Class (No.)	( )

FINAL EXAMINATION 2021-2022

FORM SIX MATHEMATICS
Compulsory Part
Paper 1A
Question-Answer Book

8:00 am – 10:15 am (2¼ hours), Jan. 2022

This paper must be answered in English

	Marker's Use Only
Question No.	Marks
1 & 2	
3 & 4	
5 & 6	
7 & 8	
9	
Total	

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Index Number, Class and Class Number in the spaces provided on Page 1 and on the top of each page.
2. This paper consists of ONE section, A(1).
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your Index Number, Class and Class Number. Mark the question number box on each sheet, and fasten them with string INSIDE this book.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

Index No.	

Answers written in the margins will not be marked.

SECTION A(I) (35 marks)

1. Simplify $\frac{(-2\alpha^3\beta)^{-2}\beta}{2(\alpha^2\beta^{-5})}$ and express your answer with positive indices. (3 marks)

2. Make b the subject of $a+\frac{1}{2-b}=3c$. (3 marks)

Answers written in the margins will not be marked.

3. Factorize

(a) $12a^2 + 8ab - 15b^2$,

(b) $5b - 6a - 12a^2 - 8ab + 15b^2$.

(3 marks)

4. (a) Find the range of values of x which satisfy both $1 - \frac{4-x}{5} \geq \frac{x}{6}$ and $5x - 20 < 0$.

(b) How many non-positive integers satisfying the inequality in (a)?

(4 marks)

Answers written in the margins will not be marked.

5. In a polar coordinate system, O is the pole. The polar coordinates of points P and Q are (8, 123°) and

(6, 213°) respectively.

(a) Find $\angle POQ$.

(b) Find the area of ΔPOQ.

(c) Find the distance between O and PQ.

(4 marks)

6. The number of game cards owned by Edan is 6 times that owned by Stanley. If Edan gives 9 game cards to Stanley, then the number of game cards owned by Edan is 3 times that owned by Stanley. Find the total number of game cards owned by Edan and Stanley.

(4 marks)

Answers written in the margins will not be marked.

7. In a shop, the marked price of a toaster is 50% above its cost price. If the toaster is sold at a discount

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of 40%, a loss of $20 is made. Find the selling price of the toaster.

(4 marks)

8. A pack of coffee beans is termed mega size if it weighs 1.0 kg correct to the nearest 0.1 kg.

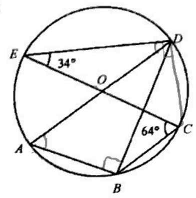
(a) Find the least possible weight of a mega size pack of coffee beans.

(b) Frankie claims that all mega size packs of coffee beans can be repackaged into exactly 18 small packs of coffee beans, where each small pack weighs 50 g correct to the nearest 1 g. Do you agree? Explain your answer.

(5 marks)

Answers written in the margins will not be marked.

9. Refer to the figure below, O is the centre of the circle. AD and CE are diameters of the circle. It is given that $\angle OCB = 64°$ and $\angle OED = 34°$.



(a) Find $\angle OAB$.

(b) Find $\frac{AB}{AD}$.

(5 marks)

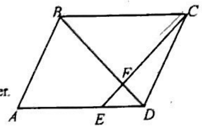
Answers written in the margins will not be marked.

SECTION A(2) (35 marks)

10. When a company employs n workers in a month, its expenditure in that month is $\$E$.
It is given that E is the sum of two parts, one part varies as n and the other part is a constant.
When $n = 5$, $E = 52\ 500$; when $n = 9$, $E = 78\ 500$.
(a) When the company employs 16 workers in a month, find its expenditure in that month. (4 marks)
(b) Is it possible that when the company employs a certain number of workers in a month, its expenditure in that month is \$200 000? Explain your answer. (2 marks)

Answers written in the margins will not be marked.

11. In the figure below, $ABCD$ is a parallelogram. E is a point on AD. BD and CE intersect at F.
(a) Prove that $\triangle BCF \sim \triangle DEF$. (2 marks)
(b) Suppose $BC = 80$ cm, $BD = 84$ cm, $DE = 25$ cm and $CF = 48$ cm.
(i) Is $\triangle BCF$ a right-angled triangle? Explain your answer.
(ii) Find the perimeter of $ABCD$. (4 marks)



Answers written in the margins will not be marked.

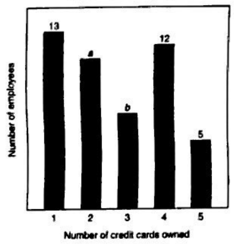
12. Let $f(x)$ be a cubic polynomial with the leading coefficient being 2. Given that $f(x)$ is divisible by $(x+2)$, and when $f(x)$ is divided by (x^2+2x-3), the remainder is $(15x+6)$. Ian claims that $f(x)=0$ has exactly one real root. Do you agree with his claim? Explain your answer.

(8 marks)

Answers written in the margins will not be marked.

13. The bar chart below shows the distribution of the numbers of credit cards owned by the employees in a company, where $5 < b < a < 12$. It is known that the median of the numbers of credit cards owned by the employees is 2.5.

