PO LEUNG KUK CELINE HO YAM TONG COLLEGE
First Term Examination
2019-2020
FORM SIX MATHEMATICS
COMPULSORY PART
Marking Scheme

1. $\dfrac{(2a^{-3})^2}{4a^{-2}}$

$= \dfrac{4a^{-6}}{4a^{-2}}$ [1M]

$= \dfrac{1}{a^4}$ [1M + 1A]

2. (a) $4x^2 - 12xy + 9y^2$

$= (2x - 3y)^2$ [1A]

(b) $25 - 4x^2 + 12xy - 9y^2$

$= 5^2 - (2x - 3y)^2$ [1M]

$= (5 - 2x + 3y)(5 + 2x - 3y)$ [1A]

3. $\dfrac{3a + 7b}{2} = 2b + 1$

$3a + 7b = 4b + 2$ [1M]

$3b = 2 - 3a$ [1M]

$b = \dfrac{2 - 3a}{3}$ [1A]

4. (a) $\dfrac{x}{3} - \dfrac{2x - 1}{4} < -1$

$4x - 3(2x - 1) < -12$ [1M]

$4x - 6x + 3 < -12$

$-2x < -15$

$x > \dfrac{15}{2}$ [1M + 1A]

(b) 8 [1A]

5. (a) (i) $25.7 + 28.2 + 36.5 + 42.6$

$\approx 25 + 28 + 36 + 42$

$= \$131$ [1A]

(ii) $25.7 + 28.2 + 36.5 + 42.6$

$\approx 26 + 29 + 37 + 43$

$= \$135$ [1A]

(b) $\because 25.7 + 28.2 + 36.5 + 42.6$

> 131 [1M]

> 130

$\therefore$ They will have enough money. [1f.t.]

6. Let the number of orange and apple be x and y respectively.

$\begin{cases} x + y = 350 \\ 400x + 550y = 158\ 000 \end{cases}$ [1M + 1A]

$y = 120$ [1M + 1A]

$\therefore$ The number of apple is 120.

7. (a) The selling price

$= 500 \times (1 + 30\%)$ [1M]

$= \$650$ [1A]

(b) The marked price

$= \dfrac{650}{1 - 25\%}$ [1M]

$= \$\dfrac{2600}{3}$ [1A]

8. (a) Least possible value of the median: 1.5 [1A]

Greatest possible value of the median: 4 [1A]

(b) $\dfrac{1 \times 8 + 2 \times 4 + 3 \times 3 + 4k}{8 + 4 + 3 + k} \le 2.5$ [1M]

$\dfrac{25 + 4k}{15 + k} \le \dfrac{5}{2}$

$50 + 8k \le 75 + 5k$

$3k \le 25$

$k \le \dfrac{25}{3}$ [1A]

$\therefore$ There are 8 possible value of k. [1A]

9. (a) $\angle BAD = 20^\circ$

$\angle DAE = 40^\circ$ [1A]

$AD = AB$

$AB = AE$

$\therefore AD = AE$

$\angle AED = \dfrac{180^\circ - 40^\circ}{2} = 70^\circ$ [1M]

$\angle DEB = 70^\circ - 60^\circ = 10^\circ$ [1A]

(b) $AB // EF$,

$\angle DEF + 60^\circ + 70^\circ = 180^\circ$ [1M]

$\angle DEF = 50^\circ$

$\angle DCF = 50^\circ$ [1A]

10. (a) $\angle POQ = 360^\circ - 268^\circ + 28^\circ = 120^\circ$

Let the coordinates of M be (r, θ)

$\cos\frac{120^\circ}{2} = \frac{r}{4}$ [1M]

$r = 2$

$\theta = 268^\circ + 60^\circ = 328^\circ$ [1M]

$\therefore$ The polar coordinates of M are $(2, 328^\circ)$. [1A]

(b) $PM = \sqrt{4^2 - 2^2} = \sqrt{12}$

Area of ΔOPQ

$\frac{1}{2} \times 2 \times \sqrt{12} \times 2 = 4\sqrt{3} / 2\sqrt{12}$ [1M]

