $x^2+y^2-8y-5=0$ touch externally.

(3 marks)

11. The table below shows the distribution of the ages of the members in a volunteer group, where $y \leq 10$.

| ***Age*** | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|
| ***Number of members*** | 7 | 9 | $x$ | 10 | $y$ | 8 |

The mode and the median of the ages of the members are 9 and 9.5 respectively.

(a) Find $x$ and $y$. (3 marks)

(b) Find the least possible standard deviation of the distribution. (2 marks)

(c) Four members now leave the group. It is given that the range of their ages is 2. Find the greatest possible mean of the ages of the remaining members in the group.

(2 marks)

12. It is given that when $10x^3 + ax^2 + bx - 24$ is divided by $x - 1$ and $x - 2$, the remainders are $-26$ and 54 respectively.

(a) Find the values of $a$ and $b$.

(4 marks)

(b) Joyce claims that $10x^3 + ax^2 + bx - 24$ is divisible by $2x - 3$. Is she correct? Explain your answer.

(2 marks)

13. It is given that $f(x)$ is partly constant and partly varies directly as $x$. Suppose that $f(7) = 22$ and $f(-2) = 4$.

(a) Find $f(x)$.

(3 marks)

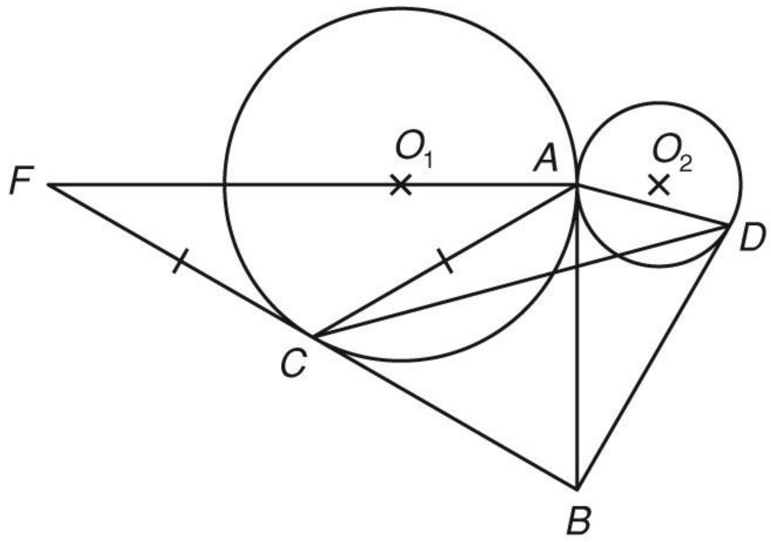
(b) The straight line $L_1$: $y = f(x)$ cuts the $x$-axis and the $y$-axis at the points $B$ and $C$ respectively. The straight line $L_2$ cuts $L_1$ and the $y$-axis at the points $M$ and $D(0, 3)$ respectively. It is given that the slope of $L_2$ is $-\frac{1}{2}$.

(i) Prove that $BD = CD$.

(ii) Let $F$ and $P$ be two points such that $C$ is the mid-point of $BF$ and $P$ does not lie on $L_1$. Find the ratio of the area of $\Delta PFM$ to the area of $\Delta PBM$.

(5 marks)

14. The figure below shows two circles with centres $O_1$ and $O_2$ respectively. $BA$ is the common tangent to the two circles at $A$. $BF$ is the tangent to the larger circle at $C$ and $BD$ is the tangent to the smaller circle at $D$. $FO_1A$ is a straight line and $FC = CA$.




(a) Find $\angle ABC$.

(4 marks)

(b) If $\angle BCD = 45°$, find $\angle ADB$.

(4 marks)

## SECTION B (35 marks)

15. 3 men and 6 women randomly form a queue. It is given that no men are next to each other in the queue. Find the probability that a man is in the middle and the other 2 men are at each end of the queue.

(3 marks)

16. In Figure 1, the equations of $L_1$ and $L_2$ are $x = 60$ and $y = 10$ respectively. The straight line $L_3$ passes through the origin. The straight line $L_4$ intersects $L_2$ and $L_3$ at (380, 10) and (240, 80) respectively.

