ST. FRANCIS XAVIER'S COLLEGE
FINAL EXAMINATION 2021-2022
FORM SIX MATHEMATICS – Compulsory Part Paper 2
MARKING SCHEME

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
B	A	A	B	C	C	D	B	B	D
11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
D	C	A	A	C	B	C	A	B	D
21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
C	C	C	A	B	A	A	D	B	B
31.	32.	33.	34.	35.	36.	37.	38.	39.	40.
D	B	A	B	D	A	D	D	C	C
41.	42.	43.	44.	45.					
C	B	D	D	A					

A : 11 B : 12 C:11 D : 11

Section A

1. B $\frac{3^{14a}\times 9^{3-2a}}{81^a}=\frac{3^{14a}\times 3^{6-4a}}{3^{4a}}=3^{14a+6-4a-4a}=3^{6a+6}=\left(3^3\right)^{2a+2}=27^{2a+2}$

2. A $k=\frac{4}{ab}-\frac{1}{c};k=\frac{4c-ab}{abc};kabc=4c-ab;kabc+ab=4c;ab(kc+1)=4c;b=\frac{4c}{a(kc+1)}$

$k=\frac{4}{ab}-\frac{1}{c};k+\frac{1}{c}=\frac{4}{ab};\frac{kc+1}{c}=\frac{4}{ab};b=\frac{4c}{a(kc+1)}$

3. A $(4p-2q)(3p+2q)+4(2p-q)=2(2p-q)(3p+2q)+4(2p-q)=2(2p-q)(3p+2q+2)$

4. B $\frac{2}{3a-2}-\frac{3}{3a+2}=\frac{2(3a+2)-3(3a-2)}{(3a-2)(3a+2)}=\frac{-3a+10}{(3a-2)(3a+2)}=\frac{3a-10}{(2-3a)(3a+2)}$

5. C Maximum absolute error $=\frac{0.1}{2}=0.05$; $9.95\le x<10.05$; $5.15\le y<5.25$; $15.1\le x+y<15.3$

6. C Let $a=x-1$, i.e. $x=a+1$; $f(a)=(a+1)^2-4=a^2+2a-3$; $f(x)=x^2+2x-3$;

$f(x)+f(-x)=x^2+2x-3+(-x)^2+2(-x)-3=2x^2-6$

7. D $p(a)=a^3-a\times a^2-3=a$; $a=-3$; $p(-(-3))=p(3)=3^3+(3)\times(3)^2-3=51$

8. B For option I: Slope of $L_1=-\frac{1}{-a}=\frac{1}{a}$; Slope of $L_2=-\frac{c}{-2}=\frac{c}{2}$;From the figure, we have slope of $L_1<$ slope of $L_2<0$; $\frac{1}{a}<\frac{c}{2}$; $ac<2$; ∴ Option I is true.

For option II: y-intercept of $L_1=-\frac{-b}{-a}=-\frac{b}{a}$; $-\frac{b}{a}>0$; $b>0$; ∴ Option II is not true.

For option III: y-intercept of $L_2=-\frac{-d}{-2}=-\frac{d}{2}$; From the figure, y-intercept of $L_1>$ y-intercept of L_2

$-\frac{b}{a}>-\frac{d}{2}$; $\frac{b}{a}<\frac{d}{2}$; $2b>ad$; ∴Option III is true.

9. B For option A: $y=3(0-2)^2+5=17\neq 5$; ∴ Option A is not true.

For option B: The graph opens downwards and the vertex is below the x-axis. ;∴ The graph has no x-intercepts.;∴Option B is true.

For option C: Coordinates of the vertex = (2, –5); ∴Option C is not true.;

For option D: The equation of the axis of symmetry is $x=2$.; ∴ Option D is not true.

