2021-22 S6 Mock Exam Paper 2 Solution

BACCD DBDBC
DADCC DCCDA
BDDAA BCCCC
CDDAB CBBDB
DBABC

1. B

$$9^{444} \cdot 16^{222} = (3^2)^{444} \cdot (2^4)^{222}$$
$$= 3^{888} \cdot 2^{888}$$
$$= 6^{888}$$

2. A

$$h = 2 - \frac{4}{k+3}$$
$$\frac{4}{k+3} = 2 - h$$
$$k + 3 = \frac{4}{2-h}$$
$$k = \frac{4 - 3(2-h)}{2-h}$$
$$= \frac{3h-2}{2-h}$$

3. C

$3x + 3y - x^2 + y^2$
$= 3(x + y) - (x^2 - y^2)$
$= 3(x + y) - (x - y)(x + y)$
$= (x + y)[3 - (x - y)]$
$= (x + y)(3 - x + y)$

4. C

$$\frac{1}{x-1} + \frac{1}{(x-1)(x-2)} = \frac{(x-2)+1}{(x-1)(x-2)}$$
$$= \frac{x-1}{(x-1)(x-2)}$$
$$= \frac{1}{x-2}$$

5. D

	Significant figure(s)	Decimal place(s)
2	0.010	0.01
3	0.0103	0.010
4	0.010 31	0.0103
5	0.010 305	0.010 31

6. D

L.H.S.

$= n(x-3)^2 - 2x$

$= n(x^2 - 6x + 9) - 2x$

$= nx^2 - 6nx + 9n - 2x$

$= nx^2 - (6n+2)x + 9n$

R.H.S.

$= 9x^2 + mx(x+2) + 18$

$= 9x^2 + mx^2 + 2mx + 18$

$= (9+m)x^2 + 2mx + 18$

$\therefore \quad nx^2 - (6n+2)x + 9n \equiv (9+m)x^2 + 2mx + 18$

By comparing the coefficients of x^2 and the constant term, we have

$$\begin{cases} n = 9 + m & \ldots\ldots(1) \\ 9n = 18 & \ldots\ldots(2) \end{cases}$$

By substituting (1) into (2), we have

$$9(9+m) = 18$$

$$9 + m = 2$$

$$m = \underline{\underline{-7}}$$

Alternative method:

Sub. $x = 3$,

$$n(3-3)^2 - 2(3) = 9(3)^2 + m(3)(3+2) + 18$$

$$-6 = 81 + 15m + 18$$

$$m = -7$$

7. B

$$f(2) = f\left(\frac{1}{2}\right)$$

$$2(2)^2 + 2k - 1 = 2\left(\frac{1}{2}\right)^2 + \frac{1}{2}k - 1$$

$$8 + 2k = \frac{1}{2} + \frac{1}{2}k$$

$$16 + 4k = 1 + k$$

$$3k = -15$$

$$k = -5$$

8. D

By the remainder theorem,

$$f(1) = 3$$

$$(1)^3 + a(1)^2 + 12(1) - 7 = 3$$

$$1 + a + 12 - 7 = 3$$

$$a = -3$$

∴ $f(x) = x^3 - 3x^2 + 12x - 7$

When $f(x)$ is divided by $x + 1$,

remainder $= f(-1)$

$= (-1)^3 - 3(-1)^2 + 12(-1) - 7$

$= -1 - 3 - 12 - 7$

$= \underline{\underline{-23}}$

9. B

Let b and h be the base and the height of the original triangle respectively.

Then, new base $= b(1 + x\%)$ and new height $= h(1 + x\%)$

$$\frac{bh}{2} \times (1 + 125\%) = \frac{b(1 + x\%) \times h(1 + x\%)}{2}$$

$$2.25 = (1 + x\%)^2$$

$$1 + x\% = 1.5$$

$$x = \underline{\underline{50}}$$

10. C

$3(2x - 1) < 8x - 13$

$6x - 3 < 8x - 13$

$-2x < -10$

$x > 5$

$2x + 1 \geq \frac{4x - 7}{3}$

$6x + 3 \geq 4x - 7$

$2x \geq -10$

$x \geq -5$

∴ $x > 5$ or $x \geq -5$

∴ $x \geq -5$

11. D

$2a = 3b$

$\frac{a}{b} = \frac{3}{2}$

$3b = 4c$

$\frac{b}{c} = \frac{4}{3}$

	a	:	b			=	3	:	2		
			b	:	c	=			4	:	3
∴	a	:	b	:	c	=	6	:	4	:	3

Alternative method:

