2021-22 Mock
MATH CP
PAPER 2

CARMEL SECONDARY SCHOOL
HONG KONG DIPLOMA OF SECONDARY EDUCATION
MOCK EXAMINATION

MATHEMATICS Compulsory Part
PAPER 2
(Written Solution)

11 : 00 am – 12 : 15 pm (1¼ hours)

S6 ( )

Name: ____________________ ( )

Please circle your Math Group			
C1	C2	C3	C4
Mr CH Wong	Mr Leung	Mr KK Wong	Mr CH Wong

Date: 28 Jan 2022

No. of pages: 15

Total marks: 45

INSTRUCTIONS

1. Read carefully the instructions on the Answer Sheet.
2. When told to open this book, you should check that all the questions are there. Look for the words '**END OF PAPER**' after the last question.
3. All questions carry equal marks.
4. **ANSWER ALL QUESTIONS**. You are advised to use an HB pencil to mark all the answers on the Answer Sheet, so that wrong marks can be completely erased with a clean rubber. You must mark the answers clearly; otherwise you will lose marks if the answers cannot be captured.
5. You should mark only **ONE** answer for each question. If you mark more than one answer, you will receive **NO MARKS** for that question.
6. No marks will be deducted for wrong answers.

There are 30 questions in Section A and 15 questions in Section B.
The diagrams in this paper are not necessarily drawn to scale.
Choose the best answer for each question.

Section A

1. $\frac{3^{4n}(27^n)}{9^{3n}} =$

$\frac{3^{4n} \cdot 3^{3n}}{3^{6n}} = \frac{3^{7n}}{3^{6n}} = 3^n$

A. 3^{4n}

B. 3^n

C. 3^{-2n}

D. 3^{-3n}

2. If $3p(p-2q)=2p-q$, then $q=$

$3p^2 - 6pq = 2p - q$
$3p^2 - 2p = 6pq - q$
$3p^2 - p = q(6p-1)$
$\frac{3p^2-p}{6p-1} = q$

A. $\frac{3p^2-2p}{6p-1}$

B. $\frac{3p^2-2p}{6p+1}$

C. $\frac{3p^2+2p}{6p-1}$

D. $\frac{3p^2+2p}{6p+1}$

3. $h^2-4hk-12k^2-3h-6k=$

$(h-6k)(h+2k) - 3(h+2k)$
$= (h+2k)(h-6k-3)$

A. $(h-2k)(h+6k+3)$

B. $(h-2k)(h-6k+3)$

C. $(h+2k)(h-6k-3)$

D. $(h+2k)(h+6k-3)$

4. $\frac{\pi^2}{222} =$ 0.044457677

A. 0.044 (correct to 2 decimal places). × 3 dec. place

B. 0.0444 (correct to 3 significant figures). × 0.0445

C. 0.0445 (correct to 4 decimal places). ✓

D. 0.04446 (correct to 5 significant figures). × 4 sig. fig.

5. Let $f(x) = (hx+10)(x-6)+k$, where h and k are constants. If $f(-2) = f(3) = 5$, find k .

A. -23

B. -3

C. 43

D. 53

$f(-2)=5$
$(-2h+10)(-2-6)+k=5$
$16h-80+k=5$
$16h+k=85$

$f(3)=5$
$(3h+10)(3-6)+k=5$
$-9h-30+k=5$
$-9h+k=35$

$h=2, \quad k=53$

6. Let a and b be constants. If $2x^2+(a-3)x+a+b \equiv (x+4)(2x-5)$, then $b =$

$\equiv 2x^2+8x-5x-20$
$\equiv 2x^2+3x-20$

A. -26 .

B. -6 .

C. 6 .

D. 26 .

$a-3=3$
$a=6$

$a+b=-20$
$6+b=-20$
$b=-26$

7. Let $f(x)=5x^2-1$. If α is a constant, then $f(\alpha)-f(\alpha-1)=$

$= 5\alpha^2-1-[5(\alpha-1)^2-1]$
$= 5\alpha^2-1-5(\alpha^2-2\alpha+1)+1$
$= 5\alpha^2-1-5\alpha^2+10\alpha-5+1$
$= 10\alpha-5$

A. 5 .

B. $2\alpha-3$.

C. $3-10\alpha$.

D. $10\alpha-5$.

8. Let $p(x) = 2x^2 - x + c$, where c is a constant. If $p(x)$ is divisible by $x+2$, find the remainder when $p(x)$ is divided by $2x-1$.

