2021-22 Mock
MATH CP

PAPER 1

CARMEL SECONDARY SCHOOL
HONG KONG DIPLOMA OF SECONDARY EDUCATION
MOCK EXAMINATION

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

8.15 am – 10.30 am ($2\frac{1}{4}$ hours)
This paper must be answered in English

Please stick your barcode label here.

Candidate Number								

S6( )

Name: ____________________ ( )

Please circle your Math Group			
C1	C2	C3	C4
Mr. CH Wong	Mr. Leung	Mr. KK Wong	Mr. CH Wong

Date: 28 January 2022

No. of pages: 24

Total marks: 105

INSTRUCTIONS

1. Write your Name, Class, Class Number and circle your Math Group in the space provided on Page 1.
2. This paper consists of THREE sections, A(1), A(2) and B.
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Name, Class and Class Number on each sheet and put them INSIDE this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

A030E001

SECTION A(1) (35 marks)

1. Simplify $(a^5b)(a^{-3}b^7)^4$ and express your answer with positive indices. (3 marks)

$(a^5b)(a^{-3}b^7)^4$

$= a^5b \times a^{-12}b^{28}$ 1M for $(ab)^m = a^m b^m$ or $(a^m)^n = a^{mn}$

$= \frac{b^{29}}{a^7}$ 1A + 1M for $a^m \times a^n = a^{m+n}$ or $a^{-m} = \frac{1}{a^m}$

2. Make p the subject of the formula $7(p-2q)=3p-22$. (3 marks)

$7(p-2q) = 3p-22$

$7p-14q = 3p-22$ 1M for removing bracket

$4p = 14q-22$ 1M for p on 1 side

$p = \frac{14q-22}{4}$

$p = \frac{7q-11}{2}$ 1A

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3. Factorize

(a) $3a^2 - 7a + 4$

(b) $3a^6 - 7a^4 + 4a^2$

(4 marks)

(a) $3a^2 - 7a + 4$
$= (3a - 4)(a - 1)$ 1A

(b) $3a^6 - 7a^4 + 4a^2$
$= a^2(3a^4 - 7a^2 + 4)$ 1M for common factor
$= a^2(3a^2 - 4)(a^2 - 1)$ 1M for using (a)
$= a^2(3a^2 - 4)(a - 1)(a + 1)$ 1A

4. The number of members of Fitness Centre A is 4 times that of Fitness Centre B. If 315 members of Fitness Centre A transfer to Fitness Centre B, the number of Fitness Centre B is 2 times that of Fitness Centre A. Find the total number of members of two fitness centres. (4 marks)

Let the original number of members of fitness centre B be y.

$2(4y - 315) = y + 315$ 1M for number $\boxed{\pm 315}$ + 1M for $\boxed{4y}$ + 1A
$8y - 630 = y + 315$
$7y = 945$
$y = 135$

The total number of members
$= 135 + 4(135) = 675$ 1A

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5. (a) Find the range of values of x which satisfy both $\frac{12(2-x)}{5} \leq 7x-14$ and $x-5<4$.

(b) Write down the greatest integer satisfying both inequalities in (a).

(4 marks)

(a)	$\frac{12(2-x)}{5} \leq 7x-14$ and	$x-5<4$	
	$24-12x \leq 35x-70$	$x<9$	1A
	$94 \leq 47x$		
	$x \geq 2$		1A
	$\therefore 2 \leq x < 9$		1M (consistent with above)
(b)	8		1A

6. The marked price of a doll is higher than its cost by 130% . A profit of \$134.4 is made by selling the doll at a discount of 20% on its marked price. Find the marked price of the doll. (4 marks)

Let the marked price be $\$y$.

$y \div (1+130\%) + 134.4 = y \times (1-20\%)$ — 1M for cost $\times (1+130\%)$ = marked price; 1M for marked price $\times (1-20\%)$ = selling price

$\frac{y}{2.3} + 134.4 = 0.8y$

$y + 309.12 = 1.84y$

$0.84y = 309.12$ — 1M for grouping y

$y = 368$

The marked price $= \$368$ — 1A

Alternative

Let marked price be $\$m$ and the cost be $\$c$

$\begin{cases} m = (1+130\%)c \\ (1-20\%)m = c + 134.4 \end{cases}$

$\begin{cases} m = 2.3c & \_(1) \\ 0.8m = c + 134.4 & \_(2) \end{cases}$

1M for cost $\times (1+130\%)$ = marked price
1M for marked price $\times (1-20\%)$ = selling price

Put (1) in to (2)

$(0.8)2.3c = c + 134.4$

$0.84c = 134.4$ — 1M for grouping

$c = 160$

Marked price $= 2.3(160) = \$368$ — 1A

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7. The coordinates of the points A and B are $(-22, -14)$ and $(-4, 18)$ respectively. A is rotated anticlockwise about the origin through $90°$ to A'. B' is the reflection images of B with respect to the x-axis.