Let $2a = 3b = 4c = 12k$,

then $a = 6k$, $b = 4k$ and $c = 3k$

∴ $a : b : c = 6 : 4 : 3$

12. A

Let $z = \frac{kx}{y^2}$, where k is a non-zero constant.

Let x_1 and y_1 be the original values of x and y respectively.

Original value of $z = \frac{kx_1}{y_1^{\ 2}}$

New value of $x = 3x_1$

New value of $y = 2y_1$

New value of $z = \frac{k(3x_1)}{(2y_1)^2}$

$= \frac{3kx_1}{4y_1^{\ 2}}$

$\therefore$ Percentage change in $z = \frac{\frac{3kx_1}{4y_1^{\ 2}} - \frac{kx_1}{y_1^{\ 2}}}{\frac{kx_1}{y_1^{\ 2}}} \times 100\%$

$= \frac{\frac{kx_1}{y_1^{\ 2}}(\frac{3}{4} - 1)}{\frac{kx_1}{y_1^{\ 2}}} \times 100\%$

$= -25\%$

13. D

By observing the patterns, we know that

$T(n) = (n+1)^2 + (n-1)$

$T(7) = (7+1)^2 + (7-1)$

$= 64 + 6$

$= \underline{\underline{70}}$

Alternative solution:

$T(2) = T(1) + [2(1)+4] = 4 + 6 = 10$,
$T(3) = T(2) + [2(2)+4] = 10 + 8 = 18$,
$T(4) = T(3) + [2(3)+4] = 18 + 10 = 28$,
$T(5) = T(4) + [2(4)+4] = 28 + 12 = 40$,
$T(6) = T(5) + [2(5)+4] = 40 + 14 = 54$,
$T(7) = T(6) + [2(6)+4] = 54 + 16 = 70$.

i.e. $T(7) = 4 + 6 + 8 + 10 + 12 + 14 + 16 = \frac{(4+16)\times 7}{2} = 70$

14. C

$y = -4 - (3 - x)^2$
$= -x^2 + 6x - 13$

For option A:
Put $x = 0$ into $y = -x^2 + 6x - 13$,
$y = -0^2 + 6(0) - 13 = -13 \neq 0$
∴ The graph does not pass through the origin.
∴ A is not true.

For option B:
Coefficient of $x^2 = -1 < 0$
∴ The graph opens downwards.
∴ B is not true.

For option C:
$\Delta = 6^2 - 4(-1)(-13)$
$= -16$
< 0
∴ The graph does not cut the x-axis.
∴ C is true.

For option D:
y-intercept of the graph is -13.
∴ D is not true.

15. C

Let V cm^3 be the capacity of the paper cup.

$$\frac{V}{180} = \left(\frac{h}{\frac{2}{5}h}\right)^3$$

$$\frac{V}{180} = \left(\frac{5}{2}\right)^3$$

$$V = 180 \times \frac{125}{8}$$

$$= 2\ 812.5$$

Required volume of water

$= (2\ 812.5 - 180)$ cm^3

$= 2\ 632.5$ cm^3

16.

D

Let $AD = h$, $DE = 2h$ and $EH = 3h$, where h is a positive constant.

$\because \quad \Delta ADP \sim \Delta AEQ \sim \Delta AHG \quad (AAA)$

$\therefore \quad DP : EQ : HG = 1 : 3 : 6$

Let $DP = k$, $EQ = 3k$ and $HG = 6k$, where k is a positive constant.

Area of quadrilateral $EQGH$: Area of quadrilateral $ABCP$

$= \frac{1}{2}(3k + 6k)(3h) : \frac{1}{2}[(6k - k) + 6k](h)$

$= 27 : 11$

17.

