Marking Scheme of F.6 Mathematics Paper 1 Final Examination (2021-2022)

1. $\dfrac{\left(-2\alpha^3\beta\right)^{-2}\beta}{2(\alpha^2\beta^{-5})}$

$= \dfrac{2^{-2}\alpha^{-6}\beta^{-2}\beta}{2\alpha^2\beta^{-5}}$ 1M

$= 2^{-2-1}\alpha^{-6-2}\beta^{-2+1-(-5)}$ 1M

$= 2^{-3}\alpha^{-8}\beta^{4} = \dfrac{\beta^4}{8\alpha^8}$ 1A

2. $a+\dfrac{1}{2-b}=3c$

$\dfrac{1}{2-b}=3c-a$ 1M

$2-b=\dfrac{1}{3c-a}$ 1M

$b=2-\dfrac{1}{3c-a}$ 1A

(or $b=\dfrac{6c-2a-1}{3c-a}$)

3.

(a) $12a^2+8ab-15b^2=(6a-5b)(2a+3b)$ 1A

(b) $5b-6a-12a^2-8ab+15b^2$

$= 5b-6a-\left(12a^2+8ab-15b^2\right)$

$= -(6a-5b)-(6a-5b)(2a+3b)$ 1M

$= -(6a-5b)(1+2a+3b)$ 1A

4.

(a) $1-\dfrac{4-x}{5}\ge\dfrac{x}{6}$

$\dfrac{1+x}{5}\ge\dfrac{x}{6}$ 1M

$x\ge-6$ 1A

Also, $5x-20<0 \Rightarrow x<4$

ANS: $-6\le x<4$ 1A

(b) The non-positive integers satisfying the inequality in (a) are $0, -1, -2, \ldots, -6$

Hence, there are 7 non-positive integers required. 1A

5.

(a) $\angle POQ = 213° - 123° = 90°$ 1A

(b) Area of $\Delta POQ = 6\times 8 \div 2 = 24$ unit sq. 1A

(c) Let h units be the distance required.

$hPQ \div 2 = 24$

$h\sqrt{6^2+8^2}=48$ 1M

$h=4.8$

$\therefore$ The distance between O and PQ is 4.8 units. 1A

6. Let x be the number of game cards owned by Stanley.

Then the number of game cards owned by Edan is $6x$. 1M

$(6x - 9) = 3(x + 9)$ 1M

$x = 12$ 1A

Total number of game cards $= 6(12) + 12 = 84$ 1A

7. Let \$$x$ be the selling price.

$(x + 20)(1 + 50\%)(1 - 40\%) = x$ 2M

$0.9x + 18 = x$ 1M

$x = 180$

The selling price is \$180 1A

8.

(a) Least possible weight

$= 1.0 - 0.1 \div 2$ 1M

$= 0.95$ kg 1A

(b) The least possible weight of 18 small packs

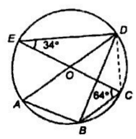
$= 18(50 - 1 \div 2)$ g 1M

$= 0.891$ kg < 0.95 kg 1M

Hence, the claim is disagreed. 1

9.

(a) Join CD.



$\angle CDE = 90°$ 1M

In ΔDCE, $\angle DCE = 180° - 34° - 90° = 56°$ 1A

Now, $\angle OAB + \angle BCD = 180°$

(either)

$\angle OAB + (64° + 56°) = 180°$

$\angle OAB = 60°$ 1A

(b) $\angle ABD = 90°$

$\dfrac{AB}{AD} = \cos 60°$ — 1M

$= \dfrac{1}{2}$ — 1A

10.

(a) Let $E = k_1 n + k_2$, where k_1 and k_2 are non-zero constants. — 1M

$52\,500 = k_1(5) + k_2$

$5k_1 + k_2 = 52\,500$ ......(1)

$78\,500 = k_1(9) + k_2$

$9k_1 + k_2 = 78\,500$ ......(2) — 1M for either one

(2) − (1): $4k_1 = 26\,000$

$k_1 = 6500$

By substituting $k_1 = 6500$ into (1), we have

$5(6500) + k_2 = 52\,500$

$k_2 = 20\,000$

$\therefore\ E = 6500n + 20\,000$ — 1A

The required expenditure = \$[6500(16) + 20000] = \$124000 — 1A

(b) Suppose the expenditure of the company in that month is \$200 000.

$200\,000 = 6500n + 20\,000$ — 1M

$180\,000 = 6500n$

$n = \dfrac{360}{13}$, which is not an integer.

