Sacred Heart Canossian College
S6 Mock Examination 2019–2020
Mathematics Paper 1 Solutions

SECTION A(1): (35 marks)

1. $\frac{b^8}{a^{-5}b^{10}}$

$= a^5 b^{8-10}$

$= \frac{a^5}{b^2}$ (3 marks)

2. $mn - 2m = kn + m$

$mn - kn = 3m$

$n(m-k) = 3m$

$n = \frac{3m}{m-k}$

(3 marks)

3. (a) $(11a+9)(11a-9)$

(b) $(11a+9)(11a-9) - 3b(11a+9)$

$= (11a+9)(11a-9-3b)$

(4 marks)

4. (a) cost $= \frac{560}{1+60\%}$

$= \$350$

(b) selling price $= 560 \times (1-35\%)$

$= \$\ 364 > \$350$

$\therefore$ There will be a gain.

(4 marks)

5. (a) maximum absolute error = 10 × 5% = 0.5 m

$\therefore\ 9.5^2 m^2 \le$ actual area $< 10.5^2 m^2$

$90.25 m^2 \le$ actual area $< 110.25 m^2$

(b) percentage error of the perimeter

$= \frac{4 \times 0.5}{40} \times 100\% = 5\% \neq 20\%$

$\therefore$ His claim is incorrect.

(4 marks)

6. (a) $\because\ AB:BC:CD:DA = 2:1:1:x$

$\therefore\ \angle ADB = 2k,\ \angle BDC = k,\ \angle CBD = k,\ \angle ABD = xk$ ($\angle$s prop. to arcs)

$2k + k + k + xk = 180^\circ$ (opp. $\angle$s, cyclic quad.)

$k(x+4) = 180^\circ$

$\therefore\ \angle BDC = k = \dfrac{180^\circ}{x+4}$

(b) $\angle ACD = \angle ABD = xk$ ($\angle$s in same segment)

$k + xk + 90^\circ = 180^\circ$ ($\angle$ sum of $\triangle$)

$k(1+x) = 90^\circ$

$\dfrac{180^\circ}{x+4}(1+x) = 90^\circ$

$x = 2$

(5 marks)

7. (a) $3y - 81 - 1 \geq 50 - 5y$

$y \geq 16.5$

(b) $y \geq 16.5$ and $y < 8$

$\therefore$ No solution.

(4 marks)

8. Let x be the number of candies owned by Fiona originally.

$80 - x + 4 = 3(x - 4)$

$84 - x = 3x - 12$

$x = 24$

The number of candies owned by Fiona originally = 24.

(4 marks)

9. (a) P(a total score of 8 in the game)

= P(4,4)

$= \dfrac{1}{6} \times \dfrac{1}{6}$

$= \dfrac{1}{36}$

(b) P(a total score of 7 or above in one game)

= P(4,4) + P(3,4) + P(4,3)

$= \dfrac{1}{36} + \dfrac{2}{6} \times \dfrac{1}{6} + \dfrac{1}{6} \times \dfrac{2}{6}$

$= \dfrac{5}{36}$

P(no prize is awarded in two games)

$= (1 - \dfrac{5}{36})(1 - \dfrac{5}{36})$

$= \dfrac{961}{1296}$ or 0.742 (cor. to 3 sig. fig.) (4 marks)

SECTION A(2): (35 marks)

10. (a) $EF^2 + FC^2 = 3^2 + 4^2 = 25$

$EC^2 = 5^2 = 25$

$\because EF^2 + FC^2 = EC^2$

$\therefore \Delta EFC$ is a rt. $\angle \Delta$. (Converse of Pyth. Theorem)

$\angle EFC = 90^\circ$

$\angle FAE = \angle CBF = 90^\circ$ (prop of square)

$\angle BCF + \angle BFC + 90^\circ = 180^\circ$ ($\angle$sum of Δ)

$\angle BCF + \angle BFC = 90^\circ$

$\angle AFE + 90^\circ + \angle BFC^\circ = 180^\circ$ (adj. $\angle$s on st. line)

$\angle AFE + \angle BFC = 90^\circ$

$\therefore \angle AFE = \angle BCF$

$\angle AEF = \angle BFC$ ($\angle$sum of Δ)

$\therefore \Delta BCF \sim \Delta AFE$ (AAA)

(3 marks)

(b) $\because \Delta BCF \sim \Delta AFE$

$\therefore \frac{CF}{FE} = \frac{BC}{AF}$ (corr. sides, $\sim\Delta$)

$\frac{4}{3} = \frac{x}{x - FB}$

$4x - 4FB = 3x$

$\therefore FB = \frac{x}{4}$ cm

(2 marks)

(c) $FB^2 + BC^2 = FC^2$

$(\frac{x}{4})^2 + x^2 = 4^2$

$\frac{17}{16}x^2 = 16$

$x^2 = \frac{256}{17}$

Area of $ABCD$ = $x^2 = \frac{256}{17}$ cm^2 < 16 cm^2. $\therefore$I don't agree. (2 marks)

