Sacred Heart Canossian College
S6 Mock Examination 2018-2019
Mathematics Paper 1 Solutions

Section A(1) (35 marks)

1. (3 marks)

$$\frac{\left(x^2y^4\right)^3}{x^8y^{-5}} = \frac{x^6y^{12}}{x^8y^{-5}} = x^{6-8}y^{12+5} = x^{-2}y^{17} = \frac{y^{17}}{x^2}$$

2. (4 marks)

(a) $x + y = \frac{1}{3}(x + 2)$

$3x + 3y = x + 2$

$2x = 2 - 3y$

$\therefore x = \frac{2-3y}{2}$

(b) The increase in the value of x is 1.5 .

3. (4 marks)

(a) $a^3 + 2a^2y - 3a^2 = a^2(a + 2y - 3)$

(b) $a^3 + 2a^2y - 3a^2 - a - 2y + 3$
$= a^2(a + 2y - 3) - (a + 2y - 3)$
$= (a^2 - 1)(a + 2y - 3)$
$= (a + 1)(a - 1)(a + 2y - 3)$

4. (5 marks)

(a) Cost = $221 \div (1 - 15\%) \div (1 + 30\%)$ = \$200

(b) Profit Percentage

$= \frac{221-200}{200} \times 100\%$

= 10.5% > 10%

$\therefore$ The claim is agreed.

5. (3 marks)

(a) Maximum absolute error = 0.5 m

(b) Least possible area :

$= 9.5 \times 5.5 + 3.5 \times 4.5$

= 68 m^2

6. (4 marks)

$x=\frac{a}{2}$

$\angle ACB = 90°$

$\angle ABC = a$

$\angle BAC = 180° - \angle ACB - \angle ABC$

$= 90° - a$

$\angle DAB + \angle DCB = 180°$

$y + 90° - a + x + 90° = 180°$

$y = \frac{a}{2}$

7. (4 marks)

(a) The required probability $=\frac{2}{5}$

(b) Note that the possible amount may be \$3 , \$4 , \$6 , \$7 and \$10.

Thus, the number of possible amount of the two coins drawn is 5.

The required probability $=\frac{10}{20}=\frac{1}{2}$

The required probability
$=\frac{1}{5}\times\frac{2}{4}\times 2+\frac{1}{5}\times\frac{2}{4}\times 2+\frac{2}{5}\times\frac{1}{4}=\frac{1}{2}$

8. (4 marks)

(a) $2(x+1)<6+x$ and $-2x-2\le 5$

$x<4$ $\quad$ $x\ge -\frac{7}{2}$

$\therefore$ The range of values of x is $-\frac{7}{2}\le x<4$

(b) Note that the integers that satisfy the compound inequalities are $-3,-2,-1,0,1,2,3$.
Thus, there are 7 integers satisfy both inequalities in (a).

9. (4 marks)

Let the numbers of male employees and female employees are x and y respectively.

Then $x+y=1092$ and $x=(1-60\%)y$

i.e. $(1-60\%)y+y=1092$

$1.4y=1092$

$y=780$

Hence, $x+780=1092$

$x=312$

Therefore, the required difference = 780 – 312 = 468

Section A(2) (35 marks)

10. (7 marks)

(a) $DE = CE$ (given)

$\angle AED = \angle BEC$ (vert. opp. $\angle$)

$\angle EDC = \angle ECD$ (base $\angle$s, isos $\triangle$)

$\angle EDC = \angle ABE$ (alt $\angle$s, $AB // DC$)

$\angle ECD = \angle BAE$ (alt $\angle$s, $AB // DC$)

$\angle ABE = \angle BAE$

$AE = EB$ (side opp. equal $\angle$s)

$\triangle ADE \cong \triangle BCE$ (SAS)

(b) (i) By (a), $AD = BC = 24$ cm (corr. $\angle$s, ~ $\triangle$s)