10. D Let $u=\frac{kv}{\sqrt{w}}$, where $k\neq 0$. ; New value of $u=\frac{k[(1-0.2)v]}{\sqrt{(1-0.36)w}}=\frac{k(0.8v)}{\sqrt{0.64w}}=\frac{0.8kv}{0.8\sqrt{w}}=\frac{kv}{\sqrt{w}}=u$

11. D Amount = $\$6000\times(1+\frac{1}{4}\times 4\%)^{4\times 5}=\$6000\times 1.01^{20}=\$7321$, *cor. to the nearest dollar*

Interest = $(7321 – 6000)= $1321

12. C $34=2a_4-a_3$ …… (1); $a_4=2a_3-10$ …… (2); Sub (2) into (1); $34=2(2a_3-10)-a_3$; $3a_3=54$; $a_3=18$

13. A $3p=2r$; $p:r=2:3$; $q:r=3:4$; $p:q:r=8:9:12$; Let $p=8k$, $q=9k$ and $r=12k$, where k is a non-zero constant. ; $(p+2q):(2r-q)=(8k+18k):(24k-9k)=26:15$

14. A Let $h(x)=(x^2+x-6)Q(x)+ax+b$. ; $h(-3)=(3^2-3-6)Q(-3)-3a+b=0$;
$-3a+b=0\sim(1)$; $h(2)=(2^2+2-6)Q(2)+2a+b=10$; $2a+b=10\sim(2)$;
$(2)-(1):5a=10$; $a=2\sim(3)$; Sub (3) into (1), ; $b=6$; i.e. the remainder $=2x+6$.

15. C $5+2x\le -5$ or $\frac{1-x}{4}\ge 1$; $2x\le -10$ or $1-x\ge 4$; $x\le -5$ or $x\le -3$; ∴ $x\le -3$

16. B Let $4k$ and $3k$ be the base radius and the height of the cylinder respectively, where $k>0$. Then the radius of the hemisphere is $8k$. ; Total surface area of the cylinder $=2\pi(4k)(3k)+2\pi(4k)^2=56\pi k^2$; Total surface area of the hemisphere $=2\pi(8k)^2+\pi(8k)^2=192\pi k^2$; Required ratio $=56\pi k^2:192\pi k^2=7:24$

17. C

$\triangle DEF\sim\triangle AEB$ (AAA) ; ∴ $\frac{\text{Area of }\triangle AEB}{\text{Area of }\triangle DEF}=\left(\frac{AE}{DE}\right)^2$; Area of $\triangle AEB=\left(\frac{3}{2}\right)^2\times 32=72\text{cm}^2$;

$\triangle AEG\sim\triangle CBG$ (AAA) ; ∴ $\frac{EG}{BG}=\frac{AE}{CB}=\frac{3}{2+3}=\frac{3}{5}$; ∴ $EG:BG=3:5$;

Area of $\triangle AEG=\frac{3}{5+3}\times 72\text{ cm}^2=27\text{ cm}^2$; Area of $\triangle ACD=\frac{5}{3}\times 72\text{ cm}^2=120\text{ cm}^2$;

Area of quadrilateral $CDEG=(120-27)\text{ cm}^2=93\text{ cm}^2$

18. A For option I: Any regular n-sided polygon has an n-fold rotational symmetry. $\therefore$ A regular 18-sided polygon has an 18-fold rotational symmetry. ; $\therefore$ Option I is true.

For option II: Exterior angle $= \frac{360°}{18} = 20°$; $\therefore$ Option II is true.

For option III: No. of diagonals $= \frac{18(18-3)}{2} = 135$; $\therefore$ Option III is not true.