C

Let h cm be the height of the cylinder.

$$\frac{1}{2} \times \frac{4}{3}\pi(6)^3 + \pi(6)^2 h = 432\pi$$

$$36h = 288$$

$$h = 8$$

Total surface area $= \left[\frac{1}{2} \times 4\pi(6)^2 + 2\pi(6)(8) + \pi(6)^2\right]$ cm^2

$= 204\pi$ cm^2

18.

C

$\because \quad ABCD$ is a kite.

$\therefore \quad \angle ABC = \angle ADC$ and AC bisects $\angle BAD$

$\angle ABC + \angle ADC = 180° \quad$ *(opp. ∠s, cyclic quad.)*

$\angle ABC = 90°$

$\therefore \quad AC$ is a diameter of the circle.

(converse of ∠ in semicircle)

In ΔABC,

$$\frac{5 \text{ cm}}{AC} = \cos\frac{138°}{2}$$

$AC \approx 13.9521$ cm

Area of the circle $= \left(\frac{AC}{2}\right)^2 \pi$

$= 152.89$ cm^2 *(cor. to 2 d. p.)*

19.

D

$\because$ $AG = DG = BE$, $AE = BC = DF$

$\angle EAG = \angle FDG = \angle CBE = 90°$ *(property of rectangle)*

$\therefore$ $\Delta AGE \cong \Delta DGF \cong \Delta BEC$ *(SAS)*

and $DF = 2DG$

$$\tan\angle DFG = \frac{DG}{DF} = \frac{1}{2}$$

$\angle DFG \approx 26.5651°$

$\angle BCE = \angle DFG \approx 26.5651°$ *(corr. ∠s, ≅ Δs)*

$\angle ECF = 90° - \angle BCE$

$\approx 63.4349°$

$\angle CFG + \angle DFG = 180°$ *(adj. ∠s on st. line)*

$\angle CFG \approx 153.4349°$

$$\angle CFH = \frac{\angle CFG}{2} \approx 76.7175°$$

In ΔCFH,

$\angle EHF = \angle CFH + \angle ECF$ *(ext. ∠ of Δ)*

$= 140°$ *(cor. to the nearest degree)*

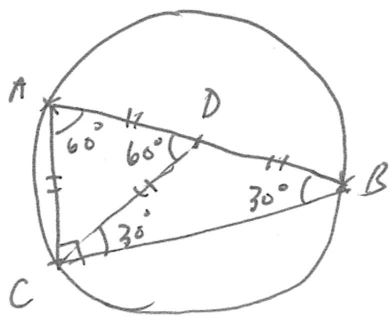
20. A

I. ΔBCD is an isosceles triangle ✓

II. $BC = \sqrt{3}AC$ ✓ $\tan 30° = \frac{AC}{BC}$

III. $\Delta ADC \sim \Delta CDB$ ✗ $BC = \sqrt{3}AC$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III



21. B

$\theta = 180° - 70° - 70°$

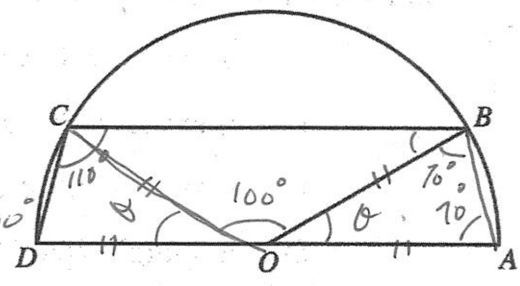
$= 40°$

$\angle BOC = 180° - 2\theta$

$= 100°$

$\therefore \angle OBC = \frac{180° - 100°}{2}$

$= 40°$



22. D

$a + b = 130^\circ$

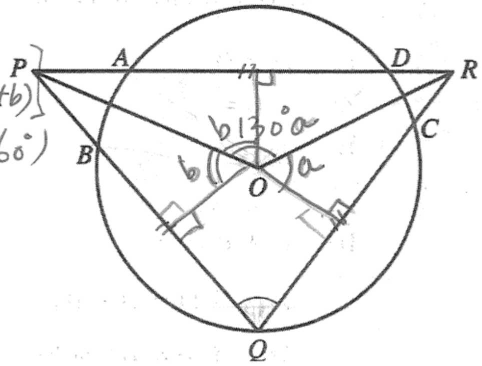
$\angle PQR =$

$= 180^\circ - [360^\circ - 2(a+b)]$

$= 180^\circ - (360^\circ - 260^\circ)$

$= 180^\circ - 100^\circ$

$= 80^\circ$ //



23.