Note that $BE^2 + BC^2 = 10^2 + 24^2 = 26^2 = EC^2$

Thus, $\triangle BCD$ is a right-angle triangle with $\angle CBD = 90°$. (Converse of Pyth. Theorem)

(ii) By (a), $AE = BE = 10$ cm

$\therefore$ $AC = 36$ cm

Note that the area of $\triangle BCE = \frac{1}{2} \times BE \times BC = \frac{1}{2}(10)(24) = 120\text{ cm}^2$

The shortest distance from B To AC

$= \frac{2(120)}{26} \approx 9.23\text{cm} > 9\text{ cm}$

Thus, there is no point F lying on AC such that the distance between B and F is less than 9 cm.

11. (7 marks)

(a) (i) The mean = 51.4 marks

The inter-quartile range $= \frac{59+64}{2} - \frac{41+44}{2} = 61.5 - 42.5 = 19$ marks

(ii) The required percentage $= \frac{11}{20} \times 100\% = 55\%$

(b) (i) The least possible value of the median $= \frac{41+44}{2} = 42.5$ marks

The greatest possible value of the median $= \frac{59+64}{2} = 61.5$ marks

(ii) Let x marks be the mean score of Class B.

$$\frac{51.4 \times 20 + 10x}{30} > 51.6$$

$$x > 52$$

Thus, it is not possible that the mean score of Class B is 52.

12. (6 marks)

(a) $f(2)=28$

$(2+1)(2-h)^2+k=28$

$12-12h+3h^2+k=28$

$3h^2-12h+k=16.....(1).$

$f(-2)=0$

$(-2+1)(-2-h)^2+k=0$

$-4-4h-h^2+k=0$

$h^2+4h-k=-4.........(2)$

$4h^2-8h=12$

(1) + (2), $h^2-2h-3=0$

$h=3$ or $h=-1$ (rejected)

From (2), $k=25$

$\therefore h=3$ and $k=25$

(b)

$$f(x)=0$$

$$(x+1)(x-3)^2+25=0$$

$$x^3-5x^2+3x+34=0$$

$$(x+2)(x^2-7x+17)=0$$

$$x=-2 \text{ or } x=\frac{7\pm\sqrt{-19}}{2}$$

Note that $\frac{7\pm\sqrt{-19}}{2}$ are not real numbers.

$\therefore$ The claim is disagreed.

13. (7 marks)

(a) Volume of empty space inside container in Figure 1

$(2\sqrt{\pi})^2(30-h)=4\pi(30-h)$

Note that

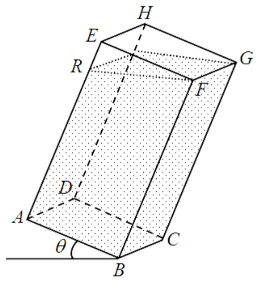
$4\pi(30-h)=\frac{4}{3}\pi(3)^3$

$h=21$

(b) (i) Volume of empty space inside container in Figure 2

= Volume of empty space inside container in Figure 1 $=4\pi(30-21)=36\pi \text{ cm}^3$

(ii) With the notations in the figure, note that $\angle EFR=\theta$



$36\pi=\frac{1}{2}\times ER\times EF\times GF$

$36\pi=\frac{1}{2}\times ER\times(2\sqrt{\pi})^2$

$ER=18\text{ cm}$

$\because \tan\theta=\frac{ER}{EF}=\frac{18}{2\sqrt{\pi}}$

$\theta\approx 78.9^\circ$

14. (8 marks)