11. (a) the numbers are 28,34,34,38,39,40,40,41,42,45,50,67

mean = 41.5

median = 40 (2 marks)

(b) mean of the bills = 41.5 × 50 + 100 = \$2175

Median of the bills = 40 × 50 + 100 = \$2100 (2 marks)

(c) (i) least possible median = $\frac{39+40}{2}$ = 39.5

greatest possible median = $\frac{40+41}{2}$ = 40.5

(ii) $\because$ mean is unchanged,

$\therefore$ $a + b = 2 \times 41.5 = 83$

$\because$ median is unchanged,

$\therefore$ possible pairs of (a, b) are (40,43), (39,44), (38,45), …

Least value of the difference between a and b = 3.

(4 marks)

12. (a) Let $g(x) = k_1x^3 + k_2x^2$, where k_1 and k_2 are non-zero constants.

$$\begin{cases} 8 = k_1(2)^3 + k_2(2)^2 \\ -4 = k_1(-1)^3 + k_2(-1)^2 \end{cases}$$

$$\begin{cases} 2 = 2k_1 + k_2 \text{ ......(1)} \\ -4 = -k_1 + k_2 \text{ ......(2)} \end{cases}$$

From (1) and (2), $k_1 = 2$ and $k_2 = -2$

$\therefore$ $g(x) = 2x^3 - 2x^2$

(3 marks)

(b) (i) $h(x) = g(x) + mx + 8$

$= 2x^3 - 2x^2 + mx + 8$

$h(2) = 0$

$16 - 8 + 2m + 8 = 0$

$m = -8$

(ii) $h(x) = 8$

$2x^3 - 2x^2 - 8x + 8 = 8$

$2x(x^2 - x - 4) = 0$

$x = 0$ or $x = \dfrac{1 \pm \sqrt{1 - 4(1)(-4)}}{2} = \dfrac{1 \pm \sqrt{17}}{2}$

Since $\dfrac{1 \pm \sqrt{17}}{2}$ are irrational roots, therefore, only 0 is a rational root of $h(x) = 8$.

Thus, the equation $h(x) = 8$ has 1 rational root.

(4 marks)

13. (a) (i) Volume of the cylinder

$= \pi \times 4^2 \times 114$

$= 1824\pi \text{ cm}^3$

(ii) $\dfrac{\text{volume of smaller cone}}{\text{volume of larger cone}} = \left(\sqrt{\dfrac{9}{25}}\right)^3 = \dfrac{27}{125}$

Volume of the smaller cone

$= 1824\pi \times \dfrac{27}{27 + 125}$

$= 324\pi \text{ cm}^3$

(3 marks)

(b) Let r cm and h cm be the base radius and the height of the smaller cone respectively.

$\frac{h}{h+8}=\frac{3}{5}$

$h = 12$

Volume of the smaller cone = 324π cm^3

$\frac{1}{3}\times\pi\times r^2\times 12=324\pi$

$r = 9$

Curved surface area of the frustum

= $\pi\times 9\times\sqrt{9^2+12^2}\times\frac{25-9}{9}$

= 240π cm^2

(3 marks)

14. (a) $2x+y\le 6$

$x+2y\le 6$

$x\ge 0$

$y\ge 0$

(3 marks)

(b) $6x+3y\le 18$ (simplify to $2x+y\le 6$)

$4x+8y\le 24$ (simplify to $x+2y\le 6$)

$x\ge 0$

$y\ge 0$

Total profit = \$($15x + 10y$)

At point	Total profit(\$)
(0,0)	0
(0,3)	30
(3,0)	45
(2,2)	50

The maximum profit = \$50 which is not greater than \$50.

$\therefore$ I don't agree. (4 marks)

SECTION B: (35 marks)

15. (a) $PR\perp PS$ (tangent $\perp$ radius)

$\therefore PR$ is a tangent to C_2 (converse of tangent $\perp$ radius)

(1 mark)

(b) $RS=\sqrt{18^2+6^2}=6\sqrt{10}$ cm (1 mark)

(c) In $\triangle PQR$, $PQ^2=PR^2-QR^2 = 18^2-(\sqrt{360}-x)^2$

In $\triangle PQS$, $PQ^2=PS^2-QS^2 = 6^2-x^2$

$18^2-(\sqrt{360}-x)^2= 6^2-x^2$

$324-360+2\sqrt{360}x-x^2=36-x^2$

$x=\dfrac{3\sqrt{10}}{5}$

(2 marks)

(d) $S=(\dfrac{3\sqrt{10}}{5},0)$

$PQ^2=6^2-(\dfrac{3\sqrt{10}}{5})^2$

$PQ=\dfrac{9\sqrt{10}}{5}$

$\therefore P=(0,\dfrac{9\sqrt{10}}{5})$

Centre of C_2 = mid-point of PS = $(\dfrac{3\sqrt{10}}{10},\dfrac{9\sqrt{10}}{10})$