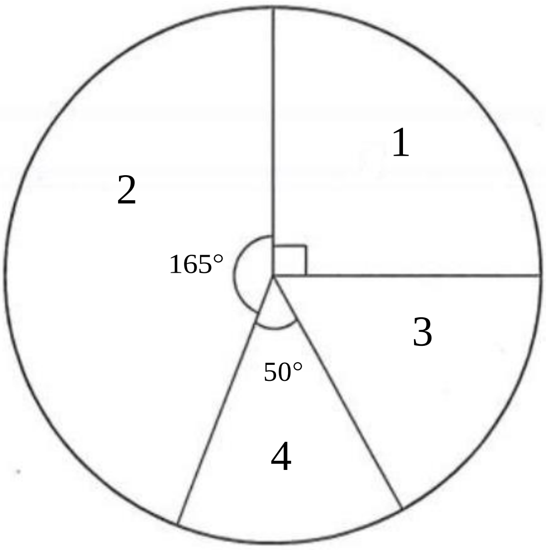
(a) Write down the coordinates of A' and B'.

(b) Prove that A, A' and B' are collinear.

(4 marks)

(a)	$A' = (14, -22)$	1A
	$B' = (-4, -18)$	1A
(b)	$m_{AA'} = \frac{-14+22}{-22-14} = -\frac{2}{9}$	1M for slope of AA' or $A'B'$ or AB'
	$m_{A'B'} = \frac{-22+18}{14+4} = -\frac{2}{9}$	
	$\because m_{AA'} = m_{A'B'}$	
	$\therefore$ A, A' and B' are collinear.	1A (f.t.)
	Alternative	
(b)	Equation of $A'B'$:	
	$\frac{y+22}{x-14} = \frac{-18+22}{-4-14}$	1M equation of straight line
	$-18y - 396 = 4x - 56$	
	$2x + 9y + 170 = 0$	
	Put A into the equation,	
	L.H.S. = $2(-22) + 9(-14) + 170$	
	$= 0 =$ R.H.S.	
	$\therefore$ A, A' and B' are collinear.	1A (f.t.)

8. The pie chart below shows the distribution of the numbers of subjects applied by a group of students in a tutorial class.



(a) Find the mean of the distribution.

(b) If 21 students have applied for at least 3 subjects, find the total number of students in the tutorial class.

(c) If 2 students left the tutorial class, is it possible that the angle of the sector representing students who have applied for 2 subjects less than 156° .

(5 marks)

(a) The required mean

$= \frac{90}{360} \times 1 + \frac{165}{360} \times 2 + \frac{360-165-50-90}{360} \times 3 + \frac{50}{360} \times 4$ 1M

$= 2.18$ 1A

(b) Let y be the total number of students in the tutorial class

$y \times \frac{50+55}{360} = 21$

$y = 72$ 1A

(c) If 2 students left the tutorial class, the new number of students $= 70$ and the smallest possible new number of students have applied for 2 subjects

$= 72 \times \frac{165}{360} - 2 = 31$ 1M for original number – 2

The required angle $= \frac{31}{70} \times 360° = 159.42° > 156°$

$\therefore$ It is impossible. 1A (f.t.)

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9. In Figure 1, $ABCD$ is a circle. It is given that $BC = DC$. AC and BD intersect at the point E.

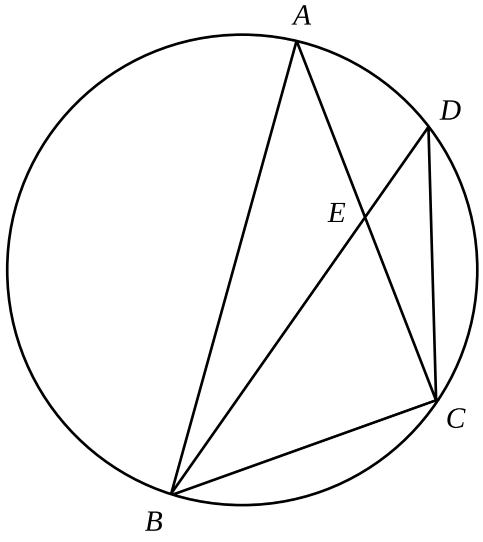

Figure 1

(a) Prove that $\Delta ABC \sim \Delta BEC$.

(b) If $BC = 12$ and $EC = 9$, find AE.

(4 marks)

(a)	$\because BC = DC$		
	$\therefore \angle CBD = \angle CDB$	(base $\angle$s, isos. Δ)	2A for correct steps and reasons
	$\angle BAC = \angle CDB$	($\angle$ in the same segment)	1A for correct steps
		or (equal $\angle s$, equal chords)	
	$\angle BAC = \angle EBC$	(proved)	
	$\angle ACB = \angle BCE$	(common $\angle$)	
	$\angle ABC = \angle BEC$	($\angle$ sum of Δ)	
	$\therefore \Delta ABC \sim \Delta BEC$	(AAA) / (AA)	
(b)	$\frac{AC}{BC} = \frac{BC}{EC}$	(corr. sides, ~Δs)	
	$\frac{AC}{12} = \frac{12}{9}$		1A
	$AC = 16$		
	$AE = 16 - 9 = 7$		1A