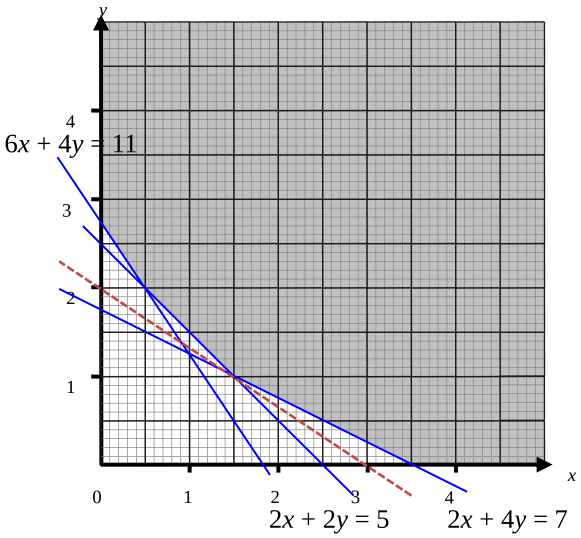
(a) The constraints are

$$\begin{cases} x \ge 0,\ y \ge 0 \\ 0.2x + 0.4y \ge 0.7 \\ 0.3x + 0.2y \ge 0.55 \\ 0.2x + 0.2y \ge 0.5 \end{cases}$$

After simplification, we have

$$\begin{cases} x \ge 0,\ y \ge 0 \\ 2x + 4y \ge 7 \\ 6x + 4y \ge 11 \\ 2x + 2y \ge 5 \end{cases}$$

(b)



(c) Let the total cost, $P(x,y) = 2kx + 3ky$ (in dollars) $k \ne 0$

By drawing parallel line of $2x + 3y = c$,

for any constant c.

Alternative method: Testing points

	$P = 2kx + 3ky$
(3.5, 0)	$7k$
(1.5, 1)	$6k$
(0.5, 2)	$7k$
$(0, \frac{11}{4})$	$8.25k$

$P(x, y)$ attains its minimum value at $(1.5, 1)$

Thus, Miss Ng should feed 1.5 kg of food X and 1 kg of food Y.

Section B (35 marks)

15. (7 marks)

(a) Let $f(x) = ax + bx^2$, where a and b are non-zero constants.

So, We have $3a + 9b = 53$ and $6a + 36b = 104$

Solving , we have $a = 18$ and $b = -\frac{1}{9}$

$\therefore f(x) = 18x - \frac{1}{9}x^2$

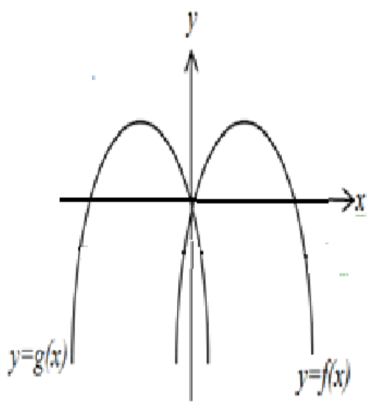
(b) $f(x) = -\frac{1}{9}x^2 + 18x$

$$= -\frac{1}{9}(x^2 - 162x) = -\frac{1}{9}(x^2 - 162x + 81^{2.} - 81^2) = -\frac{1}{9}(x-81)^2 + 729$$

$\therefore$ The coordinates of the vertex of the graph of $y = f(x)$ is $(81, 729)$.

(c) Note that the graph of $y = f(x)$ passes through the origin (0 , 0).
We have the coordinates of the vertex of the graph of $y = g(x)$ are $(-81, 729)$.

$\therefore g(x) = -\frac{1}{9}\ (x+81)^2 + 729$



15. (c)

Alternative method

Let $g(x) = f(x+k)$, where $k \neq 0.$,

$$\therefore g(x) = -\frac{1}{9}(x+k-81)^2 + 729$$

$$\therefore g(x) = -\frac{1}{9}(\text{x}+k-81)^2 + 729$$

At(0,0), $-\frac{1}{9}(k-81)^2 + 729 = 0$

$$(k-81)^2 - 81^2 = 0$$

$k = 162$ or $k = 0$ (rejected)

$$\therefore g(x) = -\frac{1}{9}(x+81)^2 + 729$$

16. (7 marks)

(a) mid-point of *A* and *B* is $(\frac{21}{2}, \frac{25}{2})$.

