2122 S.6 Mock Paper I Solution (Student)

Section A(1)

1) $\dfrac{(2x^3y^{-2})^3}{x^2y}$

$= \dfrac{8x^9y^{-6}}{x^2y}$

$= \dfrac{8x^7}{y^7}$ ※

2) $\dfrac{3-2b}{a} = 4$

$3-2b = 4a$

$-2b = 4a-3$

$b = \dfrac{4a-3}{-2}$ accept $\dfrac{3-4a}{2}$

3a) $8x^2-26xy-15y^2$

$= (4x-3y)(2x-5y)$

b) $4xy-3y^2-8x^2+26xy+15y^2$

$= (4x-3y)y-(4x-3y)(2x-5y)$

$= (4x-3y)(y-2x+5y)$

$= (4x-3y)(6y-2x)$

$= 2(4x-3y)(3y-x)$

4a) $\frac{2(x-2)}{3} - 11 < 4(x+1)$ and $-2x+4 \geq 0$

$2x - 4 - 33 < 12x + 12$ $\quad$ $-2x \geq -4$

$-10x < 59$ $\quad$ $x \leq 2$

$x > -\frac{49}{10}$

$\therefore -\frac{49}{10} < x \leq 2$

b) 7

5) Let x be the number of \$10 coins

$10x + (20 - x) \leqslant 120$

$9x \leqslant 100$

$x \leqslant 11.1$ (to 3 s.f.)

$\therefore$ max number of \$10 coins is 11

6) Let \$x be the marked price

$x(1-20\%) - 100 = 10\%(100)$

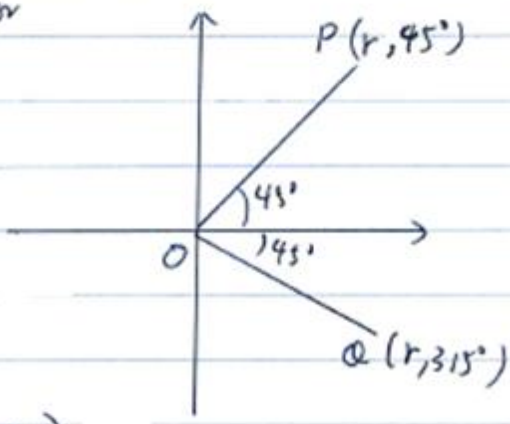
$0.8x - 100 = 10$

$0.8x = 110$

$x = 137.5$

$\therefore$ marked price is \$ 137.5

7) a) Polar axis is the angle bisector of $\angle POQ$



b) $\because \angle POQ = 90°$

$\therefore$ Area of $POQ = \frac{r^2}{2} = 32$

$r^2 = 64$

$r = 8$ or -8 (rej.)

8) a) In $\triangle ABF$ and $\triangle EDF$

$\angle AFB = \angle EFD$ (given)

$AF = FE$ (rhombus)

$BF = FD$ (equilateral $\triangle$)

$\therefore \triangle ABF \cong \triangle EDF$ (SAS)

b(i) $\because \angle DFB = 60°$ (prop. of equil. $\triangle$)

$\therefore \angle AFE = 60° + 40° + 40° = 140°$

$\angle ACE = \angle AFE = 140°$ (opp. $\angle$s of // gram)

$\because AC = CE$ (prop of rhombus)

$\because AB = DE$ (corr. sides, $\cong \triangle$s)

$\therefore BC = DC$

$\therefore \angle CBD = \angle BDC$

$\therefore \angle CBD + \angle BDC + \angle BCD = 180°$ ($\angle$ sum of $\triangle$)

$\angle CBD = \frac{180° - 140°}{2} = 20°$ ※

9 a) ∵ The three number are differnt

score at least 45 exclude the missing data

$= 15$.

Let x be the number of missing score at least 45

$$\frac{15+x}{32} = \frac{9}{16}$$

$$x = 3$$

∴ all three number are larger or equal to 45,

∵ they are all different

∴ The 3 missing numbers are 5, 6, 7

b) The least possible value of median is obtained when the 3 missing numbers are 3, 4, 5

, which is equal to $\frac{45+44}{2} = 44.5$

The greatest possible value of median is obtained when the 3 missing number are 5, 6, 7,

which is equal to $\frac{46+47}{2} = 46.5$

Section A(2)

10.(a) Consider the mid-point of AB = $\left(\frac{\frac{3}{\sqrt{2}} - \frac{3}{\sqrt{2}}}{2}, \frac{2 + \frac{3}{\sqrt{2}} + 2 - \frac{3}{\sqrt{2}}}{2} \right)$

$$= (0, 2)$$

$$r^2 = \left(\frac{3}{\sqrt{2}} - 2 \right)^2 + \left(2 + \frac{3}{\sqrt{2}} - 2 \right)^2$$
$$= 3^2$$