D

$M(3, -2)$ is reflected about the x-axis to $N(3, 2)$.

For I,

the point after rotation is $(2, 3)$.

$\therefore$ I is not true.

For II,

the point after translation is $(3 - 6, -2) = (-3, -2)$.

$\therefore$ II is not true.

For III,

the point after translation is $(3, -2 + 4) = (3, 2)$.

$\therefore$ III is true.

For IV,

the point after translation is $(3, -2 - 2) = (3, -4)$,

and then the point after reflection is $(3, 2)$.

$\therefore$ IV is true.

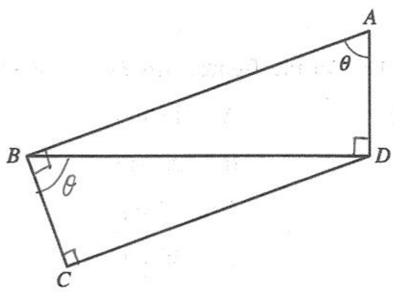
24. A

A. $\dfrac{1}{\sin\theta}$

B. $\sin\theta$

C. $\sin\theta\tan\theta$

D. $\dfrac{\cos^2\theta}{\sin\theta}$



$\tan\theta = \dfrac{BD}{AD}$

$AD = \dfrac{BD}{\tan\theta}$

$\cos\theta = \dfrac{BC}{BD}$

$BC = \cos\theta BD$

$\therefore \dfrac{AD}{BC} = \dfrac{BD}{\tan\theta} \times \dfrac{1}{\cos\theta BD}$

$= \dfrac{1}{\sin\theta}$

25.

A

The slope of $2x+3y-6=0$ is $-\frac{2}{3}$.

The slope of $2x+3y-12=0$ is $-\frac{2}{3}$.

The slope of $2x-3y-6=0$ is $-\frac{2}{-3}=\frac{2}{3}$.

The slope of $3x+2y-6=0$ is $-\frac{3}{2}$.

Since the first two lines have the same slope, they are parallel, i.e. I is correct.

On the other hand, since $-\frac{2}{3}\times\frac{2}{3}\neq-1$ and $-\frac{2}{3}\times\left(-\frac{3}{2}\right)\neq-1$, II and III are not correct.

26.

B

Mid-point of $PQ=\left(\frac{3+7}{2},\frac{-5+1}{2}\right)=(5,-2)$

Centre of C_1 = Centre of $C_2=(5,-2)$

For A,

$$\text{mid-point of } RS=\left(\frac{4+6}{2},\frac{-2+6}{2}\right)=(5,2)$$

$\therefore$ A is not true.

For B,

$$\text{mid-point of } RS=\left(\frac{0+10}{2},\frac{-1+(-3)}{2}\right)=(5,-2)$$

$\therefore$ B may be true.

For C,

$$\text{mid-point of } RS=\left(\frac{-1+5}{2},\frac{1+5}{2}\right)=(2,3)$$

$\therefore$ C is not true.

For D,

$$\text{mid-point of } RS=\left(\frac{-4+0}{2},\frac{-7+1}{2}\right)=(-2,-3)$$

$\therefore$ D is not true.

27. C

$$PX = 2PY$$

$$\sqrt{(x-0)^2+(y-5)^2} = 2\sqrt{(x-1)^2+(y-0)^2}$$

$$x^2+y^2-10y+25 = 4(x^2-2x+1+y^2)$$

$$x^2+y^2-10y+25 = 4x^2-8x+4+4y^2$$

$$3x^2+3y^2-8x+10y-21=0$$

28.

C

P (1st one is a girl and rest are boys)

= P(1st one is a girl)

×P(2nd one is a boy)

×P(3th one is a boy)

×P(4th one is a boy)

$$=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$

$$=\frac{1}{16}$$

29.