A. -10

B. -5

C. 5

D. 10

$p(-2) = 0$
$2(-2)^2 - (-2) + c = 0$
$10 + c = 0$
$c = -10$

Remainder $= p\left(\frac{1}{2}\right)$
$= 2\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 10$
$= -10$

9. A sum of $84000 is deposited at an interest rate of 8% per annum for 5 years, compound monthly. Find the interest correct to the nearest dollar.

A. $2836

B. $33600

C. $40341

D. $41147

$84000\left(1 + \frac{0.08}{12}\right)^{5 \times 12} - 84000$
$= 41147$

10. Let a , b and c be non-zero numbers. If $2a = 3b$ and $a : c = 4 : 3$, then $\frac{a+2b}{5b-c} =$

A. $\frac{16}{27}$.

B. $\frac{24}{37}$.

C. $\frac{28}{31}$.

D. $\frac{7}{6}$.

$a : b = 3 : 2 = 12 : 8$
$a : c = 4 : 3 = 12 : 9$
$a : b : c = 12 : 8 : 9$

$\frac{a+2b}{5b-c} = \frac{12+2(8)}{5(8)-9} = \frac{28}{31}$

11. The solution of $7x - 6 \geq 5(x+4)$ and $\frac{8-5x}{3} < -19$ is

A. $x \leq 13$.

B. $x \geq 13$.

C. $x < 13$.

D. $x > 13$.

$7x - 6 \geq 5x + 20$
$2x \geq 26$
$x \geq 13$ and

$8 - 5x < -57$
$65 < 5x$
$13 < x$
$x > 13$

$\therefore x > 13$

12. It is given that w varies directly as the x and inversely as square root of y. If x is decreased by 10% and y is increased by 44%, then w

A. is increased by 34%.

B. is decreased by 25%.

C. is increased by 60%.

D. is decreased by 37.5%.

$w = \frac{kx}{\sqrt{y}}$

Let $x = 1 \rightarrow 0.9$
$y = 1 \rightarrow 1.44$
$k = 1$

New $w = \frac{1(0.9)}{\sqrt{1.44}} = 0.75$

$\frac{0.75-1}{1} = -0.25$
$= -25\%$

13. Let a_n be the nth term of a sequence. If $a_1 = 2$, $a_2 = 5$ and $a_{n+2} = a_n + 2a_{n+1}$ for any positive integer n, then $a_5 =$

A. 19.

B. 29.

C. 37.

D. 70.

$a_3 = a_1 + 2a_2 = 2 + 2(5) = 12$
$a_4 = a_2 + 2a_3 = 5 + 2(12) = 29$
$a_5 = a_3 + 2a_4 = 12 + 2(29) = 70$

14. Let h and k be real constants with $h > 0$. Which of the following statements about the graph of $y = h(k-x)^2 + h$ must be true?

I. The graph opens upwards. ($\checkmark$ $\because h > 0$)
II. The vertex of the graph is (h, k). ($\times$ Vertex: (k, h))
III. The y-intercept of the graph is positive. $\checkmark$ y-int. $= h(k-0)^2 + h$
$= hk^2 + h$
> 0 ($\because h > 0$, $k^2 \geq 0$)

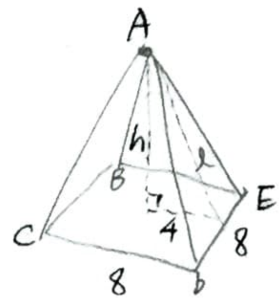
A. I and II only

B. I and III only

C. II and III only

D. I, II and III

15. The base of a solid right pyramid is a square with length 8 cm. If the total surface area is 144 cm^2, find the volume of the pyramid.



A. 64 cm^2

B. 96 cm^2

C. $\frac{320}{3}\text{ cm}^2$

D. 192 cm^2

Area of $\Delta ADE = \frac{144-64}{4}$
$= 20$

$\frac{8l}{2} = 20$
$l = 5$
$h = \sqrt{5^2-4^2} = 3$
$\therefore V = \frac{1}{3}(8^2)(3)$
$= 64$

16. In the figure, OPQ and ORS are sectors with centre O, where $OP = 10\text{ cm}$ and $OR = 18\text{ cm}$. The area of the shaded region $PQSR$ is $84\pi\text{ cm}^2$. Which of the following is/are true?