The equation of circle:

$\therefore \quad (x-0)^2 + (y-2)^2 = 3^2$

$$x^2 + y^2 - 4y - 5 = 0$$

(b) Centre of the circle $x^2 + y^2 - 8y - 5 = 0$ is

$= (-\frac{-8}{2}, -\frac{2}{2})$

$= (4, -1)$

Radius $r = 2$

Distance between centres = 3 + 2

= 5

= sum of radii

i.e. the two circles touch externally.

11. (a) $\because$ Mode = 9

$\therefore$ $x > 10$

$\because$ Median = 9.5

$\therefore$ $7 + 9 + x = 10 + y + 8$

$y = x - 2$

Note that $x > 10$ and $y \le 10$.

When $x = 11$, $y = 11 - 2 = 9$.

When $x = 12$, $y = 12 - 2 = 10$.

When $x = 13$, $y = 13 - 2 = 11$ (*rejected*).

When $x \ge 14$, y must be greater than 11.

$\therefore$ $\begin{cases} x = 11 \\ y = 9 \end{cases}$ *or* $\begin{cases} x = 12 \\ y = 10 \end{cases}$.

(b)

When $x = 11$ and $y = 9$,

standard deviation = 1.61, *cor. to 3 sig. fig.*

When $x = 12$ and $y = 10$,

standard deviation = 1.59, *cor. to 3 sig. fig.*

$\therefore$ The least possible standard deviation is 1.59.

(c)

The mean of the ages of the remaining members in the group is the greatest when the ages of the four members leaving the group are 7, 7, 7 and 9.

When $x = 11$ and $y = 9$,

mean = $\dfrac{7 \times (7-3) + 8 \times 9 + 9 \times (11-1) + 10 \times 10 + 11 \times 9 + 12 \times 8}{7 + 9 + 11 + 10 + 9 + 8 - 4}$

= 9.7

When $x = 12$ and $y = 10$,

mean = $\dfrac{7 \times (7-3) + 8 \times 9 + 9 \times (12-1) + 10 \times 10 + 11 \times 10 + 12 \times 8}{7 + 9 + 12 + 10 + 10 + 8 - 4}$

= 9.71, *cor. to 3 sig. fig.*

$\therefore$ The greatest possible mean is 9.71.

12. Let $f(x)=10x^3+ax^2+bx-24$.

(a) When $f(x)$ is divided by $x-1$,

$$f(1)=-26$$
$$10(1)^3+a(1)^2+b(1)-24=-26$$
$$a+b=-12 \quad ......(1)$$

When $f(x)$ is divided by $x-2$,

$$f(2)=54$$
$$10(2)^3+a(2)^2+b(2)-24=54$$
$$4a+2b=-2$$
$$2a+b=-1 \quad ......(2)$$

(2) – (1): $a=\underline{\underline{11}}$

By substituting a = 11 into (1), we have

$$11+b=-12$$
$$b=\underline{\underline{-23}}$$

(b) From (a), $f(x)=10x^3+11x^2-23x-24$

$$f\left(\frac{3}{2}\right)=10\left(\frac{3}{2}\right)^3+11\left(\frac{3}{2}\right)^2-23\left(\frac{3}{2}\right)-24$$
$$=\frac{135}{4}+\frac{99}{4}-\frac{69}{2}-24$$
$$=0$$

∴ By the factor theorem, $f(x)$ is divisible by $2x-3$.

∴ Joyce is correct.

13. (a) From the question, $f(x) = k_1 + k_2x$, where k_1 and k_2 are non-zero constants.

$f(7) = 22$

$k_1 + 7k_2 = 22$ .... (1)

$f(-2) = 4$

$k_1 - 2k_2 = 4$ .... (2)

(1) – (2): $9k_2 = 18$

$k_2 = 2$

Substitute $k_2 = 2$ into (1).

$k_1 + 7(2) = 22$

$k_1 = 8$

∴ $f(x) = 8 + 2x$

(b)(i) When $f(x) = 0$,

$0 = 8 + 2x$

$x = -4$

$\therefore$ The coordinates of B are $(-4 , 0)$.

$f(0) = 8 + 2(0) = 8$

$\therefore$ The coordinates of C are $(0 , 8)$.