Equation of C_2 is $(x-\dfrac{3\sqrt{10}}{10})^2+(y-\dfrac{9\sqrt{10}}{10})=9$

or $\dfrac{y-0}{x-\dfrac{3\sqrt{10}}{5}}\times\dfrac{y-\dfrac{9\sqrt{10}}{5}}{x-0}=-1$

$x^2+y^2-\dfrac{3\sqrt{10}}{5}-\dfrac{9\sqrt{10}}{5}=0$

(3 marks)

16. (a) (i) $f(x)= k(x^2-8x+4^2-4^2)-6k^2+2$

$= k(x-4)^2-6k^2-16k+2$

Vertex of y = f(x) is $(4\ ,\ -6k^2-16k+2)$

(ii) $-6k^2-16k+2=-4$

$3k^2+8k-3=0$

$(3k-1)(k+3)=0$

$k=\dfrac{1}{3}$ (rejected) or $k=-3$ (since f(x) has a maximum value)

(3 marks)

(b) (i) g(x) = –f(x + 6) + 2

S = (4 , –4) and T = (–2 , 6)

Equation of the locus of Q is

$(x-4)^2+(y+4)^2=(x+2)^2+(y-6)^2$

$3x-5y+2=0$

(ii) mid-point of OS = (2 , –2)

Equation of the ⊥ bisector of *OS* is

$$(\frac{y+2}{x-2})(\frac{-4-0}{4-0})=-1$$

$$x-y-4=0$$

Solving $\begin{cases} x-y-4=0 \\ 3x-5y+2=0 \end{cases}$,

The coordinates of the circumcentre = (11 , 7) which lies in the first quadrant.

∴ The claim is agreed.

(4 marks)

17. (a) $$\frac{PR}{\sin(180°-32°-105°)}=\frac{12}{\sin 105°}$$

PR = 8.472679887

= 8.47 cm (cor. to 3 sig. fig.)

(2 marks)

(b) (i) Let *S* be the foot of the perpendicular from *P* to *QR*

$$PS = PR\sin(180°-105°)$$

= 8.183980321 cm

$$\sin\angle PSM = \frac{4}{8.183980321}$$

$$\angle PSM = 29.25909468°$$

The angle between *PQR* and the horizontal ground = 29.3° (cor. to 3 sig. fig.)

(ii) $$\frac{QR}{\sin 32°}=\frac{12}{\sin 105°}$$

QR = 6.583353502 cm

Area of the shadow *MQR*

$$= \frac{1}{2}\times QR\times MS$$

$$= \frac{1}{2}\times QR\times PS\cos\angle PSM \quad \text{or} \quad \frac{1}{2}\times QR\times\frac{PM}{\tan\angle PSM}$$

= 23.5 cm^2

(5 marks)

18. (a) The total amount that Jimmy will get at the end of the nth year

$$= 30000(1.005)^{12n}+30000(1.005)^{12n-1}+\cdots+30000(1.005)$$

$$= 30000(1.005)[\frac{(1.005)^{12n}-1}{1.005-1}]$$

$$= 6030000[(1.005)^{12n}-1]$$

(2 marks)

(b) $$6030000[(1.005)^{12n}-1]\geq 3000000$$

$$(1.005)^{12n}\geq\frac{301}{201}$$

$12n\log 1.005 \geq \log(\frac{301}{201})$

$n \geq 6.75$

The least value of n = 7

(3 marks)

19. (a) P(the invoice was from store A)

$= \frac{70}{200} = \frac{7}{20}$ or 0.35 (1 mark)

(b) P(the invoice contained an error)

$= \frac{70\times 5\% + 80\times 3\% + 50\times 4\%}{200}$

$=$ 0.0395 or $\frac{79}{2000}$ (2 marks)

(c) P(the invoice was from store A, given that it contained an error)

$= \frac{\frac{70\times 5\%}{200}}{0.0395} = \frac{35}{79}$ or 0.443 (cor. to 3 sig. fig.)

(2 marks)

20. (a) $\log_4 y = -2x + \frac{5}{2}$

$y = 4^{-2x+\frac{5}{2}}$

$y = 32(4^{-2x})$ or $y = 32(2^{-4x})$ (2 marks)

(b) $32(2^{-4x}) \geq 2^x$

$2^{5x} \leq 32$

$2^{5x} \leq 2^5$

$5x \leq 5$

$x \leq 1$

$\therefore$ The greatest value of x = 1.

(2 marks)

Paper 2 Answers

1. B	11. D	21. B	31. C	41. C
2. B	12. D	22. B	32. A	42. B
3. D	13. C	23. B	33. A	43. D
4. C	14. D	24. B	34. A	44. D
5. B	15. D	25. C	35. A	45. D
6. A	16. A	26. C	36. C	
7. A	17. D	27. D	37. C	
8. A	18. A	28. D	38. B	
9. C	19. B	29. A	39. C	
10.B	20. C	30. A	40. C	