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SECTION A(2) (35 marks)

10. It is given that $f(x)$ is partly constant and partly varies as $(2x-3)^2$. Suppose that $f(3)=31$ and $f(-1)=111$

(a) Find $f(5)$. (4 marks)

(b) Solve $f(x)=6$. (2 marks)

(a)	Let $f(x)=h(2x-3)^2+k$	1A
	$f(3)=31$ $\quad f(-1)=111$	1M for either 1 substitution
	$9h+k=31$ _(1) $\quad 25h+k=111$ _(2)	
	By solving,	
	$h=5,\ k=-14$	1A
	$f(x)=5(2x-3)^2-14$	
	$f(5)=5(10-3)^2-14=231$	1A
(b)	$f(x)=6$	
	$5(2x-3)^2-14=6$	1M for using (a)
	$5(4x^2-12x+9)-20=0$	
	$20x^2-60x+25=0$	
	$(2x-5)(2x-1)=0$	
	$\therefore x=\frac{1}{2}$ or $x=\frac{5}{2}$	1A

Answers written in the margins will not be marked.

11. The table below shows the distribution of the numbers of points got by a group of students in a competition.

Number of points got	1	2	3	4	5
Number of students	q	6	18	10	2

It is given that q is a positive integer.

(a) If the mode of the distribution is 3 , write down the greatest possible value of q . (1 marks)

(b) It is given that the median of the distribution is 3 .

(i) Write down the least possible value of q .

(ii) Write down the greatest possible value of q .

(2 marks)

(c) It is given that the mean of the distribution is 3 .

(i) Find the value of q .

(ii) Write down the inter-quartile range and the standard deviation of the distribution.

(4 marks)

(a) 17 1A

(bi) 1 1A

(bii) 23 1A

(ci) $\frac{q+12+54+40+10}{q+6+18+10+2} = 3$

$q + 116 = 3q + 108$ 1M

$2q = 8$

$q = 4$ 1A

(cii) Inter-quartile range

$= 4 - \frac{3+2}{2} = 1.5$ 1A

Standard deviation

$= 1$ 1A

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12. The height and the base radius of a solid right circular cone are 72 cm and 30 cm respectively. The circular cone is divided into three parts by two planes which are parallel to its base. The ratio of the height of the lower part to the height of the middle part to the height of the upper part is $3:2:1$.

Express, in terms of π ,

(a) the volume of the middle part of the circular cone; (3 marks)

(b) the curved surface area of the middle part of the circular cone. (3 marks)

(a) The volume of the middle part

$=\frac{1}{3}\pi(30)^2(72)\left(\frac{3^3-1^3}{6^3}\right)$ 1M for volume of cone $V=\frac{1}{3}\pi r^2h$ + 1M for $\left(\frac{l_1}{l_2}\right)^3$

$=2600\pi\ \text{cm}^3$ 1A

(b) The curved surface area of the middle part

$=\pi(30)\sqrt{72^2+30^2}\left(\frac{3^2-1^2}{6^2}\right)$ 1M for curved surface area $=\pi rl$ (l including pyth. Thm.)

+ 1M for $\left(\frac{A_1}{A_2}\right)^2$

$=520\pi\ \text{cm}^2$ 1A

Alternative

(a) Let the radius and height of the smallest cone be r_1 and h_1 respectively.
Let the radius and height of the middle-size cone be r_2 and h_2 respectively.

$r_1=\frac{1}{6}\times 30=5\ \text{cm}$ 1M for 1 set of radius and height correct

$h_1=\frac{1}{6}\times 72=12\ \text{cm}$

$r_2=\frac{3}{6}\times 30=15\ \text{cm}$

$h_2=\frac{3}{6}\times 72=36\ \text{cm}$

The volume of the middle part

$=\frac{1}{3}\pi(15)^2(36)-\frac{1}{3}\pi(5)^2(12)$ 1M for volume of cone $V=\frac{1}{3}\pi r^2h$

$=2600\pi\ \text{cm}^3$ 1A

(b) Let l_1 and l_2 be the slant height of the smallest and middle-size cone respectively.
The curve surface area of the middle part

$=\pi(15)\sqrt{36^2+15^2}-\pi(5)\sqrt{12^2+5^2}$ 1M for 1 set of radius and slant height correct

1M for curved surface area $=\pi rl$

$=520\pi\ \text{cm}^2$ 1A

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13. $f(x)$ is a cubic polynomial. When $f(x)$ is divided by $2x^2+x-3$, the remainder is $-8x+14$. When $f(x)$ is divided by $x+1$, the remainder is 20 and $f(x)$ is divisible by $3x-2$.

(a) Find the quotient when $f(x)$ is divided by $2x^2+x-3$. (3 marks)

(b) Amy claims that the roots of the equation $f(x)=0$ are all rational. Do you agree? Explain your answer.