19. B $AB = AM$ (*given*) ; $\therefore \angle ABM = \angle AMB$ (*base* $\angle s$, *isos.* $\triangle$) ; $\angle AMC = 180° - \angle AMB$ (*adj.* $\angle s$ *on st. line*); $\angle AMC = 180° - \angle ABM = \angle BCD$ (*int.* $\angle s$, $AB \parallel DC$) ; Similarly, $\angle ANC = \angle BCD$;
In quadrilateral $AMNC$, $3\angle BCD + 15° = 360°$ ($\angle$ *sum of polygon*) ; $\angle BCD = 115°$

20. D In $\triangle ABM$, $\tan 69° = \frac{AB}{BM}$; In $\triangle CDM$, $\tan \angle CDM = \frac{CM}{CD}$; $\tan \angle CDM = \frac{BC - BM}{CD}$;

$\tan \angle CDM = \frac{AB - BM}{AB}$; $\tan \angle CDM = 1 - \frac{BM}{AB}$; $\tan \angle CDM = 1 - \frac{1}{\tan 69°}$;

$\angle CDM = 32°$, *cor. to the nearest degree*

21. C Refer to the figure on the right. ; $\because ABCD$ is a rectangle. ; $\therefore BC = a$ and $CD = b$; In $\triangle CYD$,

$\frac{CY}{CD} = \sin \angle CDY$; $CY = b \sin \theta$; $\angle BCX + \angle BCD + \angle DCY = 180°$ (*adj.* $\angle s$ *on st. line*) ;

$\angle BCX + 90° + (90° - \theta) = 180°$; $\angle BCX = \theta$; In $\triangle BCX$, $\frac{CX}{BC} = \cos \angle BCX$; $CX = a \cos \theta$

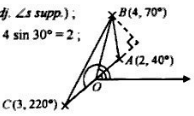
The required distance $= CX + CY = a \cos \theta + b \sin \theta$

22. C Let $x°$ and r be the angle and the radius of the original sector respectively.

$2\pi r(1 + k\%)\frac{x°}{360°}(1 - 20\%) = 2\pi r\frac{x°}{360°}$; $(1+k\%)(0.8) = 1$; $1 + k\% = 1.25$; $k\% = 0.25$; $k = 25$

23. C Refer to the figure. ; $220° - 40° = 180°$; $\therefore AOC$ is a straight line. (*adj.* $\angle s$ *supp.*) ;
$AC = AO + OC = 2 + 3 = 5$; Height of $\triangle ABC = OB \sin(70° - 40°) = 4 \sin 30° = 2$;

Area of $\triangle ABC = \frac{5 \times 2}{2} = 5$



24. A Join AD. ; $\because AC$ is a diameter. ; $\therefore \angle ADC = 90°$ ($\angle$ *in semi-circle*) ;
$\angle ADE = \angle ABE = 25°$ ($\angle s$ *in the same segment*) ; $\therefore \angle CDE = \angle ADC + \angle ADE = 90° + 25° = 115°$;
In $\triangle CDP$, $\angle CDP + \angle CPD + \angle DCP = 180°$ ($\angle$ *sum of* $\triangle$) ; $115° + \angle CPD + 40° = 180°$;
$\angle CPD = 25°$

25. B The locus of P is a pair of parallel lines.

26. A $2x^2 + 2y^2 - 6x + 12y - 135 = 0$; $x^2 + y^2 - 3x + 6y - 67.5 = 0$; $(x - 1.5)^2 + (y + 3)^2 = 78.75$;
For option A: Distance between the origin and the centre

$= \sqrt{(1.5-0)^2 + (-3-0)^2} = \sqrt{11.25} < \sqrt{78.75}$ $\therefore$ Option A is true.

For option B: Area $= 78.75\pi \approx 247.400\,421\,5 < 250$; $\therefore$ Option B is not true.
For option C: Radius $= \sqrt{78.75}$; $-3 + \sqrt{78.75} > 0$; $\therefore$ Option C is not true.
For option D: Coordinates of centre $= (1.5, -3)$; $\therefore$ Option D is not true.