I. The angle of the sector OPQ is $135°$. ✓
II. The area of the sector ORS is $108\pi\text{ cm}^2$. ×
III. The perimeter of the shaded region $PQSR$ is $(21\pi + 16)\text{ cm}$. ✓

I. $\pi(18^2)\frac{\theta}{360°} - \pi(10^2)\frac{\theta}{360°} = 84\pi$
$\theta = 135°$

A. I only

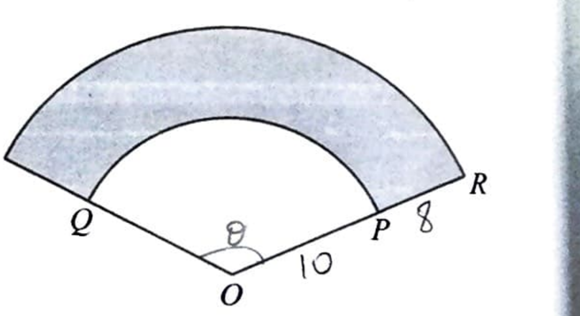
B. II only

C. I and III only

D. II and III only

II. Area of ORS
$= \pi(18^2)\frac{135°}{360°}$
$= 121.5\pi$

III. Perimeter
$= 2\pi(10)\times\frac{135°}{360°} + 2\pi(18)\times\frac{135°}{360°}$
$+ 8 + 8$
$= 21\pi + 16$



17. In the figure, $ABCD$ is a parallelogram. F is a point lying on AD such that BF produced and CD produced meet at E. It is given that $AF : FD = 5 : 3$. If the area of ΔDEF is 135 cm^2, then the area of parallelogram $ABCD$ is

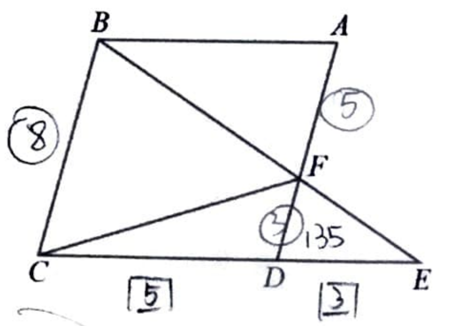
A. 720 cm^2.

B. 750 cm^2.

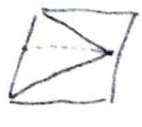
C. 1065 cm^2.

D. 1200 cm^2.

$\because AF : BC = 3 : 8$
and $\Delta EFD \sim \Delta EBC$
$DE : DC = 3 : (8-3) = 3 : 5$
$\Delta FCD : 135 \times \frac{5}{3} = 225$
$\Delta BCE : 135 \times \frac{8^2}{3^2} = 960$
$\therefore \Delta BCF = 960 - 225 - 135 = 600$
$\frac{\Delta ABF}{\Delta CDF} = \frac{5}{3}$ (same height)
$\Delta ABF = 225 \times \frac{5}{3} = 375$
$\therefore ABCD : 225 + 600 + 375 = 1200$



✓ OR by drawing
ΔBCF is half of ABCD



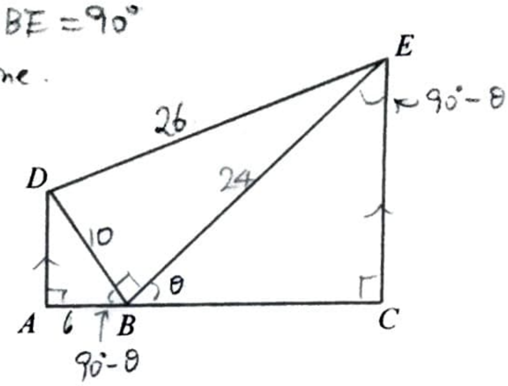
$\therefore 1200$

18. In the figure, ABC is a straight line. $AD // CE$ and $\angle DAB = 90°$. $DB = 10$ cm, $BE = 24$ cm, $DE = 26$ cm and $AB = 6$ cm. Find the perimeter of the quadrilateral $ACED$.

$\because 10^2 + 24^2 = 26^2$, $\therefore \angle DBE = 90°$

By $\angle$ sum of Δ, adj. $\angle$s on st. line.

$\Delta DAB \sim \Delta BCE$ (AAA)



A. 68 cm

B. 73.6 cm

C. 74 cm

D. 79.6 cm

$\frac{AB}{CE} = \frac{10}{24}$

$\frac{6}{CE} = \frac{10}{24}$

$CE = 14.4$

$DA = \sqrt{10^2 - 6^2} = 8$

$BC = \sqrt{24^2 - 14.4^2} = 19.2$

Perimeter $= 6 + 8 + 14.4 + 19.2 + 26$
$= 73.6$

19. The length of a ribbon is measured to be 95 cm, correct to the nearest cm. The length of a rope is measured to be 150 cm with a percentage error of 2%. Find the upper limit of the difference in the length between the rope and the ribbon.

$94.5 \sim 95.5$

A. 52.5 cm

B. 57 cm

C. 58.5 cm

D. 59.5 cm

Max. error $= 150 \times 2\% = 3$

$\Rightarrow 147 \sim 153$.

Upper limit : $153 - 94.5 = 58.5$ cm.

