2021-22 Mock
MATH CP

PAPER 1

CARMEL SECONDARY SCHOOL
HONG KONG DIPLOMA OF SECONDARY EDUCATION
MOCK EXAMINATION

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

8.15 am – 10.30 am ($2\frac{1}{4}$ hours)
This paper must be answered in English

Please stick your barcode label here.

Candidate Number

S6( )

Name: _____ ( )

Please circle your Math Group			
C1	C2	C3	C4
Mr. CH Wong	Mr. CY Leung	Mr. KK Wong	Mr. CH Wong

Date: 28 January 2022

No. of pages: 24

Total marks: 105

INSTRUCTIONS

1. Write your Name, Class, Class Number and circle your Math Group in the space provided on Page 1.
2. This paper consists of THREE sections, A(1), A(2) and B.
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name, Class and Class Number on each sheet and put them INSIDE this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

* A 0 3 0 E 0 0 1 *

SECTION A(1) (35 marks)

1. Simplify $(a^5b)(a^{-3}b^7)^4$ and express your answer with positive indices. (3 marks)

2. Make p the subject of the formula $7(p-2q)=3p-22$. (3 marks)

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3. Factorize

(a) $3a^2 - 7a + 4$

(b) $3a^6 - 7a^4 + 4a^2$

(4 marks)

4. The number of members of Fitness Centre A is 4 times that of Fitness Centre B. If 315 members of Fitness Centre A transfer to Fitness Centre B, the number of Fitness Centre B is 2 times that of Fitness Centre A. Find the total number of members of two fitness centres. (4 marks)

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5. (a) Find the range of values of x which satisfy both $\frac{12(2-x)}{5} \leq 7x - 14$ and $x - 5 < 4$.

(b) Write down the greatest integer satisfying both inequalities in (a).

(4 marks)

6. The marked price of a doll is higher than its cost by 130% . A profit of \$134.4 is made by selling the doll at a discount of 20% on its marked price. Find the marked price of the doll. (4 marks)

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7. The coordinates of the points A and B are $(-22, -14)$ and $(-4, 18)$ respectively. A is rotated anticlockwise about the origin through $90°$ to A'. B' is the reflection images of B with respect to the x-axis.

(a) Write down the coordinates of A' and B'.

(b) Prove that A, A' and B' are collinear.

(4 marks)

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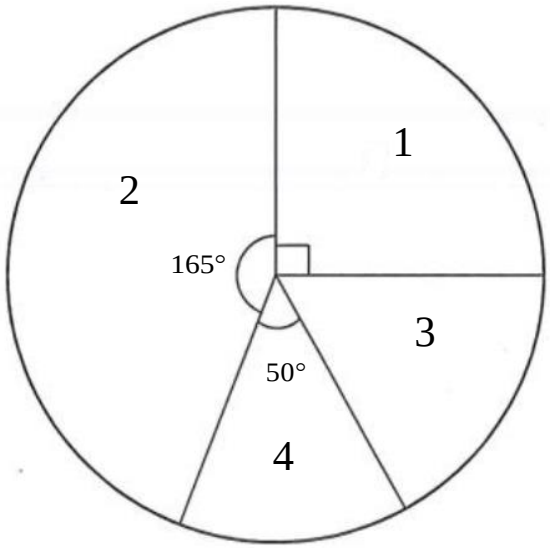
8. The pie chart below shows the distribution of the numbers of subjects applied by a group of students in a tutorial class.



(a) Find the mean of the distribution.

(b) If 21 students have applied for at least 3 subjects, find the total number of students in the tutorial class.

(c) If 2 students left the tutorial class, is it possible that the angle of the sector representing students who have applied for 2 subjects less than $156°$.

(5 marks)

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9. In Figure 1, $ABCD$ is a circle. It is given that $BC = DC$. AC and BD intersect at the point E.