Distribution of the numbers of credit cards owned by the employees



(a) Find all the possible values of a and b.

(2 marks)

(b) Five more employees now join the company. It is found that the mode of the numbers of credit cards owned by the company is increased by 1. Consider $a = 11$.

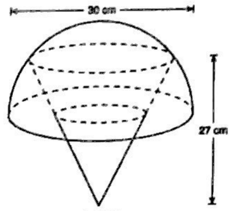
(i) Find the greatest possible inter-quartile range of the numbers of credit cards owned by employees of the company.

(ii) Anson claims that the standard deviation of the numbers of credit cards owned by the employees of the company may be less than 1.32. Do you agree? Explain.

(4 marks)

Answers written in the margins will not be marked.

14. The Solid X in the following figure consists of an inverted right circular cone with height 27 cm and the base radius 9 cm, fitted in a hemisphere with diameter 30 cm. Y is another solid similar to Solid X. It is known that the total surface area of Solid X and that of Solid Y are in the ratio 25 : 36.



Solid X

(a) (i) Find the volume of Solid X in terms of π.

(ii) Find the volume of Solid Y in terms of π.

(6 marks)

(b) A large cubical tank of side 90 cm is 85% filled with water. 10 Solid X and 5 Solid Y are then put into the tank so that they are totally submerged into the water.

Alton claims that the volume of water overflowed from the tank in the process is less than 0.03 m^3. Is his claim correct? Explain your answer.

(3 marks)

Answers written in the margins will not be marked.

SECTION B (35 marks)

15. There are four classes in the sixth form, namely 6A, 6B, 6C and 6D. Three representatives will be selected from each class to form a committee for organizing the graduation party. Four members of the committee will be randomly chosen from the committee to be the masters of ceremony of the party. Find the probabilities of the following events.
    (a) All masters of ceremony are from different classes.
    (b) Exactly three masters of ceremony are from the same class.
    (c) At least two masters of ceremony are from the same class.

(4 marks)

Answers written in the margins will not be marked.

16. Let n be a positive integer such that

$$1.44 + 1.44^2 + 1.44^3 + ... + 1.44^n > 1.2 + 1.2^2 + 1.2^3 + ... + 1.2^{n+6} + 1.2^{n+7} .$$

Find the minimum value of n.

(5 marks)

Answers written in the margins will not be marked.

17. The equations of lines L_1 and L_2 are $2x + 3y + 9 = 8a$ and $3x - 4y + 22a = 12$, where a is a real number. Let P be the point of intersection of L_1 and L_2.

(a) Does P lie on the graph of L: $2x + y + 3 = 0$? Explain your answer.

(b) If 4 distinct points P, $Q(-1, -1)$, $R(0, 2047)$ and $S(2022, -1997)$ are concyclic, find the length of the line segment PS.

(6 marks)

Answers written in the margins will not be marked.

18. Let $f(x) = \frac{2}{3(k+1)}\left[x^2 + 6(k-1)x + 12(1-2k)\right]$ where k is a real number with $k \neq -1$.

Let V be the vertex of the graph of $y = f(x)$.

(a) Using the method of completing square, find the coordinates of V in terms of k.

(3 marks)

(b) Consider $P(6, 8)$ and let W be the vertex of $y = 2 - f(x-4)$.

(i) Find the coordinates of W in terms of k.

(iii) If the circumcentre of ΔWVP lies on WV, find the equation of the circle which passes through W, V and P.

(6 marks)

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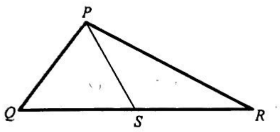
19. (a) Refer to the figure below. S lies on QR and PS is the angle bisector of $\angle QPR$.



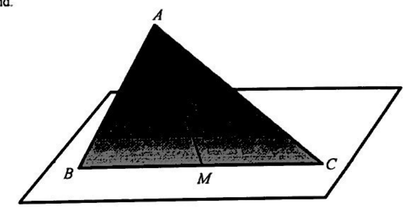
(i) By considering ΔPSR, prove that $\dfrac{PR}{SR} = \dfrac{\sin \angle PSQ}{\sin \angle SPR}$.

(ii) Prove that $\dfrac{PR}{SR} = \dfrac{PQ}{SQ}$.

(3 marks)

Answers written in the margins will not be marked.

(b) Refer to the figure below. An inclined plastic sheet ABC is placed on a horizontal ground with BC on the horizontal plane and M lies on BC. Let H be the incentre of ΔABC. Given that $AB = 260$ cm, $BM = 65$ cm, $AH = 52\sqrt{13}$ cm and $HK = 30$ cm. where K be the projection of H on the ground.



(i) Prove that $AM = 65\sqrt{13}$ cm.

(ii) Let L be the foot of perpendicular of A on BC. Find the distance between A and L.

(iii) Find the angle between ΔABC and the ground.

(iv) Anson claims that the area of ΔKBC is less than 20% of that of ΔABC. Do you agree? Explain you answer.

(8 marks)

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