PO LEUNG KUK CELINE HO YAM TONG COLLEGE

First Term Examination

2019-2020

FORM SIX MATHEMATICS

COMPULSORY PART

PAPER 1

Question-Answer Book

INSTRUCTIONS

1. Write your name, class, class no., date and time in the space provided on this cover.
2. This paper consists of THREE sections, A(1), A(2) and B.
3. Answer **ALL** questions in Section A(1), A(2) and B. Write your answer in the space provided. Supplementary answer sheets will be supplied on request.
4. Unless otherwise specified, all working must be shown clearly.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in the paper are not necessarily drawn to scale.

Time allowed: 2 hours 15 minutes

Name:	
Class:	
Class Number:	
Date:	
Time:	

Mark:	

SECTION A(1) (35 marks)

Answer ALL questions in this section and write your answers in the space provided.

1. Simplify $\frac{(2a^{-3})^2}{4a^{-2}}$ and express your answer with positive indices. (3 marks)

2. Factorize

   (a) $4x^2 - 12xy + 9y^2$,

   (b) $25 - 4x^2 + 12xy - 9y^2$.

(3 marks)

3. Make b the subject of the formula $\frac{3a+7b}{2} = 2b+1$. (3 marks)

4. (a) Solve the inequality $\frac{x}{3} - \frac{2x-1}{4} < -1$.

(b) Write down the least integral value of x that satisfies the inequality in (a).

(4 marks)

5. Four boys, Tom, Patrick, Scott and William have \$25.7, \$28.2, \$36.5 and \$42.6 respectively.

   (a) Estimate the total amount that they have by

      (i) rounding down each datum to the nearest dollar,

      (ii) rounding up each datum to the nearest dollar.

   (b) If the four boys want to buy a box of board game of price \$130, will they have enough money? Explain your answer.

(4 marks)

6. The weight of an orange and an apple are 400 g and 550 g respectively. The total number of oranges and apples sold was 350 and the total weight of the oranges and apples sold was 158 kg. Find the number of apples sold. (4 marks)

7. The cost of a bag is $500. The bag is now sold and the percentage profit is 30%.

   (a) Find the selling price of the bag.

   (b) If the bag is sold at a discount of 25% on its marked price, find the marked price of the bag.

(4 marks)

8. The following table shows the distribution of the numbers of revision time (in hour) spent by a group of students on a certain day.

Number of revision time (in hour)	1	2	3	4
Number of students	8	4	3	k

It is given that k is a positive number.

(a) Write down the least possible value and the greatest possible value of the median of the distribution.

(b) If the mean of the distribution is at most 2.5, how many possible values of k are there? Explain your answer.

(5 marks)

9.

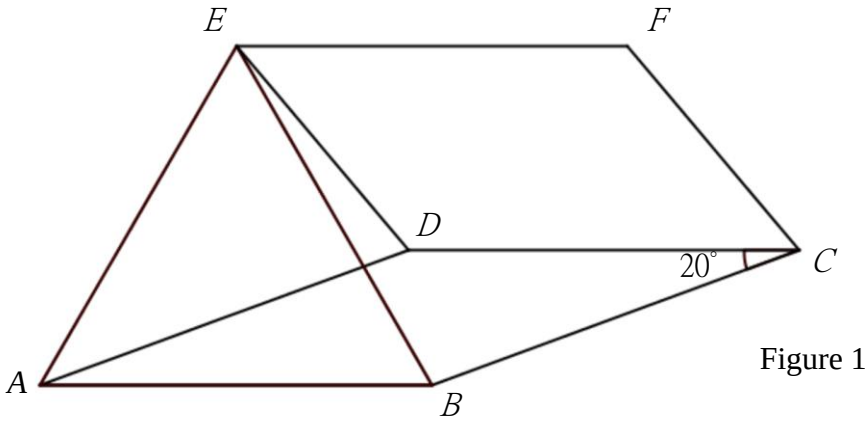

Figure 1

In Figure 1, *ABCD* is a rhombus, *CDEF* is a parallelogram and *ABE* is an equilateral triangle. If $\text{Đ}BCD = 20^{\circ}$, find

(a) $\text{Đ}DEB$, and

(b) $\text{Đ}DCF$.

(5 marks)

SECTION A(2) (35 marks)

Answer ALL questions in this section and write your answers in the space provided.

10. In a polar coordinate system, O is the pole. The polar coordinates of the points P and Q are $\left(4, 28^\circ\right)$ and $\left(4, 268^\circ\right)$ respectively. M is the mid-point of PQ.