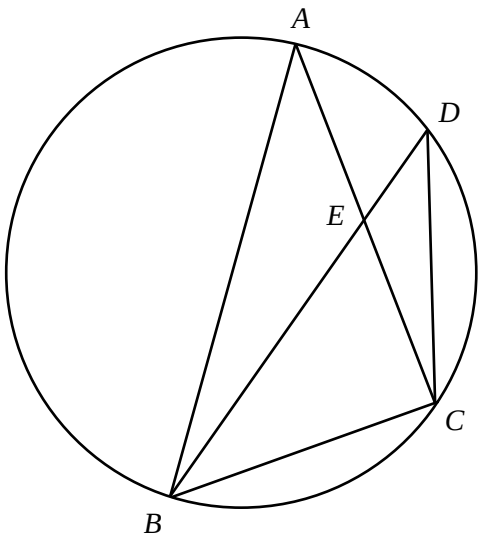

Figure 1

(a) Prove that $\Delta ABC \sim \Delta BEC$.

(b) If $BC = 12$ and $EC = 9$, find AE.

(4 marks)

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SECTION A(2) (35 marks)

10. It is given that $f(x)$ is partly constant and partly varies as $(2x-3)^2$. Suppose that $f(3)=31$ and $f(-1)=111$

(a) Find $f(5)$. (4 marks)

(b) Solve $f(x)=6$. (2 marks)

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11. The table below shows the distribution of the numbers of points got by a group of students in a competition.

Number of points got	1	2	3	4	5
Number of students	q	6	18	10	2

It is given that q is a positive integer.

(a) If the mode of the distribution is 3, write down the greatest possible value of q. (1 marks)

(b) It is given that the median of the distribution is 3.

(i) Write down the least possible value of q.

(ii) Write down the greatest possible value of q.

(2 marks)

(c) It is given that the mean of the distribution is 3.

(i) Find the value of q.

(ii) Write down the inter-quartile range and the standard deviation of the distribution.

(4 marks)

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12. The height and the base radius of a solid right circular cone are 72 cm and 30 cm respectively. The circular cone is divided into three parts by two planes which are parallel to its base. The ratio of the height of the lower part to the height of the middle part to the height of the upper part is $3:2:1$.

Express, in terms of π,

(a) the volume of the middle part of the circular cone; (3 marks)

(b) the curved surface area of the middle part of the circular cone. (3 marks)

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13. $f(x)$ is a cubic polynomial. When $f(x)$ is divided by $2x^2 + x - 3$, the remainder is $-8x + 14$. When $f(x)$ is divided by $x + 1$, the remainder is 20 and $f(x)$ is divisible by $3x - 2$.

(a) Find the quotient when $f(x)$ is divided by $2x^2 + x - 3$. (3 marks)

(b) Amy claims that the roots of the equation $f(x) = 0$ are all rational. Do you agree? Explain your answer.

(4 marks)

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14. The coordinates of the points A and B are $(-8, 15)$ and $(-4, 7)$ respectively. Let P be a moving point in the rectangular coordinate plane such that $AP = AB$. Denote Γ the locus of P.

(a) Find the equation of Γ. (2 marks)

(b) Let Q be a moving point in the rectangular coordinate plane such that $AQ = BQ$. Denote the locus of Q by Φ.

(i) Describe the geometric relationship between Φ and line segment AB.

(ii) Find the equation of Φ.

(iii) Suppose that Φ cuts Γ at C and D. Someone claims that the area of $ACBD$ exceeds 70. Is the claim correct? Explain your answer.

(7 marks)

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SECTION B (35 marks)

15. There are 7 vans and 5 motorbikes in a car park. If 6 vehicles are randomly selected from the car park at the same time.

    (a) Find the probability that equal number of vans and motorbikes are selected. (2 marks)

    (b) Find the probability that more motorbike(s) are selected than van(s) . (2 marks)

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16. Let $A(n)$ be the nth term of an arithmetic sequence. It is given that $A(4) = 19$ and $A(15) = 52$.

(a) Express $A(1) + A(2) + A(3) + \cdots + A(n)$ in terms of n. (4 marks)

(b) It is given that $B(n) = 9^{A(n)}$ for any positive integer n. Find the least value of n such that $\log_3 \left(B(1)\, B(2)\, B(3) \cdots B(n) \right) > 6000$. (3 marks)

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17. In Figure 2 , A , B and C are points on a circle. TA and TB are tangent to the circle at A and B respectively. D is a point on TA such that $DB \perp TA$ and DB intersects the circle at E . Let $\angle ATB = \theta$.