Equation of the perpendicular bisector of *AB*

$$\frac{\text{y}-\frac{25}{2}}{x-\frac{21}{2}} = \frac{-1}{\frac{23-2}{21-0}}$$

$$x + y - 23 = 0$$

(b) (i) Let the centre *G* be (*a*, *b*)

Note the centre *G* lies on the perpendicular bisector of *AB*.

We have $a + b - 23 = 0$ …(1)

Note that tangent $\perp$ radius.

Thus, $-\frac{3}{4} \times \frac{b-2}{a-0} = -1$

$4a - 3b + 6 = 0$ …(2)

By solving (1) and (2), we have the coordinates of G =(9, 14).

(ii) Let *D*= (0, *d*)

$$\frac{2+d}{2} = 14$$

$$d = 26$$

The coordinates of *D* are $(0, 26)$.

Let the coordinates of *E* be $(e, 14)$.

$$\frac{14-2}{e-0} = -\frac{}{4}$$

$$e = -16$$

The area of the quadrilateral *AGDE*

$$= \frac{1}{2}[9-(-16)](14-2) 2\times 2 =$$

17. (4 marks)

(a) $\frac{7}{m+7}\times\frac{6}{m+6}=\frac{7}{15}$

$m^2+13m-48=0$

$(m+16)(m-3)=0$

$m=\underline{\underline{3}}$ or $m=-16$ (reject

(b) The required probability

$\frac{7!\times P^8_3}{10!}=\underline{\underline{\frac{7}{15}}}$

18. (5 marks)

(a) The required total cost

$=\$[3.2\times10^5\times0.05+3.2\times10^5\times0.9\times0.05+3.2\times10^5\times0.9^2\times0.05]$

$=\$[3.2\times10^5\times0.05(1+0.9+0.9^2)]$

$=\$43360$

(Accept $\$4.336\times10^4$ or $\$4.34\times10^4$)

(b) Note that the total cost of the insurance in the first n years is

$\$[3.2\times10^5\times0.05+3.2\times10^5\times0.9\times0.05+3.2\times10^5\times0.9^2\times0.05+\ldots\ldots+3.2\times10^5\times0.9^{n-1}\times0.05]$

We have

$$\frac{3.2\times10^5(0.05)\left[1-(0.9)^n\right]}{1-0.9}>1.4\times10^5$$

$$1-0.9^n>\frac{1.4}{1.6}$$

$$0.9^n<0.125$$

$$\log0.9^n<\log(0.125)$$

$$n\log0.9<\log(0.125)$$

$$n>\frac{\log(0.125)}{\log0.9}$$

$$n>19.7\ \text{(cor. to 3 sig. fig.)}$$

$\therefore$ In 2038, the total cost of the insurance will first exceed $\$1.4\times10^5$.

19. (7 marks)

(a) Let E be a point on BC such that $VE\perp BC$ and F be a point on AD such that $VF\perp AD$.

The angle between the plane VBC and the plane $ABCD$ is $\angle VEF$.

$VE=\sqrt{45^2-(7.5)^2}=\sqrt{1968.75}\approx44.37059837\text{cm}$

$VA^2=27^2+45^2-2(27)(45)\cos36^\circ$

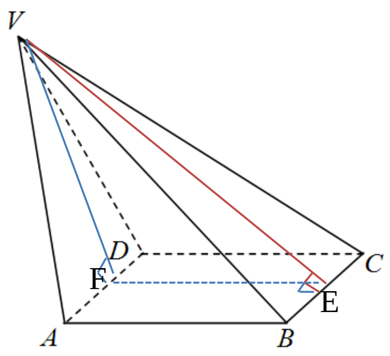
$VA\approx28.07291762$ cm

$VF\approx\sqrt{28.07291762^2-(7.5)^2}=27.05251751\text{cm}$

$\cos\angle VEF=\frac{VE^2+FE^2-VF^2}{2(VE)(FE)}=\frac{44.37059837^2+27^2-27.052517}{2(44.37059837)(27)}$

$\angle VEF\approx34.86582597^\circ$

$\therefore$ The required angle is 34.9°.