20. $ABCDE$ is a regular pentagon. The diagonals AC and BD intersects each other at F. Which of the following are true?

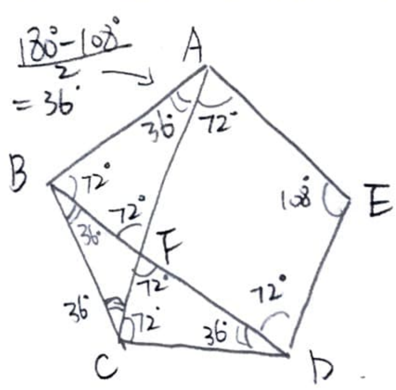
$\rightarrow$ Each angle $= \frac{(5-2)(180°)}{5} = 108°$

$\frac{180°-108°}{2} = 36°$

I. $AF = FC$. ✗ (By observation)

II. $\Delta ABF \sim \Delta ACD$. ✓ (By angles found)

III. $AEDF$ is a rhombus. ✓ $\leftarrow$ By the angles found, AEDF is a // gram, with AE = ED, it is a rhombus



A. I and II only

B. I and III only

C. II and III only

D. I, II and III

21. In the figure, $ABCDE$ is a circle. It is given that $\overset{\frown}{AE}:\overset{\frown}{DE}=5:4$, $AB \,//\, DC$, $\angle ADC = 108°$ and $\angle BCE = 80°$. Find $\angle EBC$.

A. 37°

B. 39°

C. 40°

D. 42°

$\angle DAB = 180° - 108°$ (int. ∠s, AB // DC)
$= 72°$

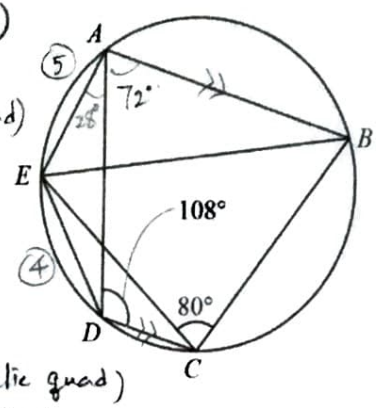
$\angle EAB = 180° - 80°$ (opp. ∠s, cyclic quad ABCE)
$= 100°$

$\angle EAB = 100° - 72° = 28°$

$\dfrac{\angle ABE}{\angle EAB} = \dfrac{5}{4}$, $\angle ABE = 35°$

$\angle EBC + 35° + 108° = 180°$ (opp. ∠s, cyclic quad ABCD)

$\angle EBC = 37°$



22. $\dfrac{\sin(270° + \theta)\cos\theta}{\cos(360° - \theta)} - \dfrac{1}{\cos(180° - \theta)} =$

A. $\sin\theta$.

B. $-\sin\theta\tan\theta$.

C. $\sin\theta\tan\theta$.

D. $\dfrac{1 - \sin\theta\cos\theta}{\cos\theta}$.

$= \dfrac{-\cos\theta \cancel{\cos\theta}}{\cancel{\cos\theta}} - \dfrac{1}{-\cos\theta}$

$= -\cos\theta + \dfrac{1}{\cos\theta}$

$= \dfrac{-\cos^2\theta + 1}{\cos\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} \cdot \sin\theta$

$= \tan\theta \sin\theta$

23. The figure shows the straight lines L_1: $ax + y = b$ and L_2: $x + cy = 1$, where a , b and c are constants. Which of the following must be true?

$m_1 = -a$
$x_1 = \dfrac{b}{a}$
$y_1 = b$

$m_2 = -\dfrac{1}{c} \Rightarrow c > 0$ to have $-$ve slope
$x_2 = 1$
$y_2 = \dfrac{1}{c}$

I. $a = b$ ✓

II. $c < 0$ ✗

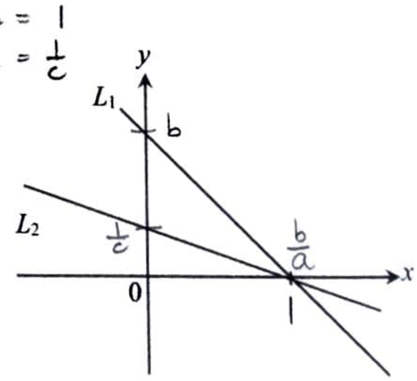
III. $ac < 1$ ✗

A. I only

B. II only

C. I and III only

D. II and III only



$x_1 = x_2$
$\dfrac{b}{a} = 1$
$a = b$

$\because y_1 > y_2$
$b > \dfrac{1}{c}$
$bc > 1$ ($\because c > 0$)
$ac > 1$ ($\because a = b$)

$122^\circ - 32^\circ = 90^\circ$ $\quad$ $302^\circ - 122^\circ = 180^\circ$

24. The polar coordinates of the points A, B and C are $(30, 32^\circ)$, $(18, 122^\circ)$ and $(24, 302^\circ)$ respectively. Find the area of ΔABC.