(a) Find the polar coordinates of M. (3 marks)

(b) $\triangle OPQ$ is enlarged to form $\triangle OP'Q'$. The polar coordinates of the mid-point of $P'Q'$ are $\left(3, 328^\circ\right)$. Find the area $\triangle OP'Q'$. (3 marks)

11. Let $f(x) = 2x^3 - 7x^2 + 6x - 5$.

(a) Find the quotient and the remainder when $f(x)$ is divided by $x^2 - 4x + 3$. (2 marks)

(b) Let $g(x) = f(x) - (rx + s)$, where r and s are constants. It is given that $g(x)$ is divisible by $x^2 - 4x + 3$.

(i) Write down the values of r and s.

(ii) Hence, factorize $g(x)$ completely.

(4 marks)

12. The stem-and-leaf diagram below shows the distribution of the hourly wages (in dollars) of the workers in a group.

Stem (tens)	Leaf (units)								
4	1	1	2	3	4	5	5	8	9
5	*a*	5	6	6					
6	2	*b*							

It is given that the mean and the range of the above distribution are \$50 and \$28 respectively.

(a) Find the median and the standard deviation of the above distribution. (5 marks)

(b) If a worker is randomly selected from the group, find the probability that the hourly wage of the selected worker exceeds \$50. (2 marks)

13. A wooden model is made from a right cylinder by drilling a hole in the middle. The hole is in the shape consisting of a right circular frustum and a hemisphere with the common base. The radii of the upper base and the lower base of the frustum are 2 cm and 4 cm respectively. The depth of the hole is h cm. The width of the ring of the top base of the model is 3 cm and the height of the model is 12 cm (See Figure 2). It is known that the volume of the model is $192p$ cm^3.

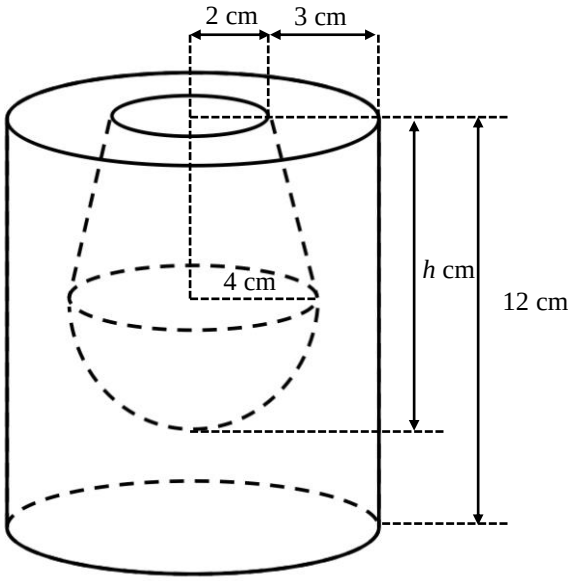

Figure 2

(a) Find the value of h. (5 marks)

(b) If the model is cut into two identical parts, is the increase in the total surface area greater than 100 cm^2? Explain your answer. (3 marks)

14. The profit $\$P$ of product *T* is the sum of two parts, one part varies directly as x and the other part varies directly as x^3. It is known that when $x = 2$, $P = 28$ and when $x = 1$, $P = -10$.

(a) Express P in terms of x. (3 marks)

(b) If $P = Ax(Ax - 1)^2 + 2Ax(Ax - B)$ for all values of *x*, where A and B are constants, find the values of A and B. (2 marks)

(c) Meanwhile, another product *Q* suffers a loss of $\$(4x - 10)$. If the profit of product *T* is equal to the loss of product *Q*, using the result of (a) and (b), or otherwise, find the values of x.

(3 marks)

SECTION B (35 marks)

Answer ALL questions in this section and write your answers in the space provided.

15.

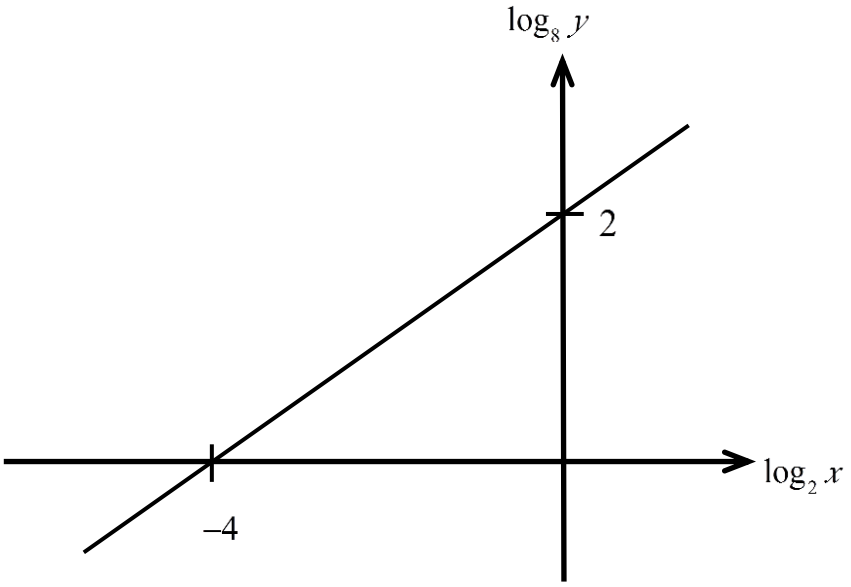

Figure 3

The graph in Figure 3 shows the linear relation between $\log_2 x$ and $\log_8 y$. The intercepts on the horizontal axis and the vertical axis of the graph are -4 and 2 respectively. It is given that $y = ax^b$, where a and b are constants. Find the values of a and b. (4 marks)