(4 marks)

(a)

Let $Ax+B$ be the required quotient

$f(x)=(Ax+B)(2x^2+x-3)-8x+14$ **1A**

$f(-1)=20$

$(-A+B)\left[2(-1)^2-1-3\right]-8(-1)+14=20$

$(-A+B)(-2)=-2$

$-A+B=1$ ....(1)

$f\left(\frac{2}{3}\right)=0$

1M either (for either of the two substitutions above)

$\left(\frac{2}{3}A+B\right)\left[2\left(\frac{2}{3}\right)^2+\frac{2}{3}-3\right]-8\left(\frac{2}{3}\right)+14=0$

$\left(\frac{2}{3}A+B\right)\left(\frac{-13}{9}\right)=\frac{-26}{3}$

$\frac{2}{3}A+B=6$ ....(2)

Solving (1) *and* (2),

$A=3,\ B=4$

Required quotient : $3x+4$ **1A**

(b)

$f(x)=(3x+4)(2x^2+x-3)-8x+14$

$=6x^3+8x^2+3x^2-9x+4x-12-8x+14$

$=6x^3+11x^2-13x+2$ **1M for expanded cubic equation**

$=(3x-2)(2x^2+5x-1)$ **1M for $(3x-2)(px^2+qx+r)$**

For $f(x)=0$

$3x-2=0$ *or* $2x^2+5x-1=0$

$x=\frac{2}{3}$ *or* $x=\frac{-5\pm\sqrt{5^2-4(2)(-1)}}{2(2)}$ **1M quad. formula**

$x=\frac{2}{3}$ *or* $x=\frac{-5\pm\sqrt{33}}{4}$

$\because \frac{-5\pm\sqrt{33}}{4}$ are not rational. **1 f.t. (short explanation needed)**

$\therefore$ *No*

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14. The coordinates of the points A and B are $(-8, 15)$ and $(-4, 7)$ respectively. Let P be a moving point in the rectangular coordinate plane such that $AP = AB$. Denote Γ the locus of P.

(a) Find the equation of Γ. (2 marks)

(b) Let Q be a moving point in the rectangular coordinate plane such that $AQ = BQ$. Denote the locus of Q by Φ.

(i) Describe the geometric relationship between Φ and line segment AB.

(ii) Find the equation of Φ.

(iii) Suppose that Φ cuts Γ at C and D. Someone claims that the area of $ACBD$ exceeds 70. Is the claim correct? Explain your answer.

(7 marks)

(a) The equation of Γ:

$(x+8)^2 + (y-15)^2 = (-8+4)^2 + (15-7)^2$ — 1M for standard form (only for mistake in +/-)

$(x+8)^2 + (y-15)^2 = 80$ — 1A

$x^2 + y^2 + 16x - 30y + 209 = 0$

(bi) Φ is the perpendicular bisector of line segment AB. — 1A

(bii) mid-point of AB $= (-6, 11)$ — 1A for mid point

The equation of Φ:

$\frac{y-11}{x+6} \times \frac{15-7}{-8+4} = -1$ — 1M for slope x slope = -1

$-2y + 22 = -x - 6$

$x - 2y + 28 = 0$ — 1A

Alternative

Let Q be (x, y).

$AQ = BQ$ — 1A + 1M for distance AQ = distance BQ

$\sqrt{(x+8)^2 + (y-15)^2} = \sqrt{(x+4)^2 + (y-7)^2}$

$16x - 30y + 289 = 8x - 14y + 65$

$8x - 16y + 224 = 0$

$x - 2y + 28 = 0$ — 1A

(biii) radius of circle Γ: $\sqrt{80}$

$AB = \sqrt{80}$

$\frac{AB}{2} = \sqrt{20}$

Let M be the mid-point of AB.

$CM = DM = \sqrt{80 - 20} = \sqrt{60}$ — 1M for AM or CM

Area of $ACBD$

$= 4 \times \frac{1}{2}(\sqrt{20})(\sqrt{60})$ — 1M for area of triangle → rhombus

$= 69.2820$

< 70

$\therefore$ The claim is disagreed. — 1A (f.t.)

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SECTION B (35 marks)

15. There are 7 vans and 5 motorbikes in a car park. If 6 vehicles are randomly selected from the car park at the same time.

(a) Find the probability that equal number of vans and motorbikes are selected. (2 marks)

(b) Find the probability that more motorbike(s) are selected than van(s) . (2 marks)

Required Probability $= \dfrac{C_3^7 \times C_3^5}{C_6^{12}}$

$= \dfrac{25}{66}$

1M for $\dfrac{?}{C_6^{12}}$

1A

Required Probability $= \dfrac{C_5^5 \times C_1^7 + C_4^5 \times C_2^7}{C_6^{12}}$

$= \dfrac{4}{33}$

1M for $p_1 + p_2$

1A

16. Let $A(n)$ be the nth term of an arithmetic sequence. It is given that $A(4)=19$ and $A(15)=52$.

(a) Express $A(1)+A(2)+A(3)+\cdots+A(n)$ in terms of n . (4 marks)