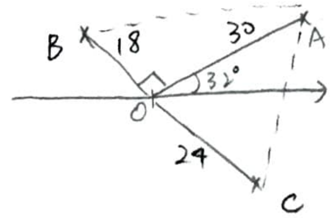
A. 216

B. 486

C. 576

D. 630

$$\text{Area} = \frac{(18+24)(30)}{2}$$
$$= 630.$$



25. It is given that A and B are two distinct points on the straight line $x - 2y + k = 0$, where k is a constant. Let P be a moving point in the rectangular coordinate plane such that $AP^2 + BP^2 = AB^2$. If the equation of the locus of P is $x^2 + y^2 - (44 + k)x + 2y + 17 = 0$, $k =$

$\downarrow$
$\angle APB = 90^\circ$
$\downarrow$
Circle with diameter AB
$\downarrow$
Centre lies on St. line passing AB

A. -20.

B. -16.

C. 16.

D. 20.

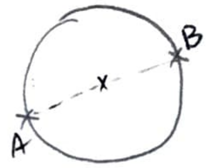
Centre: $\left(\frac{44+k}{2}, -1\right)$

$\frac{44+k}{2} - 2(-1) + k = 0$

$44 + k + 4 + 2k = 0$

$3k = -48$

$k = -16$



26. Let h and k be constants. The coordinates of the points A and B are $(3, k)$ and $(20, 8)$ respectively. The straight line $hx + 2y - 29 = 0$ is an altitude of ΔOAB that passes through A, where O is the origin. $k =$

$\hookrightarrow$ Slope $= \frac{-h}{2}$, Slope of $OB = \frac{8-0}{20-0} = \frac{2}{5}$

$\frac{-h}{2}\left(\frac{2}{5}\right) = -1$

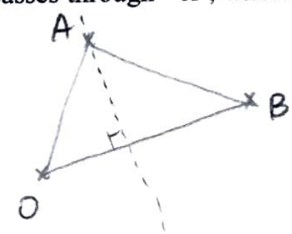
$h = 5$

Subs. $A(3, k)$ into St. line.

$5(3) + 2k - 29 = 0$

$2k = 14$

$k = 7$



A. -5.

B. 7.

C. 14.

D. 22.

27. The equation of the circle C is $2x^2 + 2y^2 - 12x - 4y + 15 = 0$. Which of the following ~~are~~ is/ are true?

I. The coordinates of the centre of C is $(6, 2)$. ×

II. The area of the circle is 2.5π. ✓

III. C cuts the y-axis at two distinct points. ×

$x^2 + y^2 - 6x - 2y + 7.5 = 0$

Centre: $(3, 1)$

Radius: $\sqrt{3^2 + 1^2 - 7.5} = \sqrt{2.5}$

Area $= \pi r^2 = 2.5\pi$

A. I only

B. II only

C. I and III only

D. II and III only

Subs. $x = 0$.

$y^2 - 2y + 7.5 = 0$

$\Delta = (-2)^2 - 4(7.5) < 0$

$\therefore$ No y-int.

28. Two numbers are randomly drawn at the same time from six cards numbered 1 , 2 , 4 , 5 , 7 , 8 respectively. Find the probability that the sum of the two numbers drawn is less than 10 .

	1	2	4	5	7	8
1	x	3	5	6	8	9
2	3	x	6	7	9	10
4	5	6	x	9	11	12
5	6	7	9	x	12	13
7	8	9	11	12	x	15
8	9	10	12	13	15	x

A. $\frac{3}{10}$

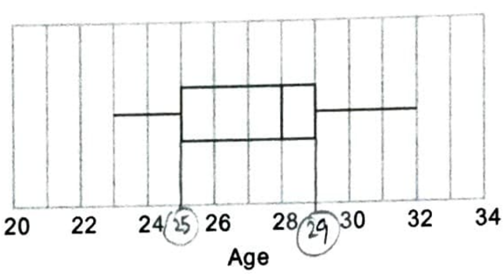
$\frac{18}{30} = \frac{3}{5}$

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{3}{5}$

29. The box-and-whisker diagram below shows the distribution of the ages of students in a baking class. Find the inter-quartile range of the distribution.



A. 4

$29 - 25 = 4$

B. 5

C. 6

D. 9

30. The stem-and-leaf diagram below shows the distribution of the ages of the workers in a company, where x and y are integers with $0 \le x, y \le 9$.