C

∵ The shape of II is the narrowest and the shape of I is the widest.

∴ The standard deviation of II is the least and that of I is the greatest.

30.

C

Median $= x - 1 = a$

∴ $x = a + 1$

$$\text{Mean} = \frac{6(x-3)+5(x-1)+7(x)+2(x+5)}{20}$$

$$= \frac{6(a+1-3)+5(a+1-1)+7(a+1)+2(a+1+5)}{20}$$

$$= \frac{20a+7}{20}$$

$$= a + 0.35$$

Section B

31. C	First expression = $a^3b^2c^4$ Second expression = $a^3b^4c^7$ Third expression = a^5b^6c
32. D	8^5+8^{13} $=\left(2^3\right)^5+\left(2^3\right)^{13}$ $=2^{3+4\times3}+2^{3+4\times9}$ $=2^3\times16^3+2^3\times16^9$ $=8000008000_{16}$
33. D	$\log\left(a^2-b^2\right)=8$ $\log\left[\left(a-b\right)\left(a+b\right)\right]=8$ $\log\left(a-b\right)+\log\left(a+b\right)=8$ $\log\left(a+b\right)=5$ $\log\frac{1}{\sqrt[3]{(a+b)^2}}$ $=\log\left(a+b\right)^{-\frac{2}{3}}$ $=-\frac{2}{3}\log\left(a+b\right)$ $=-\frac{2}{3}(5)$ $=-\frac{10}{3}$
34. A	$\log_4 y=\log_4 64-3\log_4 x$ $\log_4 y=3-\frac{\log_2 x}{\log_2 4}$ $\log_4 y=3-\frac{3}{2}\log_2 x$ ...(1) The intercept on the vertical axis = 3 Sub. $\log_4 y=0$ into (1), we have $\log_2 x=2$ So, area = $\frac{1}{2}(2)(3)=3$

35. B	$\dfrac{5k}{1+2i}-4i$ $=\dfrac{5k}{1+2i}\times\dfrac{1-2i}{1-2i}-4i$ $=\dfrac{5k(1-2i)}{5}-4i$ $=k-2ki-4i$ $=k-2(k+2)i$ As $\dfrac{5k}{1+2i}-4i$ is a real number, $k+2=0$ $k=-2$
36. C	$a=-94$ $d=-86-(-94)=8$ $T(n)=a+(n-1)d$ $=-94+(n-1)(8)$ $=8n-102$ Let $T(m)$ be the largest negative term in the sequence. $T(m)<0$ $8m-102<0$ $8m<102$ $m<12.75$ There are 12 negative terms in the sequence. The minimum value of $S(k)$ = Sum of the first 12 terms of the sequence $S(12)=\dfrac{12}{2}\left[-94+(-6)\right]=-600$ So the minimum value of $S(k)$ is -600
37. B	$\Delta\le 0$ $p^2-4(2)(p+6)\le 0$ $p^2-8p-48\le 0$ $(p+4)(p-12)\le 0$ $\therefore -4\le p\le 12$

38.
B

We put the figure to the rectangular coordinates plan such that
$A(0,382), E(350,382), F(600,382), G(150,0)$ and $D(0,0)$

Equation of AF:

$$\frac{y-382}{x-0}=\frac{282-382}{600-0}=\frac{-1}{6}$$

$$6y-2292=-x$$

$$x+6y=2292 \qquad ...(1)$$

Equation of EG:

$$\frac{y-382}{x-350}=\frac{0-382}{150-350}=\frac{191}{100}$$

$$100y-38200=191x$$

$$191x-100y=28650 \qquad ...(2)$$

By solving (1), (2), $\begin{cases} x\approx 321.9101124 \\ y\approx 328.3483146 \end{cases}$