(b) It is given that $B(n)=9^{A(n)}$ for any positive integer n . Find the least value of n such that $\log_3\left(B(1)\,B(2)\,B(3)\cdots B(n)\right)>6000$. (3 marks)

(a)	$a+3d=19$	1M for either 1
	$a+14d=52$	
	$\therefore d=3$ and $a=10$	1A
	$A(n)=10+(n-1)(3)=3n+7$	
	$A(1)+A(2)+A(3)+\cdots+A(n)$	
	$=\frac{n}{2}[2(10)+(n-1)(3)]$	1M for sum of AS
	$=\frac{3n^2+17n}{2}$	1A (simplified / factorized)
		Can score back from (b)
(b)	$\log_3\left(B(1)B(2)B(3)\ldots B(n)\right)>6000$	
	$\log_3 B(1)+\log_3 B(2)+\log_3 B(3)+\cdots\log_3 B(n)>6000$	1M for property of log
	$2[A(1)+A(2)+A(3)+\cdots+A(n)]>6000$	1M 2 x result in (a) > 6000
	$3n^2+17n-6000>0$	
	$n<-47.6444$ (rej.) or $n>41.9777\ldots$	
	$\therefore$ The least value of n is 42 .	1A

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17. In Figure 2 , A , B and C are points on a circle. TA and TB are tangent to the circle at A and B respectively. D is a point on TA such that $DB \perp TA$ and DB intersects the circle at E . Let $\angle ATB = \theta$.

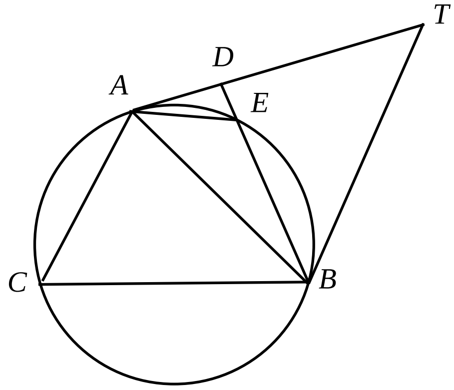

Figure 2

(a) Prove that $\angle ABE = \dfrac{\theta}{2}$. (2 marks)

(b) TA and BC are produced to meet at F . Fred claims that if $\angle DAE = \angle AFC = 18°$, ΔABC is an isosceles triangle. Do you agree? Explain your answer. (3 marks)

(a)

$\because TA = TB$ (tangent properties)

$\therefore \angle TAB = \angle TBA$ (base $\angle$s, isos. Δ)

Accept just tangent properties

In ΔTAB,

$\angle TBA = \dfrac{180° - \theta}{2}$ ($\angle$ sum of Δ)

$= 90° - \dfrac{\theta}{2}$

In ΔTDB,

$\angle TBD = 180° - 90° - \theta$ ($\angle$ sum of Δ)

$= 90° - \theta$

$\angle ABE = \angle TBA - \angle TBD$

$= \left(90° - \dfrac{\theta}{2}\right) - (90° - \theta)$

$= \dfrac{\theta}{2}$

Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

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(b) No need reasons

$\angle ABE = \angle DAE = 18°$ ($\angle$ in alt. segment)

$\frac{\theta}{2} = 18°$

$\theta = 36°$

1M (or equivalent skills)

In ΔTDB,

$\angle EBT = 180° - 90° - 36°$ ($\angle$ sum of Δ)

$= 54°$

$\angle ACB = \angle ABT$ ($\angle$ in alt. segment)

$= 54° + 18°$

$= 72°$

In ΔTFB,

$\angle ABC = 180° - 18° - 72° - 36°$ ($\angle$ sum of Δ)

$= 54°$

1A any one

In ΔABC,

$\angle CAB = 180° - 54° - 72°$

$= 54°$

$\because \angle ABC = \angle CAB = 54°$

$\therefore AC = AB$ (sides opp. equal $\angle$s)

$\therefore$ Yes, ΔABC is an isosceles. Δ

1 f.t.

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18. Let $f(x) = 3x^2 + 9kx + 9k^2 + 1$, where k is a positive constant.

(a) Using the method of completing the square, express, in terms of k, the coordinates of the vertex P of the graph of $y = f(x)$. (2 marks)

(b) The graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ with respect to the y-axis and then translating the resulting graph rightwards by $7k$ units. The graph of $y = h(x)$ is obtained by reflecting the graph of $y = f(x)$ with respect to the x-axis and then translating the graph leftwards by 5 units. Let Q and R be the vertices of the graphs of $y = g(x)$ and $y = h(x)$ respectively.