Stem (tens)	Leaf (units)
2	2 2 6
3	5 7 7 x
4	2 y

If the range and the mode of the above distribution are 22 kg and 37 kg respectively, find the standard deviation of the distribution correct to 3 significant figures.

$40 + y = 22 + 22$
$y = 4$

$x = 7$

A. 7.24 kg .

B. 7.76 kg .

C. 8.13 kg .

D. 8.23 kg .

Use SD Mode,
Standard deviation = 7.76

Section B

31. The H.C.F. and the L.C.M. of three expressions are $2a^2b^2$ and $20a^4b^5$ respectively. If the first expression and the second expression are $4a^4b^2$ and $20a^3b^3$ respectively, then the third expression is

HCF of 4 and 20 is 4 ⇒ third exp must have a 2

Power of $a \geq 2$

Power of $b = 5$ as LCM has b^5 while $4a^4b^2$, $20a^3b^3$ do not have

A. $2a^2b^5$

B. $2a^4b^2$

C. a^2b^5 X

D. a^4b^2 X

32. $7\times16^2+5\times4^2+3=$ $7 \times 2^8 + 5 \times 2^4 + 3$

$7: 111_2$

$5: 101_2$

$3: 11_2$

A. 1110101011_2

B. 11101010110_2

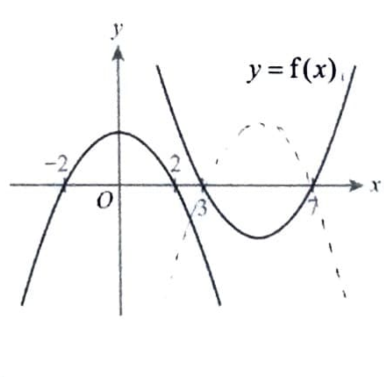
C. 11101010011_2

D. 111001010011_2

$\therefore 111010100 11_2$

$2^8\ 2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

33. Let $f(x)$ be a quadratic function. The figure below represents the graph of $y=f(x)$ and the graph of



f(x)

Reflect about x-axis.

$-f(x)$

Translate leftwards by 5 units

$-f(x+5)$

A. $y=-f(x+5)$.

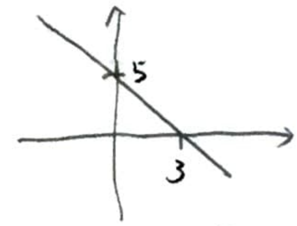
B. $y=f(-x+5)$.

C. $y=-5f(x)$.

D. $y=f(-5x)$.

34. It is given that $\log_4 y$ is a linear function of $\log_2 x$. The intercepts on the vertical axis and on the horizontal axis of the graph of the linear function are 5 and 3 respectively. Which of the following must be true?

$\log_4 y = \frac{-5}{3}\log_2 x + 5$



Slope $= \frac{-5}{3}$

A. $x^5y^3 = 2^{30}$

B. $x^3y^5 = 2^{30}$

C. $x^{10}y^3 = 2^{30}$

D. $x^3y^{10} = 2^{30}$

$\frac{\log_2 y}{\log_2 4}$

$\frac{\log_2 y}{2}$

$\therefore \log_2 y = \frac{-10}{3}\log_2 x + 10$

$= \log_2 x^{\frac{-10}{3}} + \log_2 2^{10}$

$= \log_2 (x^{\frac{-10}{3}} \cdot 2^{10})$

$y = x^{\frac{-10}{3}} 2^{10} \Rightarrow x^{\frac{10}{3}} y = 2^{10} \Rightarrow x^{10}y^3 = 2^{30}$

35. For $0° < x \leq 360°$, how many roots does the equation $7\cos x + 4\sin^2 x = 7$ have?

$7\cos x + 4(1-\cos^2 x) = 7$

$7\cos x + 4 - 4\cos^2 x = 7$

$-4\cos^2 x + 7\cos x - 3 = 0$

A. 1

B. 2

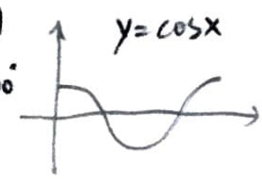
C. 3

D. 4

$\cos x = \frac{3}{4}$ or $\cos x = 1$

2 roots. $x = 0°$ (rej) or $360°$

1 roots



$\therefore$ 3 roots.

36. Let a_n be the nth term of a geometric sequence. Given that $a_1 + a_2 + a_3 + \ldots + a_8 = \sqrt{2} + 1$ and $\frac{a_5}{a_4} = \sqrt{2}$, which of the following must be true?