19.

(b) Let R be the projection of M on the plane $ABCD$.
The distance from R to the plane $ABCD$

$$\frac{VE}{2}\sin\angle VEF \approx \frac{\sqrt{1968.75}}{2}\sin 34.86582597^\circ \approx 12.68237254 \text{ cm}$$

Let T be a point on PQ such that $MT \perp PQ$.

.

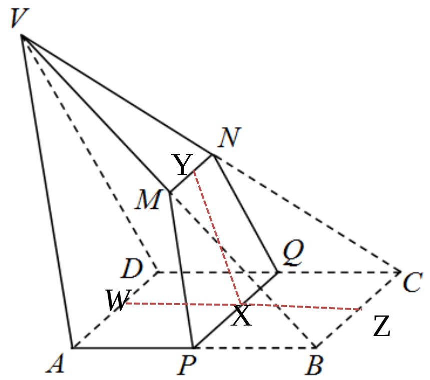
$$MT = \sqrt{MP^2 - PT^2} = \sqrt{(\frac{VA}{2})^2 - \left(\frac{PQ-MN}{2}\right)^2} = \sqrt{(\frac{28.07291762}{2})^2 - \left(\frac{15-7.5}{2}\right)^2} \approx 13.52625876 \text{ cm}$$

Note that the angle between the plane $PQNM$ and the plane $ABCD$ is $\angle MTR$.

$$\sin\angle MXR = \frac{MR}{MX} = \frac{12.68237254}{13.52625876}$$

$\angle MXR \approx 69.7^\circ < 70^\circ$

Thus, the claim is disagreed.



20. (5 marks)

(a) $A = \frac{4-0}{0-(-6)} = \frac{2}{3}$

Note that $\log_4 B = 4$

$B = 4^4 = 256$

(b) $\log_8 y = \frac{2}{3}\log_2 x + 4$

$\frac{\log_2 y}{\log_2 8} = \frac{2}{3}\log_2 x +$

$\log_2 y = 2\log_2 x + 12$

$\log_2 y = \log_2 x^2 + \log_2$

$\log_2 y = \log_2(4096x^2)$

$y = 4096x^2$

OR Q19 (b)

Let X ,Y and Z be the mid-points of PQ, MN and BC respectively.

Note that $YX \perp PQ$ and $ZX \perp PQ$.

$$XY = \sqrt{(\frac{28.07291762}{2})^2 - \left(\frac{15-7.5}{2}\right)^2} \approx 13.52625876 \text{ cm}$$

$YZ = \frac{44.37059837}{2} \approx 22.18529919\text{cm}$, XZ=13.5 cm

$$\cos\angle YXZ = \frac{XY^2 + XZ^2 - YZ^2}{2(XY)(XZ)}$$

$$= \frac{13.52625876^2 + 13.5^2 - 22.18529919^2}{2(13.52625876)(13.5)}$$

$\angle YXZ \approx 110.3458076^\circ$

Let W be the mid-point of AD.

The angle between the plane $PQNM$ and the plane $ABCD$ is $\angle YXW$.

$\angle YXW = 180^\circ - \angle YXZ \approx 69.7^\circ < 70^\circ$

Thus, the claim is disagreed.

Paper 2 Solution

1. A	11. C	21. B	31. B	41. B
2. C	12. A	22. D	32. A	42. D
3. B	13. C	23. D	33. D	43. A
4. C	14. C	24. B	34. A	44. B
5. C	15. B	25. A	35. C	45. A
6. B	16. A	26. B	36. C	
7. D	17. D	27. D	37. B	
8. B	18. C	28. D	38. A	
9. A	19. A	29. D	39. A	
10. D	20. C	30. B	40. A	

~ End ~