(i) Express Q and R in terms of k. (2 marks)

(ii) Denote D and E be the circumcentre and orthocentre of ΔPQR respectively. Find the values of k such that D, E and P are collinear. (3 marks)

(a) $f(x) = 3x^2 + 9kx + 9k^2 + 1$

$f(x) = 3(x^2 + 3kx) + 9k^2 + 1$

$f(x) = 3\left[x^2 + 2(x)\left(\frac{3k}{2}\right) + \left(\frac{3k}{2}\right)^2\right] - 3\left(\frac{3k}{2}\right)^2 + 9k^2 + 1$ 1M

$f(x) = 3\left(x + \frac{3k}{2}\right)^2 + \left(\frac{9}{4}k^2 + 1\right)$

∴ The required vertex $P = \left(-\frac{3k}{2}, \frac{9k^2+4}{4}\right)$ 1A

(bi) $Q = \left(\frac{17k}{2}, \frac{9k^2+4}{4}\right)$ 1A

$R = \left(-\frac{3k+10}{2}, -\frac{9k^2+4}{4}\right)$ 1M for corr. from P

(bii) If D, E, F, P are collinear, ΔPQR is an isosceles triangles where $PQ = PR$

$PQ = PR$ 1M

$\left(\frac{17k}{2} + \frac{3k}{2}\right) = \sqrt{\left(-\frac{3k}{2} + \frac{3k+10}{2}\right)^2 + \left(\frac{9k^2+4}{4} + \frac{9k^2+4}{4}\right)^2}$ 1M

$(10k)^2 = 5^2 + \left(\frac{9k^2+4}{2}\right)^2$

$0 = \frac{81k^4}{4} - 82k^2 + 29$ 1M for $ak^4 + bk^2 + c = 0$

$k^2 = 3.65787118$ or 0.391511536

$k = 1.91255619$ or -1.91255619 (rej.) or 0.625708826 or -0.625708826 (rej.)

$k = 1.91$ or 0.626 1A

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19. Figure 3(a) shows a pentagonal paper card where $AE = 18$, $BC = 33$ and $CD = 65$. It is given that $\angle ECD = 71°$, $\angle CDE = 62°$ and $\angle ABC = \angle EAB = 90°$.

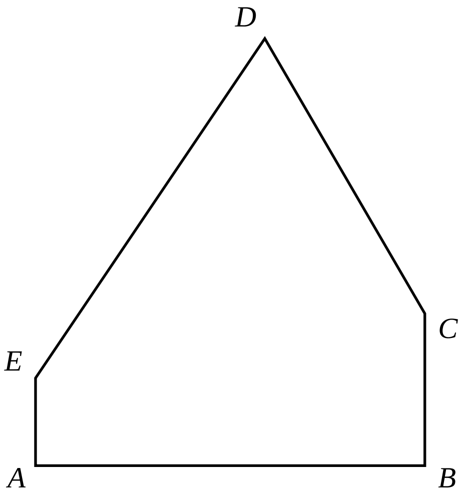

Figure 3(a)

(a) Find CE and $\angle BCE$. (3 marks)

(b) The paper card is folded along CE and placed such that the plane $ABCE$ is vertical to the horizontal ground with AB on the horizontal ground and the vertex D touches the horizontal ground as shown in Figure 3(b). CE produced meets the horizontal ground at F .

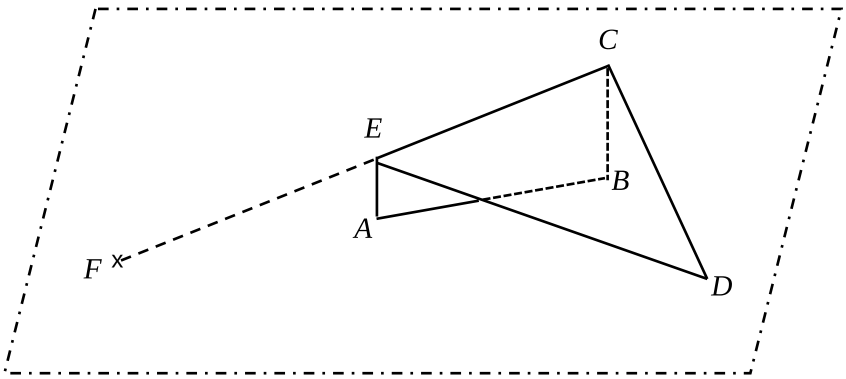

Figure 3(b)

(i) Find EF .

(ii) Denote α the angle between the plane CED and the horizontal ground . Find α .

(iii) Denote β the angle between the plane CED and the plane $ABCE$. Cherry claims that $\beta > 60°$. Do you agree? Explain your answers.

(9 marks)

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(a)

$$\frac{CE}{\sin 62^\circ} = \frac{65}{\sin(180^\circ - 62^\circ - 71^\circ)}$$ **1M**