$r = \sqrt{2}$, $\frac{a(\sqrt{2}^8 - 1)}{\sqrt{2} - 1} = \sqrt{2} + 1$

$a(15) = (\sqrt{2}+1)(\sqrt{2}-1)$

$15a = (\sqrt{2})^2 - 1^2$

$15a = 1$

$a = \frac{1}{15}$

I. a_1 is rational. ✓

II. $a_{20} < 50$ ✓

III. $a_1 + a_2 + a_3 + \ldots + a_{20} < 150$ ✗

$a_{20} = \frac{1}{15}(\sqrt{2})^{20-1} = 48.27 < 50$

A. I only

B. III only

C. I and II only

D. II and III only

$a_1 + a_2 + \cdots a_{20} = \frac{\frac{1}{15}(\sqrt{2}^{20} - 1)}{\sqrt{2} - 1}$

$= 164.65$

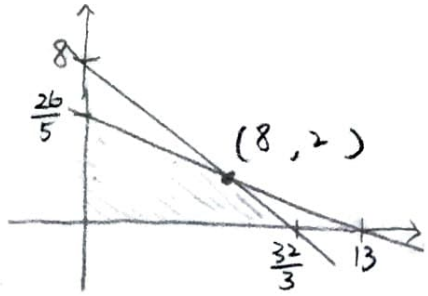
> 150

37. Consider the following system of inequalities:

$$\begin{cases} 3x+4y-32\le 0 \\ 2x+5y-26\le 0 \\ x\ge 0 \\ y\ge 0 \end{cases}$$

x-int : $\frac{32}{3}$ y-int : 8

x-int : 13 y-int : $\frac{26}{5}$



Let R be the region which represents the solution of the above system of inequalities. Find the constant k such that the greatest value of $9x+10y-k$ is 55, where (x, y) is a point lying in R.

A. −3

B. 37

C. 41

D. 43

	$9x+10y-k$
$(8,2)$	$92-k$
$(\frac{32}{3},0)$	$96-k$
$(0,\frac{26}{5})$	$52-k$
$(0,0)$	$-k$

← Greatest : $96-k=55$

$k=41$

38. Let $u=\dfrac{1}{\cos\theta - i\sin\theta}$ and $v=\sin\theta + i$ where $0°\le\theta\le 360°$. Define $z=u^2+v^2$. Which of the following must be true?

✓ I. The imaginary part of u is equal to the real part of v. ($\sin\theta$, $\sin\theta$)

✗ II. The imaginary part of z is equal to $2\sin\theta\cos\theta$.

✓ III. The real part of z is equal to $-\sin^2\theta$.

A. I and II only

B. I and III only

C. II and III only

D. I , II and III

$u=\dfrac{1}{\cos\theta - i\sin\theta}\cdot\dfrac{\cos\theta+i\sin\theta}{\cos\theta+i\sin\theta}$

$=\dfrac{\cos\theta+i\sin\theta}{\cos^2\theta+\sin^2\theta}$

$=\cos\theta+i\sin\theta$

$z=(\cos\theta+i\sin\theta)^2+(\sin\theta+i)^2$

$=\cos^2\theta+2i\sin\theta\cos\theta-\sin^2\theta+\sin^2\theta+2i\sin\theta-1$

$=\underbrace{\cos^2\theta-1}_{\text{Real part}}+\underbrace{(2\sin\theta\cos\theta+2\sin\theta)i}_{\text{Imag. part}}$

$=-(1-\cos^2\theta)$

$=-\sin^2\theta$

39. Let k be a constant. If the straight line $x+2y-k=0$ and the circle $x^2+y^2-8x+12y-48=0$ intersect at two distinct points A and B, then the y-coordinate of the mid-point of AB is

A. $\dfrac{28-4k}{5}$.

B. $\dfrac{4k-28}{5}$.

C. $\dfrac{14-2k}{5}$.

D. $\dfrac{2k-14}{5}$.

$x=k-2y$, $(k-2y)^2+y^2-8(k-2y)+12y-48=0$

$k^2-4ky+4y^2+y^2-8k+16y+12y-48=0$

$5y^2+(28-4k)y+k^2-8k-48=0$

y-coord. Mid point $=\frac{1}{2}(\text{Sum of roots})$

$=\frac{1}{2}\left(\dfrac{-(28-4k)}{5}\right)$

$=\dfrac{2k-14}{5}$

40. The figure shows a sector AOB with centre O. P is a point on $\overset{\frown}{AB}$ with $\overset{\frown}{AP}:\overset{\frown}{PB}=3:4$. AB and OP intersect at Q. $OA = 10$ cm and $\angle OAB = 27°$. Find the shaded area correct to the nearest $0.1\ \text{cm}^2$.