16. A committee consists of 4 teachers and 6 parents. Four people are selected randomly from the committee. Find the probabilities that

(a) at least 2 parents are selected, (2 marks)

(b) different number of teachers and parents are selected. (2 marks)

17. Let T_n be the n^{th} term of a geometric sequence. It is given that the sum of the first 2 terms of the sequence is 8 and $T_3 + T_4 = 72$.

(a) Find T_7. (4 marks)

(b) Determine whether the sum of the 3rd term to the k^{th} term of the sequence can be equal to 59 040, where k is a positive integer greater than 3. Explain your answer. (3 marks)

18. (a)

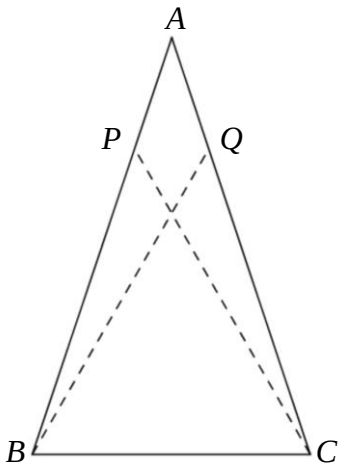

Figure 4 (a)

Figure 4 (a) shows a piece of triangular paper card ABC with $AB = AC = 21$ cm and $BC = 16$ cm. Let P and Q be the points lying on AB and AC respectively such that $\angle BCP = \angle CBQ = 60^\circ$.

(i) Find $\angle BPC$.

(ii) Find BP.

(4 marks)

(b) The paper card described in (a) is fold along PQ and the plane $BCQP$ lies on the horizontal ground as shown in Figure 4 (b).

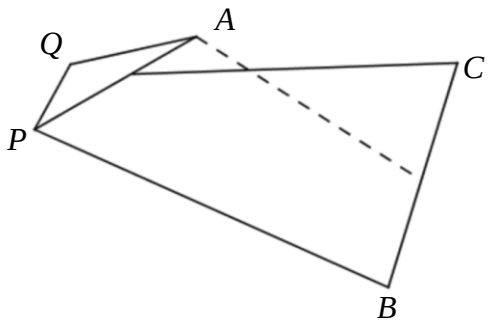

Figure 4 (b)

It is given that the shortest distance from A to BC is 14 cm. Find the shortest distance from A to the plane $BCQP$. (4 marks)

19. (a) Prove that $\tan^2 q = \frac{1}{\cos^2 q} - 1$. (2 marks)

(b) In Figure 5, the equation of the circle is $\left(x - \frac{125}{8}\right)^2 + \left(y - \frac{51}{2}\right)^2 = \left(\frac{325}{8}\right)^2$. The circle cuts the *y*-axis at the points *A* and *C*, and cuts the *x*-axis at the points *B* and *D*. *L* is the tangent to the circle at *A*.

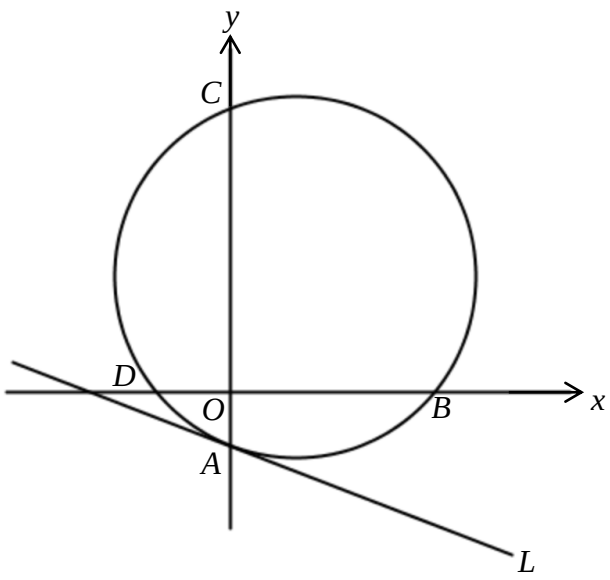

Figure 5

(i) Find the coordinates of *A*, *B*, *C* and *D*. (4 marks)

(ii) By considering ΔABC, using the result of (a), or otherwise, prove that $\tan\angle ABC = \frac{12}{5}$. (3 marks)

(iii) Using the result of (b)(ii), or otherwise, find the equation of *L*. (3 marks)

The End