$CE = 78.47310188$

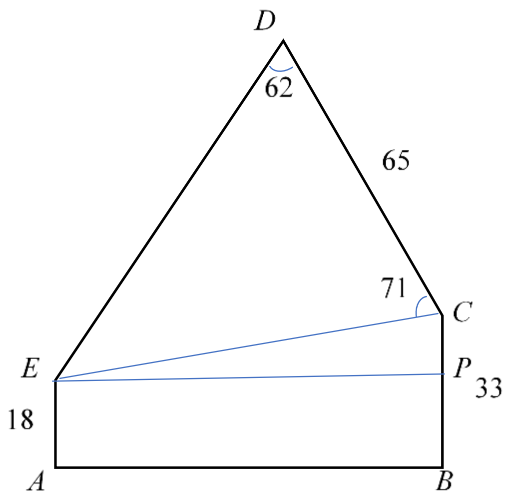
$CE = 78.5$ **1A**

Let P be the point on BC with $EP \perp BC$

$$\cos \angle BCE = \frac{33 - 18}{78.47310188}$$

$\angle BCE = 78.98019476^\circ$

$\angle BCE = 79.0^\circ$ **1A**



(bi)

$\Delta FEA \sim \Delta FCB$

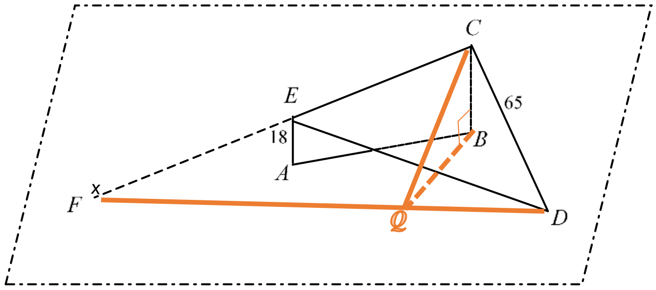
$$\frac{EF}{CF} = \frac{EA}{CB}$$

$$\frac{EF}{EF + 78.47310188} = \frac{18}{33}$$ **1M for $\frac{18}{33}$**

$15EF = 18(78.47310188)$

$EF = 94.16772225$

$EF = 94.2$ **1A**



(bii)

Let Q be a point on FD s.t. CQ is the altitude of ΔFCD.
the required angle α is $\angle BQC$. **1M**

$CF = 78.47310188 + 94.1672225 = 172.6408241$

$FD^2 = CF^2 + CD^2 - 2(CF)(CD)\cos \angle FCD$

$FD^2 = (172.6408241)^2 + 65^2 - 2(172.6408241)(65)\cos 71^\circ$

$FD = 163.471796$

$\because$ Area of $\Delta FCD = \frac{1}{2}(CF)(CD)\sin 71^\circ$

$\frac{1}{2}(FD)(CQ) = \frac{1}{2}(CF)(CD)\sin 71^\circ$ **1M**

$163.471796\, CQ = (172.6408241)(65)\sin 71$

$CQ = 64.90588688$

$$\sin \alpha = \frac{33}{64.90588688}$$

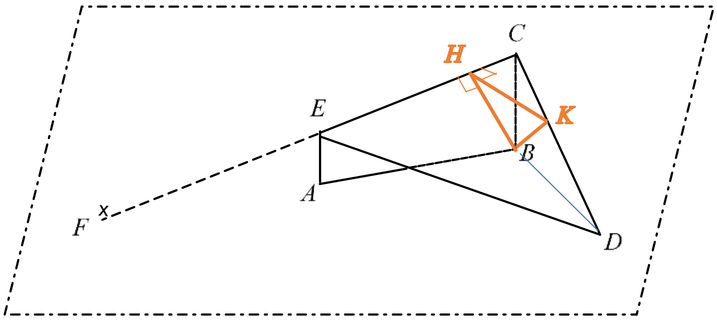
$\alpha = 30.55920673^\circ$

$\alpha = 30.6^\circ$ **1A**

(biii)

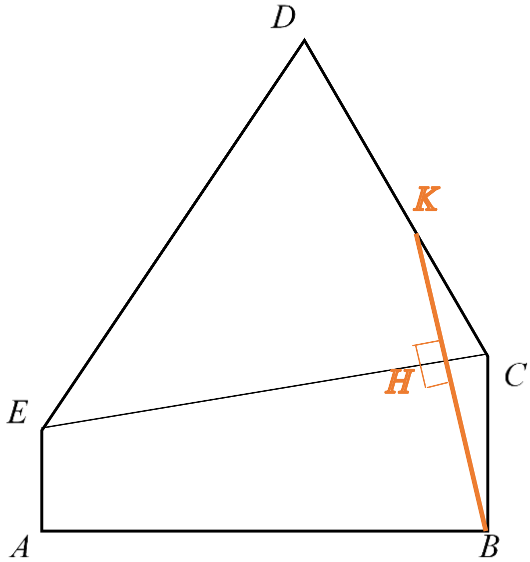
Let H be the point on CE with $BH \perp CE$
Let K be the point on CD with $HK \perp CE$
Required angle β is $\angle BHK$. **1M**

$BH = 33\sin 78.98019476° = 32.39151856$
$CH = 33\cos 78.98019476° = 6.307893892$
In ΔCHK,
$\tan 71° = \dfrac{HK}{CH}$
$HK = 6.307893892\tan 71°$
$\quad = 18.31945406$ **1M**