A. $19.1\ \text{cm}^2$

B. $21.9\ \text{cm}^2$

C. $39.1\ \text{cm}^2$

D. $41.0\ \text{cm}^2$

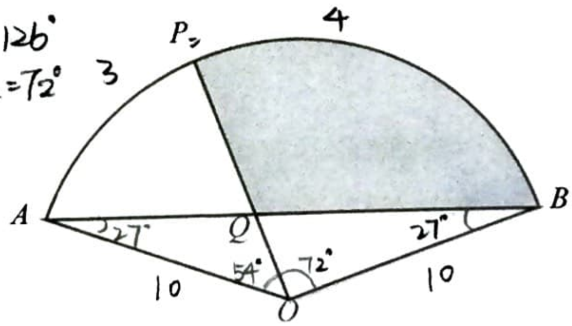
$\angle AOQ : \angle BOQ = 3:4$

$\angle AOB = 180° - 2(27°) = 126°$

$\therefore \angle AOQ = 54°,\ \angle BOQ = 72°$

$\dfrac{10}{\sin(180°-72°-27°)} = \dfrac{QB}{\sin 72°}$

$QB = 9.6291156$



Shaded Area

$= \pi(10^2)\left(\frac{72°}{360°}\right) - \frac{1}{2}(10)(9.6291156)\sin 27°$

$= 40.974\ \text{cm}^2$

x-int: 12k, y-int.: −5k

41. Let k be a positive constant. The straight line $5x-12y-60k=0$ cuts the x-axis and y-axis at A and B respectively. Denote O the origin and C the inscribed circle of ΔOAB. If the length of the radius of C is 52 units, find the coordinates of the intersection of C and AB.

A. $(72,100)$

B. $(72,-100)$

C. $(78,-108)$

D. $(100,-72)$

$BR = QB = 5k-52$

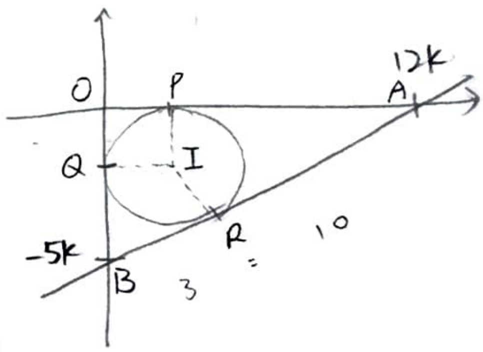
$AR = PA = 12k-52$

$5k-52+12k-52 = AB$

$17k-104 = \sqrt{(-5k)^2+(12k)^2}$

$17k-104 = 13k$

$k = 26$



$B(0,-130)$, $A(312,0)$

$BR : RA = 5(26)-52 : 12(26)-52$

$= 3 : 10$

$R: \left(\dfrac{3(312)+0}{3+10}, \dfrac{0+10(-130)}{3+10}\right) = (72,-100)$

42. A queue is formed by 6 adults and 3 children. If no children stand next to each other and no children are at any of the two ends, how many different queues can be formed?

6 adult

6!

children

P^5_3

or $C^5_3 \times 3!$

A. 43 200

B. 86 400

C. 151 200

D. 332 640

$6! \times P^5_3 = 43200$

or

$6! \times C^5_3 \times 3! = 43200$

43. Peter takes part in three different mathematics competitions. The probabilities for him to get a distinction in the three competitions are 0.2 , 0.3 and 0.15 respectively. Find the probability that he gets a distinction in at least 1 competitions.

A. 0.407

B. 0.524

C. 0.65

D. 0.991

$1-(1-0.2)(1-0.3)(1-0.15)$

$= 0.524$

44. In a test, the scores of Carol and David are 72 marks and 56 marks respectively. If the standard deviation of the test scores is 8 marks, then the difference of the standard scores of Carol and David is

A. 2 .

B. 8 .

C. 16 .

D. 128 .

Let μ be mean

$\frac{72-\mu}{8} - \frac{56-\mu}{8} = \frac{72-\mu-56+\mu}{8}$

$= 2$

45. The mean and variance of a group of numbers $\{x_1, x_2, x_3, \ldots, x_{20}\}$ are 14 and a respectively. Let k be a constant. If the mean and variance of another group of numbers $\{kx_1-3, kx_2-3, kx_3-3, \ldots, kx_{20}-3\}$ are 25 and $a+9$ respectively, $a =$

A. 2 .

B. 3 .

C. 9 .

D. 12 .

New mean : $k(14)-3$

$k(14)-3=25$

$k=2$

New variance : $k^2 a$

$2^2 a = a+9$

$3a = 9$

$a = 3$

END OF PAPER