So,

HF

$$\approx\sqrt{(321.9101124-600)^2+(328.3483146-282)^2}$$

$$\approx 281.9257914$$

39.
D

$\because AM=MC$ *(given)*

$\therefore AC\perp EM$ *(line joining centre and mid-pt. of chord ⊥ chord)*

$\therefore \angle CME=90^\circ$

$\angle BEA=\angle BAT$ *(∠ in alt. segment)*

$\angle MEA+\angle MAE=\angle CME$ *(ext. ∠ of Δ)*

$\angle BAT+\angle MAE=90^\circ$

$\angle MAE=90^\circ-\angle BAT$

$\angle BAE=90^\circ$ *(∠ in semi-circle)*

$\angle BAC+(90^\circ-\angle BAT)=90^\circ$

$\angle BAC=\angle BAT$

$\angle BDA=\angle BAT$ *(∠ in alt. segment)*

In ΔABD,

$\angle ABD+\angle BDA+\angle BAD=180^\circ$ *(∠ sum of Δ)*

$80^\circ+\angle BAT+(38^\circ+\angle BAT)=180^\circ$

$2\angle BAT=62^\circ$

$\angle BAT=31^\circ$

40. B	$\begin{cases} x^2+y^2-2x+8y+k=0 \ldots (1) \\ x+y+1=0 \ldots (2) \end{cases}$ From (2), $x=-(y+1) \ldots (3)$ Sub (3) into (1), $(y+1)^2+y^2+2(y+1)+8y+k=0$ $2y^2+12y+(k+3)=0 \ldots (4)$ y-coordinate of the mid-point of $PQ = \dfrac{\textit{sum of roots of } (4)}{2}$ $= \dfrac{-\frac{12}{2}}{2}$ $=-3$ Sub $y=-3$ into (3) $\therefore$ The coordinates of M are $(2, -3)$.
41. D	For I: Slope of $OB=-3$ Equation of line $\perp OB$ passes via A: $y-k=\frac{1}{3}(x+8)$ $3y-3k-8=x \quad \ldots(1)$ Slope of $OA = -\frac{k}{8}$ Equation of line $\perp OA$ passes via B: $y+6=\frac{8}{k}(x-2)$ $\frac{ky+6k+16}{8}=x \quad \ldots(2)$ (1) − (2): $y=\frac{80+20k}{24-k}$ $\therefore$ The y-coordinate of the orthocentre of ΔOAB is $\frac{80+30k}{24-k}$. $\therefore$ I is correct.

	For II: mid-pt of $OB = (1, -3)$ Equation of line $\perp OB$ passes via mid-pt of OB: $y+3=\frac{1}{3}(x-1)$ $3y+10=x \quad \ldots(3)$ mid-pt of $OB = (-4, \frac{k}{2})$ Equation of line $\perp OA$ passes via mid-pt of OA: $y-\frac{k}{2}=\frac{8}{k}(x+4)$ $\frac{2ky-k^2-64}{16}=x \quad \ldots(4)$ (3) − (4): $y=\frac{k^2+224}{2k-48}$ $\therefore$ The y-coordinate of the circumcentre of ΔOAB is $\frac{k^2+224}{2k-48}$. $\therefore$ II is correct. For III: The y-coordinate of the centroid of $\Delta OAB = \frac{k+(-6)+0}{3}$ $= \frac{k-6}{3}$ III is also true. $\therefore$ D is the answer.
42. B	${}_{25}C_5\left({}_{10}C_3\right)+{}_{25}C_6\left({}_{10}C_2\right)+{}_{25}C_7\left({}_{10}C_1\right)+{}_{25}C_8$ $=20233675$
43. A	$\frac{{}_6C_3\left({}_8C_4\right)5!3!+{}_6C_4\left({}_8C_3\right)4!4!+{}_6C_5\left({}_8C_2\right)3!5!+{}_8C_1 2!6!}{{}_{14}P_7}$ $=\frac{94}{1001}$
44. B	$\sigma=\frac{66-45}{3}=7$ Boy's mark $=45+7(-5)=10$

45. C	∵ Mean of $\{a,b,c,d,e,f\}=c$ ∴ the mean remains unchanged after removing the datum c. ∴ $m_2=c$ ∴ I is true. ∵ When a datum equal to the mean is removed, neither the meximum vlaue nor the minimum value changes. ∴ The range remains unchanged. ∴ $r_2=r_1$ ∴ II is true. ∵ When a datum equal to the mean is removed(number of date reduced from 6 to 5), the standard deviation will increase. Variance will increase too. ∴ III is not true.