$\because \angle BCD = 90°$
$BD = \sqrt{65^2 - 33^2} = 56$
In ΔBCD,
$\tan \angle BCK = \dfrac{56}{33}$
$\angle BCK = 59.48976259°$
$CK = \sqrt{HK^2 + CH^2}$
$\quad = \sqrt{18.31945406^2 + 6.307893892^2}$
$\quad = 19.37503348$
$BK^2 = BC^2 + CK^2 - 2(BC)(CK)\cos \angle BCK$ **1M**
$BK^2 = 33^2 + 19.37503348^2 - 2(33)(19.37503348)\cos 59.48976259°$
$BK = 28.55134431$

$\cos\beta = \dfrac{BH^2 + HK^2 - BK^2}{2(BH)(HK)}$
$\cos\beta = \dfrac{32.39151856^2 + 18.31945406^2 - 28.55134431^2}{2(32.39151856)(18.31945406)}$
$\beta = 61.3160029°$
$\therefore YES, \quad \beta > 60°$ **1A f.t.**



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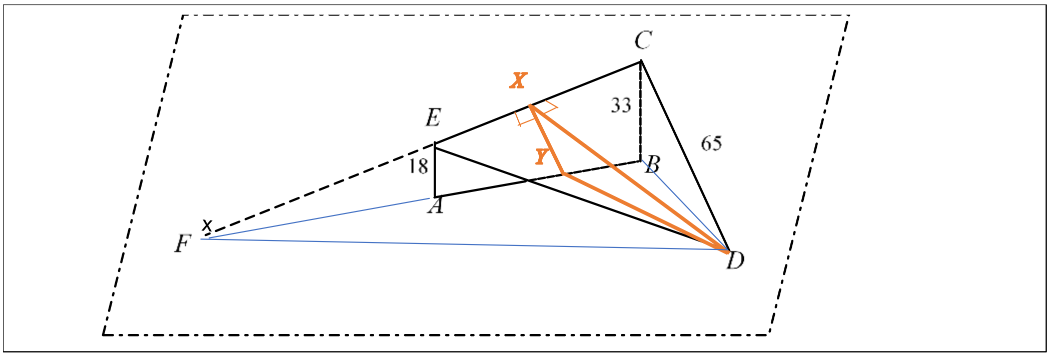
Alternative solutions



Let X be the point on CE with $DX \perp CE$

Let Y be the point on AB with $XY \perp CE$ 1M

Required angle β is $\angle DXY$.

$DX = 65\sin 71°$, $CX = 65\cos 71°$

$FB = \sqrt{CF^2 - CB^2} = \sqrt{172.6408241^2 - 33^2} = 169.4575291$

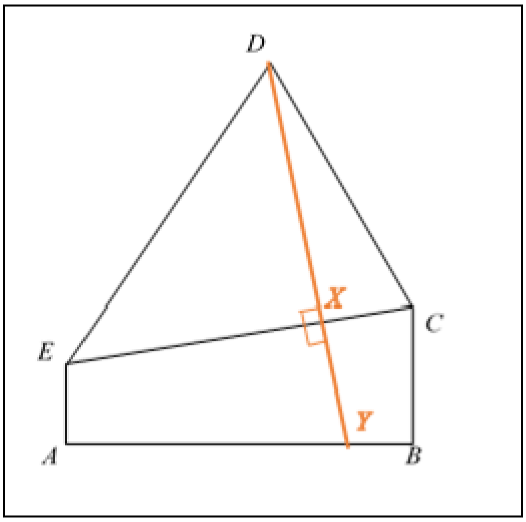
$FX = 172.6408241 - 65\cos 71° = 151.4788941$

$\because \Delta FXY \sim \Delta FBC \quad (Note: \angle FXY = \angle FBC = 90°)$

$\frac{XY}{33} = \frac{FX}{FB}$

$\frac{XY}{33} = \frac{151.4788941}{169.4575291}$ 1M

$XY = 29.4988575$



$BD = \sqrt{65^2 - 33^2} = 56$

$\cos \angle FBD = \frac{FB^2 + BD^2 - FD^2}{2(FB)(BD)}$

$\cos \angle FBD = \frac{169.4575291 + 56^2 - 163.471796^2}{2(169.4575291)(56)}$

$\angle FBD = 74.321842$

$FY = \sqrt{FX^2 + XY^2} = \sqrt{151.4788941^2 + 29.4988575^2}$

$= 154.3244567$

$BY = 169.4575291 - 154.3244567 = 15.13307239$

$YD^2 = BY^2 + BD^2 - 2(BY)(BD)\cos \angle FBD$ 1M

$YD^2 = 15.13307239^2 + 56^2 - 2(15.13307239)(56)\cos 74.321842$

$YD = 53.91651067$

$\cos \beta = \frac{XY^2 + DX^2 - YD^2}{2(XY)(DX)}$

$\cos \beta = \frac{29.4988575^2 + (65\sin 71°)^2 - 53.91651067^2}{2(29.4988575)(65\sin 71°)}$

$\beta = 61.3160029°$ 1A f.t.

$\therefore YES, \quad \beta > 60°$

END OF PAPER

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