$\frac{\text{Area of } \Delta OPQ}{\text{Area of } \Delta OP'Q'} = \left(\frac{2}{3}\right)^2$ [1M]

Area of $\Delta OP'Q' = 9\sqrt{3}$ [1A]

11. (a) The quotient: $2x+1$ [1A]

The remainder: $4x-8$ [1A]

(b) (i) $r = 4$

$s = -8$ [1M + 1A]

(ii) $g(x) = f(x) - (4x - 8)$

$= (2x+1)(x^2 - 4x + 3)$ [1M]

$= (2x+1)(x-3)(x-1)$ [1A]

12. (a) $60 + b - 41 = 28$

$b = 9$

$$\frac{41\times 2 + 42 + 43 + 44 + 45\times 2 + 48 + 49 + 50 + a + 55 + 56\times 2 + 62 + 69}{15} = 50$$

$a = 4$ [1M + 1A]

[Remarks: 1M for either finding range or mean + 1A for both *a* and *b* correct]

Median $= \$48$ [1A]

s.d. $= \$8.10$ [1M + 1A]

(b) The probability $= \frac{6}{15} = \frac{2}{5}$ [1M + 1A]

13. (a) Height of the frustum $=(h-4)$ cm

Volume of the frustum

$=\frac{1}{3}\times\pi\times4^2\times2(h-4)-\frac{1}{3}\times\pi\times2^2\times(h-4)$ [1M + 1A]

$=\frac{28}{3}(h-4)\pi$ cm^3 [1A]

$\pi\times5^2\times12-\frac{28}{3}(h-4)\pi-\frac{4}{3}\pi\times4^3\times\frac{1}{2}=192\pi$ [1M]

$h=11$ [1A]

(b) The increase in total surface area

$=\left(12\times10-\frac{(4+8)\times7}{2}-\frac{1}{2}\pi\times(4)^2\right)\times2$ [1M + 1A]

$=106$ cm^2

>100 cm^2

$\therefore$ It is greater than 100 cm^2 [1f.t.]

14. (a) Let $P=k_1x+k_2x^3$, where k_1, k_2 are constants [1A]

$\begin{cases}28=2k_1+8k_2\\-10=k_1+k_2\end{cases}$ [1M]

$k_1=-18$, $k_2=8$

$P=8x^3-18x$ [1A]

(b) By compare coefficients

$A^3=8$

$A=2$

$A-2AB=-18$

$B=5$ [1M + 1A]

(c) $8x^3-18x=4x-10$

$2x(2x-1)^2+4x(2x-5)=4x-10$ [1M]

$2x(2x-1)^2+4x(2x-5)-2(2x-5)=0$

$2x(2x-1)^2+2(2x-1)(2x-5)=0$ [1M]

$2(2x-1)(2x^2+x-5)=0$

$x=\frac{1}{2}$ or $x=\frac{-1\pm\sqrt{41}}{4}$ [1A]

15. $y = ax^b$

$\log_8 y = \log_8 a + b\log_8 x$ [1A]

$\log_8 y = \log_8 a + b\dfrac{\log_2 x}{\log_2 8}$ [1M]

$\log_8 y = \log_8 a + \dfrac{b}{3}\log_2 x$

$\log_8 a = 2$

$a = 64$ [1A]

$\dfrac{b}{3} = \dfrac{1}{2}$

$b = \dfrac{3}{2}$ [1A]

16. (a) $\dfrac{C_2^6 C_2^4 + C_3^6 C_1^4 + C_4^6}{C_4^{10}}$ [1M]

$= \dfrac{37}{42}$ [1A]

(b) $1 - \dfrac{C_2^6 C_2^4}{C_4^{10}}$ [1M]

$= \dfrac{4}{7}$ [1A]

17. (a) Let $T_n = aR^{n-1}$ [1M]

$\begin{cases} T_1 + T_2 = 8 \\ T_3 + T_4 = 72 \end{cases}$

$\begin{cases} a + aR = 8 \\ aR^2 + aR^3 = 72 \end{cases}$ [1M]

$R^2 = 9$

$R = \pm 3$ [1A]

When $R = 3$

$a = 2$

$T_7 = 2 \times 3^6 = 1458$

When $R = -3$

$a = -4$

$T_7 = -4 \times (-3)^6 = -2916$ [1A]

(b) Let $S_k - S_2 = 59\ 040$ [1M]

When $R = 3$

$$\frac{2\left(3^k - 1\right)}{3-1} - 8 = 59\ 040$$ [1M]

$k = 10$

$\therefore$ It can be equal to 59040. [1f.t.]