$\therefore$ It is not possible that when the company employs a certain number of workers in a month, its expenditure in that month is \$200 000. — 1A

11.

(a) In $\triangle BCF$ and $\triangle DEF$,

$\angle BCF = \angle DEF$ (*alt. ∠s, BC // AD*)

$\angle BFC = \angle DFE$ (*vert. opp. ∠s*)

$\therefore \triangle BCF \sim \triangle DEF$ (*AA*)

Case 1	Any correct proof with correct reasons.	2 marks
Case 2	Any correct proof without reasons.	1 mark

(b) (i) Let $BF = x$ cm.

$\because \triangle BCF \sim \triangle DEF$

$\therefore \dfrac{BF}{DF} = \dfrac{BC}{DE}$

$\dfrac{x \text{ cm}}{(84 - x) \text{ cm}} = \dfrac{80 \text{ cm}}{25 \text{ cm}}$ — 1M

$25x = 6\,720 - 80x$

$x = 64$

$BF^2 + CF^2 = (x^2 + 48^2) = (64^2 + 48^2) = 6\,400$

$BC^2 = 80^2 = 6\,400$

$\therefore\ BC^2 = BF^2 + CF^2$

$\therefore\ \triangle BCF$ is a right-angled triangle. (Converse of Pythagoras' theorem) — 1

(ii) Note that $BD \perp CF$.

In $\triangle CDF$, by Pythagoras' theorem,

$CD^2 = CF^2 + DF^2$

$CD = \sqrt{48^2 + (84 - 64)^2} = 52$ cm — 1M

$\therefore$ Perimeter of $ABCD$

$= 2(BC + CD) = 2(80 + 52) = 264$ cm — 1A

12. $\because$ f(x) be a cubic polynomial with the leading coefficient being 2 and divisible by $(x + 2)$

$\therefore\ f(x) \equiv (x + 2)(2x^2 + ax + b)$ .... (1) — 1A for (x+2) 1A for $2x^2$

$\because$ when f(x) is divided by $(x^2 + 2x - 3)$, the remainder is $(15x + 6)$

$\therefore\ f(x) \equiv (x^2 + 2x - 3)Q(x) + 15x + 4 \equiv (x - 1)(x + 3)Q(x) + 15x + 6$ .... (*) — 1M

Put $x = 1$ into (*), $f(1) = 15(1) + 6 = 21$ — 1A

By (1), $(1 + 2)(2(1)^2 + a(1) + b) = 21 \Rightarrow a + b = 5$ .... (2)

Put $x = -3$ into (*), $f(-3) = 15(-3) + 4 = -39$

By (1), $(-3 + 2)(2(-3)^2 + a(-3) + b) = -39 \Rightarrow -3a + b = 21$ .... (3) — 1M

Solve (2) and (3), yield $a = -4, b = 9$

Hence, $f(x) \equiv (x + 2)(2x^2 - 4x + 9)$

$\Delta = (-4)^2 - 4(2)(9) = -56 < 0$ — 1M

$\therefore\ 2x^2 - 4x + 9 = 0$ has no real root and hence $f(x) = 0$ has exactly one real root −2. — 1M

The claim is agreed. — 1

13.

(a) $\because$ median = 2.5

$\therefore\ 13 + a = b + 12 + 5 \Rightarrow a = b + 4$ — 1M

$\because\ 5 < b < a < 12$

$\therefore\ (a = 11$ and $b = 7)$ or $(a = 10$ and $b = 6)$ — 1A

(b) (i) $\because$ New mode = 2

$\therefore$ At least three data 2 should be added.

There are only 5 employees having 5 cards, interquartile range $\le 4 - 1 = 3$

Consider {1,2,2,2,2}. — 1M

The greatest possible inter-quartile range = $Q_3 - Q_1$

$= \dfrac{40^{th}\ datum + 41^{th}\ datum}{2} - \dfrac{13^{th}\ datum + 14^{th}\ datum}{2}$

$= \dfrac{4+4}{2} - \dfrac{1+1}{2} = 3$ — 1A

(ii) $\because$ New mode = 2

$\therefore$ At least three data 2 should be added.

Original mean = $\dfrac{1\times13+2\times11+3\times7+4\times12+5\times5}{13+11+7+12+5}=2.6875$

To reduce the dispersion of the datra set, we may try to add the five new data with their mean being close to 2.6875.