$BD = \sqrt{(-4-0)^2 + (0-3)^2}$

$= 5$

$CD = 8 - 3 = 5$

$\therefore$ $BD = CD$

(ii) Slope of L_1 × slope of L_2 $= 2 \times \left(-\frac{1}{2}\right)$

$= -1$

$\therefore$ $L_1 \perp L_2$

$\because$ $BD = CD$ and $DM \perp BC$.

$\therefore$ $BM = CM$

$\because$ C is the mid-point of BF.

$\therefore$ $CF : CM : BM = 2 : 1 : 1$

$\therefore$ $FM : BM = 3 : 1$

Note that when FM and BM are considered as the bases of $\triangle PFM$ and $\triangle PBM$ respectively, the heights of the two triangles are the same.

Area of $\triangle PFM$: area of $\triangle PBM$

$= FM : BM$

$= 3 : 1$

14. (a) Let $\angle ABC = a$.

$\angle FAB = 90°$ (tangent $\perp$ radius)

In $\triangle AFB$,

$$\begin{aligned}\angle BFA &= 180° - \angle FAB - \angle ABC \quad (\angle \text{ sum of } \triangle)\\ &= 180° - 90° - a\\ &= 90° - a\end{aligned}$$

$\because$ $CA = CF$ (given)

$\therefore$ $\angle CAF = \angle BFA$ (base $\angle$s, isos. $\triangle$)

$= 90° - a$

$$\begin{aligned}\angle BAC &= \angle FAB - \angle CAF\\ &= 90° - (90° - a)\\ &= a\end{aligned}$$

$\because$ $BC = BA$ (tangent properties)

$\therefore$ $\angle BCA = \angle BAC$ (base $\angle$s, isos. $\triangle$)

$= a$

In $\triangle ACB$,

$$\begin{aligned}\angle BCA + \angle BAC + \angle ABC &= 180° \quad (\angle \text{ sum of } \triangle)\\ a + a + a &= 180°\\ 3a &= 180°\\ a &= 60°\end{aligned}$$

$\therefore$ $\angle ABC = \underline{\underline{60°}}$

(b) $\because$ $BA = BD$ …… (1) (tangent properties)

$\therefore$ $\angle DAB = \angle ADB$ (base $\angle$s, isos. $\triangle$)

$BC = BA$ …… (2) (tangent properties)

From (1) and (2), we have

$BC = BD$

$\therefore$ $\angle BDC = \angle BCD$ (base $\angle$s, isos. $\triangle$)

$= 45°$

In $\triangle BCD$,

$$\begin{aligned}\angle CBD &= 180° - \angle BDC - \angle BCD \quad (\angle \text{ sum of } \triangle)\\ &= 180° - 45° - 45°\\ &= 90°\end{aligned}$$

$$\angle ABD = \angle CBD - \angle ABC$$
$$= 90° - 60°$$
$$= 30°$$

In $\triangle BDA$,

$$\angle ADB + \angle DAB + \angle ABD = 180° \quad (\angle \text{ sum of } \triangle)$$
$$2\angle ADB + 30° = 180°$$
$$\angle ADB = \underline{\underline{75°}}$$

<u>Section B</u>

15. Required probability $= \dfrac{P_3^3 P_6^6}{P_3^7 P_6^6}$ or $\dfrac{1}{C_3^7}$

$$= \underline{\underline{\frac{1}{35}}}$$

16. (a)

The system of inequalities are:

$$\begin{cases} x \geq 60 \\ y \geq 10 \\ x - 3y \geq 0 \\ x + 2y - 400 \leq 0 \end{cases}$$

(b)

At (240, 80),

$= [4000(240) + 15\ 000(80)]$

$= 2\ 160\ 000$

At (380, 10),

$= [4000(380) + 15\ 000(10)]$

$= 1\ 670\ 000$

At (60, 10),

$= [4000(60) + 15\ 000(10)]$

$= 390\ 000$

At (60, 20),

$= [4000(60) + 15\ 000(20)]$

$= 540\ 000$

$\therefore$ The max value is 2 160 000

17.

(a) $T_1 = 3, T_2 = 2(3) - 0 = 6,$
$T_3 = 2(6) - 1 = 11, T_4 = 2(11) - 2 = 20$
$2^1 + 1 = 3 = T_1, 2^2 + 2 = 6 = T_2,$
$2^3 + 3 = 11 = T_3, 2^4 + 4 = 20 = T_4$

(b) $T_1 +$ [illegible] $+ \ldots + T_{19}$
$= 2 +$ [illegible] $+ 2^3 + 3 + \ldots + 2^{19} + 19$
$= (2 + 2 \cdot 4 + \ldots + 2 \cdot 4^9) + (1 + 3 + \ldots + 19)$
$\frac{2(4^{10} - 1)}{4 \text{[illegible]} 1} + \frac{(1+19)(10)}{2}$
$= 699$[illegible]0

18.