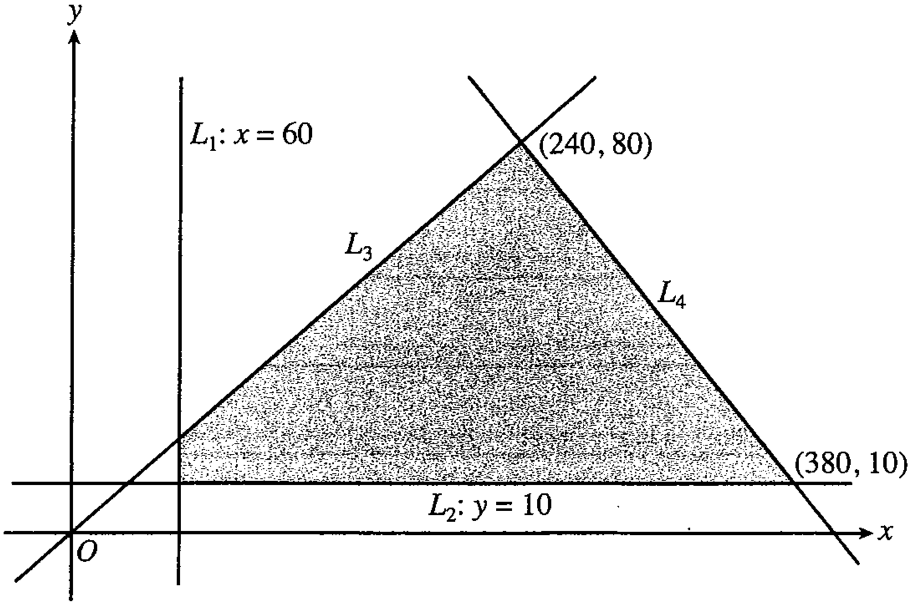



Figure 1

(a) In Figure 1, the shaded region (including the boundary) represents the solution of a system of inequalities. Write down the system of inequalities.

(4 marks)

(b) Find the maximum value of $4000x + 15000y$, where $(x, y)$ is a point lying in the shaded region.

(2 marks)

17. Let $T_1, T_2, T_3, \ldots.$ be a sequence of number where $T_1 = 3$ and $T_n = 2T_{n-1} - (n - 2)$ for $n \geq 2$.

(a) Verify that $T_n = 2^n + n$ for $n = 1, 2, 3, 4$.

(2 marks)

(b) Find $T_1 + T_3 + T_5 + \ldots + T_{19}$, given that the formula for $T_n$ in (a) is true for all positive integers $n$.

(4 marks)

18. In Figure 2(a), $O$ is the circumcentre of $\Delta ABC$ with $AB = AC = 13$ cm and $BC = 10$ cm.

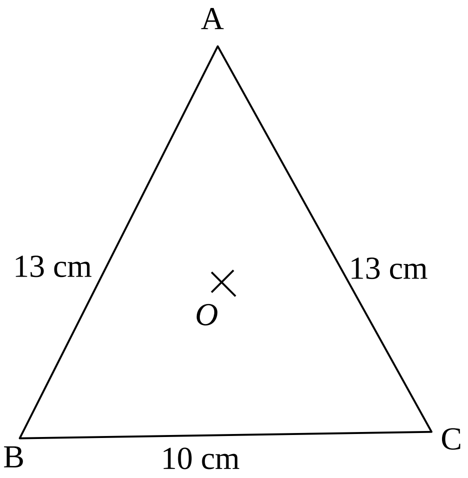



Figure 2(a)

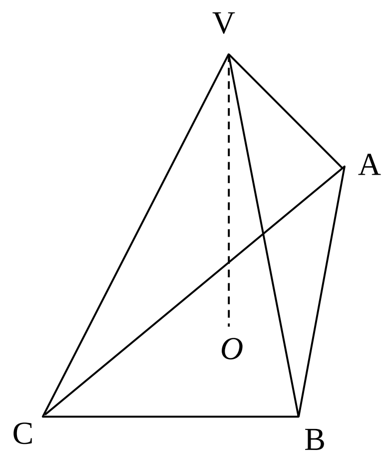



Figure 2(b)

(a) Find the length of $OA$.

(3 marks)

(b) In Figure 2(b), $VABC$ is a tetrahedron with the $\Delta ABC$ described in Figure 2(a) as the base. $O$ is the foot of the perpendicular from $V$ to the plane $ABC$. It is given that the angle between the planes $VAC$ and $ABC$ is $\alpha$ and the angle between the planes $VBC$ and $ABC$ is $\beta$.

(i) Express $VO$ in terms of $\alpha$.

(ii) Is $\beta$ greater than, less than or equal to $\alpha$? Explain your answer.

(5 marks)

19. The vertex of a triangle are $A(-9, -8)$, $B(0, 10)$ and $C(4, 8)$. Denote the circle passing through $A$, $B$ and $C$ by $\Omega$.

(a) Is $\Delta ABC$ a right-angled triangle? Explain your answer.

(2 marks)

(b) Find the equation of $\Omega$.

(3 marks)

(c) $B$ is rotated clockwise about the origin $O$ through 90° to $D$.

(i) Is $ABCD$ a cyclic quadrilateral? Explain your answer.

(ii) If the straight line $L$ is a tangent to $\Omega$ and passes through $D$, find the equation(s) of $L$.

(7 marks)

**END OF PAPER**