Take the new data set of 5 employee = {2,2,2,3,3} (say) with mean 2.4.

New mean = $\dfrac{1\times13+2\times14+3\times9+4\times12+5\times5}{13+14+9+12+5}=2.622641509$

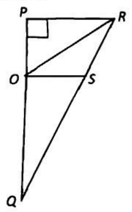
New S.D. = $\dfrac{(1-2.623)^2\times13+(2-2.623)^2\times14+\ldots+(5-2.623)^2\times5}{13+14+9+12+5}$

$=1.316705383<1.32$ 1A

∴ The claim is agreed. 1

14.

(a) (i) Let O and P be the centres of the hemisphere and the base circle of the cone respectively.



$OP=\sqrt{15^2-9^2}=12$

$OQ=27-OP=27-12=15$ } 1M

$OS=9\times(15\div27)=5$

Volume of Solid X

$=\frac{2}{3}\pi(15)^3+\frac{1}{3}\pi(5)^2(15)$ 2M

$=2375\pi\ (\text{cm}^3)$ 1A

(ii) Volume of Solid Y $=\left(\sqrt{\frac{36}{25}}\right)^3\times2375\pi$ 1M

$=4104\pi\ (\text{cm}^3)$ 1A

(b) Volume of water overflowed

$=10(2375\pi)+5(4104\pi)+0.85(90^3)-90^3$ 1M

$=29728.30677\text{ cm}^3<30000\text{ cm}^3=0.03\text{ m}^3$ 1A

The claim is agreed. 1

15.

(a) P(all different) = $\dfrac{({}_3C_1)^4}{{}_{12}C_4}=\dfrac{9}{55}$ 1A

(b) P(exactly 3) = $\dfrac{3\times4\times3}{{}_{12}C_4}=\dfrac{4}{55}$ 1A

(c) P(at least 2) = 1 − P(all different) = $1-\dfrac{9}{55}=\dfrac{46}{55}$ 1M + 1A

16. $\dfrac{1.44(1.44^n-1)}{1.44-1}>\dfrac{1.2(1.2^{n+7}-1)}{1.2-1}$ 1M for sum of G.S.

$\dfrac{36}{11}(x^2-1)>6(kx-1)$; where $x=1.2^n$, $k=1.2^7$

$36x^2-36>66kx-66$

$36x^2-66kx+30>0$ 1M for forming quad. ineq.

$6x^2-11kx+5>0$

$6x^2-39.4149888x+5>0$

$x>6.4397604$ or $x<0.1294044$ 1M for solving quad. ineq.

$1.2^n>6.4397604$ or $1.2^n<0.1294044$

$n>\dfrac{\log6.4397604}{\log1.2}$ or $n<\dfrac{\log0.1294044}{\log1.2}$ 1M for taking logarithm

$n>10.21542031$ or $n<-11.21542033$ (rejected)

The minimum value of $n=11$. 1A

17.

(a) $\begin{cases}2x+3y+9=8a\ldots\ldots\ldots\ldots(1)\\3x-4y+22a=12\ldots\ldots\ldots(2)\end{cases}$

$4(1)+3(2)\Rightarrow 8x+36+9x+66a=32a+36 \Rightarrow 8x+36+9x+66a=32a+36$

$x=-2a$

By (1), $y=\dfrac{8a-9-2(-2a)}{3}=4a-3$ 1A for both coord.

∵ $2(-2a)+(4a-3)+3=0$ 1M

∴ Yes, P lie on L for any real a. 1

(b) ∵ slope of $RS=\dfrac{2047+1997}{0-2022}=-2=$ slope of L and both P and Q lying on L (by (a))

∴ $RS\,//\,PQ$. 1M

If P, Q, R, S are concyclic, and $P\neq Q$, then $PQRS$ is an isosceles trapezium and thus 1M

$PS=QR=\sqrt{(2022+1)^2+(-1997+1)^2}=\sqrt{8076545}$ 1A

18.