.et h cm be the height of ΔABC with base BC.

$$\left(\frac{10}{2}\right)^2 + h^2 = 13^2 \quad \textit{(Pyth. theorem)}$$

$$h^2 = 144$$

$$h = 12$$

Let r cm be the radius of the circumcircle of ΔABC.

$$(12-r)^2 + \left(\frac{10}{2}\right)^2 = r^2 \quad \textit{(Pyth. theorem)}$$

$$144 - 24r + r^2 + 25 = r^2$$

$$24r = 169$$

$$r = \frac{169}{24}$$

$$\therefore \quad OA = \frac{169}{24} \text{ cm}$$

(b) (i) Let ℓ cm be the height of ΔAOC with base AC.

$$\ell^2 + \left(\frac{13}{2}\right)^2 = \left(\frac{169}{24}\right)^2 \quad \textit{(Pyth. theorem)}$$

$$\ell^2 = \frac{4225}{576}$$

$$\ell = \frac{65}{24} \text{ or } -\frac{65}{24} \text{ (rejected)}$$

$$\therefore \quad \tan\alpha = \frac{VO}{\ell \text{ cm}}$$

$$VO = \frac{65}{24}\tan\alpha \text{ cm}$$

(ii) $\tan\beta = \dfrac{VO}{(12-r)\text{ cm}}$

$$\tan\beta = \frac{VO}{\left(12 - \frac{169}{24}\right)\text{ cm}}$$

$$VO = \frac{119}{24}\tan\beta \text{ cm}$$

$$\because \quad VO = \frac{65}{24}\tan\alpha \text{ cm and } VO = \frac{119}{24}\tan\beta \text{ cm}$$

$$\therefore \quad \frac{\tan\alpha}{\tan\beta} = \frac{119}{65}$$

$\therefore \quad \tan\beta < \tan\alpha$

$\therefore \quad \beta$ is less than α when α and β are acute.

19. (a) Slope of AB × Slope of $BC = \left[\dfrac{10-(-8)}{0-(-9)}\right]\left(\dfrac{10-8}{0-4}\right)$

$= -1$

∴ $AB \perp BC$

∴ ΔABC is a right-angled triangle.

(b) ∵ $\angle ABC = 90°$ *(proved)*

∴ AC is a diameter. *(converse of ∠ in semicircle)*

Centre of $\Omega = \left(\dfrac{-9+4}{2}, \dfrac{-8+8}{2}\right) = \left(-\dfrac{5}{2}, 0\right)$

Radius of $\Omega = \sqrt{\left(-\dfrac{5}{2}-4\right)^2 + (0-8)^2} = \sqrt{\dfrac{425}{4}}$

∴ The equation of Ω:

$$\left[x - \left(-\frac{5}{2}\right)\right]^2 + (y-0)^2 = \left(\sqrt{\frac{425}{4}}\right)^2$$

$$\left(x + \frac{5}{2}\right)^2 + y^2 = \frac{425}{4}$$

(or $x^2 + y^2 + 5x - 100 = 0$)

(c) (i) The coordinates of D are $(10, 0)$.

Substituting $(10, 0)$ into equation of Ω,

$$\text{L.H.S.} = \left(10 + \frac{5}{2}\right)^2 + 0^2$$

$$= \frac{625}{4}$$

$$\neq \text{R.H.S.}$$

∴ D does not lie on the circle passing through A, B and C.

∴ $ABCD$ is not a cyclic quadrilateral.

(ii) Let the equation of L be $y - 0 = m(x - 10)$.

i.e., $y = m(x - 10)$.

Substituting $y = m(x - 10)$ into the equation of Ω,

$$\left(x + \frac{5}{2}\right)^2 + [m(x-10)]^2 = \frac{425}{4}$$

$$(1 + m^2)x^2 + (5 - 20m^2)x + 100m^2 - 100 = 0$$

∵ The tangent meets the circle at only 1 point.

∴ $\Delta = 0$

$$(5 - 20m^2)^2 - 4(1 + m^2)(100m^2 - 100) = 0$$

$$25 - 200m^2 + 400m^4 - 400m^4 + 400 = 0$$

$$200m^2 - 425 = 0$$

$$m^2 = \frac{17}{8}$$

$$m = \pm\sqrt{\frac{17}{8}}$$

∴ The equations of L:

$$y = \sqrt{\frac{17}{8}}(x - 10) \text{ or } y = -\sqrt{\frac{17}{8}}(x - 10)$$