27. A Coordinates of mid-point of $BC = \left(\frac{6+10}{2}, \frac{5+7}{2}\right) = (8,6)$

The equation of the straight line : $\frac{y-2}{x-1} = \frac{6-2}{8-1}$; $7y - 14 = 4x - 4$; $4x - 7y + 10 = 0$

28. D The required probability $= \frac{C_1^3 C_1^4 + C_1^2 C_1^1}{C_2^{10}} = \frac{14}{45}$

The required probability $= \frac{2(3)(4) + 2(2)(1)}{(10)(9)} = \frac{14}{45}$

29. B Inter-quartile range $= \$(700 - 400) = \$300$

30. B Mean $= \frac{4 \times 6 + 5 \times 2 + 6 \times 3 + 9 + k}{13} = \frac{61 + k}{13}$; Mode $= 4$; Median $= \begin{cases} 4 & \text{for } k = 4 \\ 5 & \text{for } k = 5, 6, 7, 8, 9 \end{cases}$

For option I: When $k = 4$, Mean $= 5 > 4 =$ median ; When $k \neq 4$, Mean $> 5 =$ median ;
$\therefore$ Option I must be true.

For option II: When $k = 4$, Median $=$ mode ; When $k \neq 4$, Median $>$ mode
$\therefore$ Option II may not be true.

For option III: $4 < \frac{61 + k}{13}$ for $k = 4, 5, 6, 7, 8, 9$; i.e. Mode $<$ mean. $\therefore$ Option III must be true.

31. D

1st expression:	a^3	b^5	c
2nd expression:	a^5	b^5	c^3
H.C.F.:	a^3	b^4	c
L.C.M.:	a^6	b^5	c^5

By considering the power of a in the given expressions and the L.C.M., the 3rd expression contains a^6.

By considering the power of b in the given expressions and the H.C.F., the 3rd expression contains b^4.

By considering the power of c in the given expressions and the L.C.M., the 3rd expression contains c^5.

$\therefore a^6b^4c^5$ is the 3rd expression.

32. B $1110\,1000\,1010\,1110\,1101_2 = 14 \times 16^4 + 8 \times 16^3 + 10 \times 16^2 + 14 \times 16^1 + 13 \times 16^0$
$= 14 \times 16^4 + (8 \times 16 + 10) \times 16^2 + 12 \times 16 + 2 \times 16 + 13 \times 16^0$
$= 14 \times 16^4 + 138 \times 16^2 + 12 \times 16^1 + 45$

33. A $\log_4 y = \left(-\frac{4}{2}\right)\log_2 x + 4$; $\log_4 y = -2\log_2 x + 4$; $2\log_2 x + \log_4 y = 4$; $\frac{2\log_4 x}{\log_4 2} + \log_4 y = 4$;
$4\log_4 x + \log_4 y = \log_4 4^4$; $\log_4 x^4 y = \log_4 256$; $x^4 y = 256$

34. B $\log_8 y = 4(\log_4 x - 1)$; $\log_8 y = 4\log_4 x - 4$; $\log_8 y = \log_4 x^4 - 4$; $\frac{\log_2 y}{\log_2 8} = \frac{\log_2 x^4}{\log_2 4} - \log_2 16$;

$\frac{\log_2 y}{3} = \frac{\log_2 x^4}{2} - \log_2 16$; $\frac{\log_2 y}{3} = \log_2 \frac{x^{\frac{4}{2}}}{16}$; $\log_2 y = \log_2 \frac{x^{2\times3}}{16^3}$; $y = \frac{x^6}{4096}$

35. D

$(2+i^5)(1+2i^2-i^3)=(2+i)[1+2(-1)-(-i)]=(2+i)(-1+i)=-2+i+i^2 \ =-2+i+(-1)$

$=-3+i$

36. A

$\frac{k}{2}(-3+72)=897; k=26$ ∴ Option I is true.

Let d be the common difference of the sequence. ∵ $x_{26}=72$; ∴ $-3+(26-1)d=72$;

$d=3$; ∴ $x_2=-3+3=0$; ∴ Option II is true.

Sum of the first $2n$ terms $=\frac{2n}{2}[2(-3)+(2n-1)(3)]=6n^2-9n$; ∴ Option III is not true.