18. (a) (i) In ΔABC

$$\cos\angle ABC = \frac{21^2 + 16^2 - 21^2}{2\times 21\times 16}$$ [1M]

$\angle ABC \approx 67.60731219^\circ$

In ΔBCP

$\angle BPC = 180^\circ - \angle ABC - 60^\circ \approx 52.4^\circ$ [1A]

(ii) In ΔBCP

$$\frac{BP}{\sin 60^\circ} = \frac{16}{\sin\angle BPC}$$ [1M]

$BP \approx 17.5$ cm [1A]

(b) Let the mid-points of PQ and BC be X and Y respectively.

In ΔABC in Figure 4(a)

$\angle BAC \approx 180^\circ - 67.60731219^\circ \times 2 = 44.78537561^\circ$

$AP = 21 - BP \approx 3.509214074$ cm

$$\cos\frac{\angle BAC}{2} = \frac{AX}{AP}$$

$AX \approx 3.24460059$ cm [1M]

$$\cos\frac{\angle BAC}{2} = \frac{AY}{21}$$

$AY \approx 19.41648784$ cm

In ΔAXY in Figure 4(b)

$XY \approx 19.41648784 - 3.24460059 = 16.17188725$ cm [1M]

By Heron's formula,

Area of $\Delta AXY = 18.07661854\ \text{cm}^2$ [1A]

The required distance

$$= \frac{18.07661854\times 2}{16.17188725} = 2.24\ \text{cm}$$ [1A]

19. (a) R.H.S.

$= \dfrac{1}{\cos^2\theta} - 1$

$= \dfrac{1-\cos^2\theta}{\cos^2\theta}$

$= \dfrac{\sin^2\theta}{\cos^2\theta}$ [1M]

$= \tan^2\theta$ [1A]

$=$ L.H.S.

$\therefore \tan^2\theta = \dfrac{1}{\cos^2\theta} - 1$

(b) (i) When $y = 0$,

$x = \dfrac{189}{4}$ or $x = -16$ [1M]

$\therefore$ The coordinates of B and D are $\left(\dfrac{189}{4}, 0\right)$ and $(-16, 0)$ respectively. [1A]

When $x = 0$,

$y = 63$ or $y = -12$ [1M]

$\therefore$ The coordinates of B and D are $(0, 63)$ and $(0, -12)$ respectively. [1A]

(ii) $AC = 63 + 12 = 75$

$AB = \sqrt{\left(\dfrac{189}{4}\right)^2 + (12)^2} = \dfrac{195}{4}$

$BC = \sqrt{\left(\dfrac{189}{4}\right)^2 + (63)^2} = \dfrac{315}{4}$ [1M]

$\cos\angle ABC = \dfrac{\left(\dfrac{195}{4}\right)^2 + \left(\dfrac{315}{4}\right)^2 - 75^2}{2\left(\dfrac{195}{4}\right)\left(\dfrac{315}{4}\right)} = \dfrac{5}{13}$

By (a)

$\tan^2\angle ABC = \dfrac{1}{\left(\dfrac{5}{13}\right)^2} - 1$ [1M]

$\tan\angle ABC = \dfrac{12}{5}$ [1A]

(iii) Let α be the inclination of L

The angle between AC and $L = \angle ABC$

$\therefore \alpha = 90^\circ + \angle ABC$

$\tan\alpha = \tan\left(90^\circ + \angle ABC\right)$

$= -\dfrac{1}{\tan\angle ABC}$ [1M]

$= -\dfrac{5}{12}$

$\therefore$ The equation of L is

$y = -\dfrac{5}{12}x - 12$ [1M + 1A]

The End