(a) $\dfrac{2}{3(k+1)}\left[x^2+6(k-1)x+12(1-2k)\right]$

$=\dfrac{2}{3(k+1)}\left[x^2+6(k-1)x+9(k-1)^2-9(k-1)^2+12(1-2k)\right]$ 1M

$=\dfrac{2}{3(k+1)}\left[(x+3(k-1))^2-9(k^2-2k+1)+12-24k\right]$

$=\dfrac{2}{3(k+1)}[x+3(k-1)]^2+\dfrac{2}{3(k+1)}(-9k^2-6k+3)$

$= \frac{2}{3(k+1)}[x+3(k-1)]^2 + \frac{-2(3k^2+2k-1)}{(k+1)}$ 1M

$= \frac{2}{3(k+1)}[x+3(k-1)]^2 + \frac{-2(k+1)(3k-1)}{(k+1)}$

$= \frac{2}{3(k+1)}[x+3(k-1)]^2 + 2(1-3k)$

Thus, the coordinates of V are $(3(1-k), 2(1-3k))$. 1A

(b) (i) W is obtained by translating V to the right 4 units, reflecting along the x-axis and followed by translating 2 units upwards,

$W = (3(1-k)+4, -2(1-3k)+2)$ 1M

$= (7-3k, 6k)$ 1A

(ii) If the circumcentre of ΔWVP lies on WV, $\angle WPV = 90°$.

slope of WP × slope of $VP = -1$

$\frac{6k-8}{7-3k-6} \times \frac{2(1-3k)-8}{3(1-k)-6} = -1$ 1M

$\frac{6k-8}{1-3k} \times \frac{-6-6k}{-3-3k} = -1$

$\frac{6k-8}{1-3k} \times 2 = -1$

$12k-16 = 3k-1$

$k = \frac{5}{3}$ 1A

Centre of the required circle is the mid-point of W and V

$= \left(\frac{3(1-k)+7-3k}{2}, \frac{2(1-3k)+6k}{2}\right)$

$= (5-3k, 1)$ 1M

$= \left(5-3\left(\frac{5}{3}\right), 1\right) = (0, 1)$

Radius $= \sqrt{(0-6)^2+(1-8)^2} = \sqrt{85}$

Thus, the equation of the required circle is $x^2+(y-1)^2 = 85$ 1A

19.

(a) (i) In ΔPSR, by sine law,

$\frac{SR}{\sin\angle SPR} = \frac{PR}{\sin\angle PSR} = \frac{PR}{\sin(180°-\angle PSR)} = \frac{PR}{\sin\angle PSQ}$ 1M for $\sin(180°-\theta) = \sin\theta$

i.e. $\frac{PR}{SR} = \frac{\sin\angle PSQ}{\sin\angle SPR}$

(ii) In ΔPSQ, by sine law,

$\frac{PQ}{\sin\angle PSQ} = \frac{SQ}{\sin\angle QPS} = \frac{SQ}{\sin\angle SPR}$ 1M for $\angle QPS = \angle QPS$

$\frac{PQ}{SQ} = \frac{\sin\angle PSQ}{\sin\angle SPR} = \frac{PR}{SR}$ (by (a)) 1 f.t.

(b) (i) ∵ BH is the angle bisector of $\angle BAC$

∴ $\frac{BM}{HM} = \frac{BA}{HA}$ (by (a)(ii)) 1M

$HM = \frac{65 \times 52\sqrt{13}}{260} = 13\sqrt{13}$

Thus, $AM = 52\sqrt{13}+13\sqrt{13} = 65\sqrt{13}$ 1 f.t.

(ii) In ΔABM, by cosine law,

$AM^2 = AB^2+BM^2-2AB\cdot BM\cos\angle ABM$

$\cos\angle ABM = \frac{260^2+65^2-65^2\times 13}{2\times 260\times 65} = \frac{1}{2}$ 1M

∴ $\angle ABM = 60°$

$AL = AB\sin\angle ABM = 260\sin 60° = 130\sqrt{3}$ cm. (or 225 cm) 1A

(iii) Let J be the projection of A on the ground and θ be the required angle.

∵ $\Delta AJM \sim \Delta HKM$

∴ $\frac{AJ}{HK} = \frac{AM}{HM} \Rightarrow AJ = \frac{52\sqrt{13}}{13\sqrt{13}} \times 30 = 120$ cm 1M

$\sin\theta = \frac{AJ}{AL} = \frac{120}{130\sqrt{3}}$

$\theta \approx 32.2042275° = 32.2°$ (correct to 3 significant figures) 1A

(iv) $\frac{\text{Area of } \Delta KBC}{\text{Area of } \Delta ABC}$

$= \frac{HK}{JK} \cdot \frac{\text{Area of } \Delta JBC}{\text{Area of } \Delta ABC}$

$= \frac{1}{4}\cos\theta = \frac{1}{4}\cos(32.2042275°)$

$= 0.211538461$ 1A

$> 20\%$

∴ The claim is disagreed. 1

END