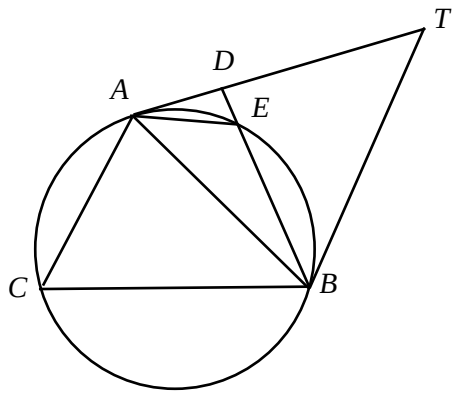

Figure 2

(a) Prove that $\angle ABE = \frac{\theta}{2}$. (2 marks)

(b) TA and BC are produced to meet at F . Fred claims that if $\angle DAE = \angle AFC = 18°$, ΔABC is an isosceles triangle. Do you agree? Explain your answer. (3 marks)

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18. Let $f(x) = 3x^2 + 9kx + 9k^2 + 1$, where k is a positive constant.

(a) Using the method of completing the square, express, in terms of k, the coordinates of the vertex P of the graph of $y = f(x)$. (2 marks)

(b) The graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ with respect to the y-axis and then translating the resulting graph rightwards by $7k$ units. The graph of $y = h(x)$ is obtained by reflecting the graph of $y = f(x)$ with respect to the x-axis and then translating the graph leftwards by 5 units. Let Q and R be the vertices of the graphs of $y = g(x)$ and $y = h(x)$ respectively.

(i) Express Q and R in terms of k. (2 marks)

(ii) Denote D and E be the circumcentre and orthocentre of ΔPQR respectively. Find the values of k such that D, E and P are collinear. (3 marks)

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19. Figure 3(a) shows a pentagonal paper card where $AE = 18$, $BC = 33$ and $CD = 65$. It is given that $\angle ECD = 71°$, $\angle CDE = 62°$ and $\angle ABC = \angle EAB = 90°$.

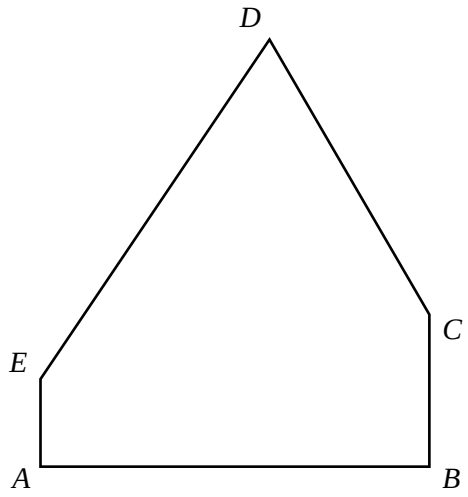

Figure 3(a)

(a) Find CE and $\angle BCE$. (3 marks)

(b) The paper card is folded along CE and placed such that the plane $ABCE$ is vertical to the horizontal ground with AB on the horizontal ground and the vertex D touches the horizontal ground as shown in Figure 3(b). CE produced meets the horizontal ground at F .

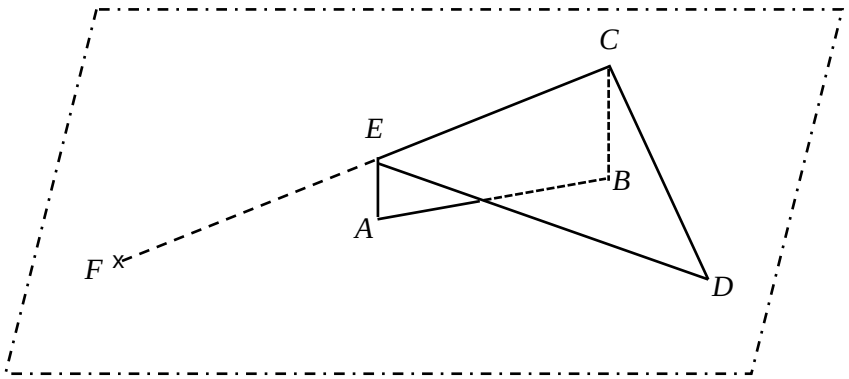

Figure 3(b)

(i) Find EF .

(ii) Denote α the angle between the plane CED and the horizontal ground . Find α .

(iii) Denote β the angle between the plane CED and the plane $ABCE$. Cherry claims that $\beta > 60°$. Do you agree? Explain your answers.

(9 marks)

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END OF PAPER

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