37. D The coordinates of the vertices of D are (10, 6), (10, 10), (20, 15) and (32, 6).

At (10, 6), $2x+3y+20=2(10)+3(6)+20=58$

At (10, 10), $2x+3y+20=2(10)+3(10)+20=70$

At (20, 15), $2x+3y+20=2(20)+3(15)+20=105$

At (32, 6), $2x+3y+20=2(32)+3(6)+20=102$

∴ The greatest value of $2x+3y+20$ is 105.

38. D The graph of $y=\mathrm{h}(x)$ may be obtained by reflecting the graph of $y=\mathrm{g}(x)$ about the x-axis and translating the resulting graph upwards. i.e. $\mathrm{h}(x)=-\mathrm{g}(x)+k$, where $k>0$.

39. C

$\angle DAT=\angle DEA$ ($\angle$ in alt. segment); $\angle ADT=\frac{1}{2}\angle EDT=\frac{1}{2}(68°)=34°$ (angle bisector);

In ΔDAT, $\angle ADT+\angle DAT+\angle ATD=180°$ ($\angle$ sum of Δ); $34°+\angle DEA+40°=180°$; $\angle DEA=106°$; $\angle AEP+\angle ARP=192°\neq 180$; A, E, P and Q are not concyclic.;

∴Option I is not correct.

$\angle RTA=\frac{1}{2}\angle DTA$ (angle bisector); $\angle RTA=\frac{1}{2}(40°)=20°$; $\angle RAT+\angle RTA=86°$ (ext. $\angle$ of Δ) ; $\angle RAT+20°=86°$; $\angle RAT=66°$; $\angle RAQ+\angle RAT=\angle QAT$; $\angle RAQ+66°=106°$; $\angle RAQ=40°=\angle ATD$; Option II is correct.

$\angle ADE=\angle ADS$ (angle bisector) ; $AD=AD$ (common side); $\angle ADE+\angle DAE+\angle AED=180°$; $34°+\angle RAT+106°=180°$; $\angle DAE=40°=\angle DAS$; ∴$\Delta ADE\cong\Delta ADS$ (A.S.A.);

Option III is correct.

40. C $x+2y-2=0$; $x=2-2y \sim (1)$.; Sub (1) into the equation of the circle, ;

$(2-2y)^2+y^2-14(2-2y)+10y+1=0$; $4y^2-8y+4+y^2-28+28y+10y+1=0$;

$5y^2+30y-23=0$; Let y_1 and y_2 are the y-coordinates of M and N respectively. ;

∴ The y-coordinate of the mid-point of $MN=\frac{y_1+y_2}{2} \ =\frac{-\frac{30}{5}}{2}=-3$; Sub. $y=-3$ into (1), ;

$x=2-2(-3)=8$

41. C $2\cos^2\theta+3\sin\theta=3$; $2(1-\sin^2\theta)+3\sin\theta=3$; $-2\sin^2\theta+3\sin\theta-1=0$; $(2\sin\theta-1)(\sin\theta-1)=0$; $\sin\theta=\frac{1}{2}$ or $\sin\theta=1$; $\theta=30°, 150°$ or $90°$ ∴ There are 3 roots.

42. B The required number $={}_{30}C_5+{}_{30}C_4\times{}_{18}C_1+{}_{30}C_3\times{}_{18}C_2=1\ 256\ 976$

43. D

Required probability $=\frac{5}{9}\times\frac{3}{8}+\frac{4}{9}\times\frac{2}{8}=\frac{23}{72}$

44. D

Let x marks be the score of John in the examination.; $1.5=\frac{x-65}{6}$; $x=74$;

The score of John is 74 marks.

45. A

$\frac{(x_1-m)^2+(x_2-m)^2+(x_3-m)^2+\cdots+(x_9-m)^2}{9}=v$

$(x_1-m)^2+(x_2-m)^2+(x_3-m)^2+\cdots+(x_9-m)^2=9v$

∴ Required variance

$=[(x_1-m)^2+(x_2-m)^2+(x_3-m)^2+\cdots+(x_9-m)^2+(m-m)^2]\div 10=\frac{9v}{10}$

END OF MARKING SCHEME