2021-2022
S6 Mock Exam
MATH CP
Paper 1

SACRED HEART CANOSSIAN COLLEGE

2021-2022 MOCK EXAMINATION

SECONDARY 6

MATHEMATICS Compulsory Part

Paper 1

Time allowed: 2 hours 15 minutes

This paper must be answered in English

Name: _____

Class: _____ No.: _____

Question No.	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
Section A Total	

Question No.	Marks
15	
16	
17	
18	
19	
Section B Total	

Total	

GENERAL INSTRUCTIONS

(1) The full mark of this paper is 105.

(2) After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on Page 1.

(3) This paper consists of THREE sections, A(1), A(2) and B.

(4) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.

(5) Graph paper and supplementary answer sheets will be supplied on request. Write your name, class and class number and the question number on each sheet.

(6) Unless otherwise specified, all working must be clearly shown.

(7) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(8) The diagrams in this paper are not necessarily drawn to scale.

(9) No extra time will be given to candidates for writing name, class and class number after the 'Time is up' announcement.

SECTION A(1) (35 marks)

1. Simplify $\frac{(xy^2)^{-1}}{-5x^4}$ and express your answer with positive indices. (3 marks)

2. Make y the subject of the formula $x-2=\frac{y+3}{4y}$. (3 marks)

3. Factorize

(a) $4x^2 - 28xy + 49y^2$,

(b) $81z^2 - 4x^2 + 28xy - 49y^2$.

(3 marks)

4. The marked price of an air conditioner is $\$2800$. The air conditioner is sold at a discount of 20% on its marked price.

(a) Find the selling price of the air conditioner.

(b) If the air conditioner is sold at a percentage profit of 40% , find the cost of the air conditioner.

(4 marks)

5. (a) Round up 2022.6139 to 3 significant figures.

(b) Round down 2022.6139 to 3 decimal places.

(c) Round off 2022.6139 to the nearest integer.

(3 marks)

6. (a) Find the range of values of x which satisfy both $\frac{5-2x}{-3} < \frac{3x+5}{2}$ and $3x-10 \le 0$.

(b) How many integers satisfy both inequalities in (a)?

(4 marks)

7. Let $f(x) = (p+2)x^2 + 12x + 9$, where $p \neq -2$. The graph of $y = f(x)$ touches the x-axis.

(a) Find p .

(b) Find the x-intercept(s) of the graph of $y = f(x) - 4$.

(5 marks)

8. In a polar coordinate system, O is the pole. The polar coordinates of the points A and B are $(6, 40°)$ and $(6, 160°)$ respectively. Let $S(r, \theta)$ be a point on AB such that OS is an axis of symmetry of ΔAOB.

(a) Find $\angle AOS$.

(b) Find r and θ.

(c) Let R be a point on OA such that $OB // RS$. Write down the number of axes of symmetry of ΔROS.

(5 marks)

9. In Figure 1, D is a point lying on AC such that $\angle ABD = \angle ACB$.

(a) Prove that $\Delta ABC \sim \Delta ADB$.

(b) Suppose that $AB = 136$ cm , $AD = 64$ cm and $BC = 255$ cm . Is ΔBCD a right-angled triangle? Explain your answer.

(5 marks)

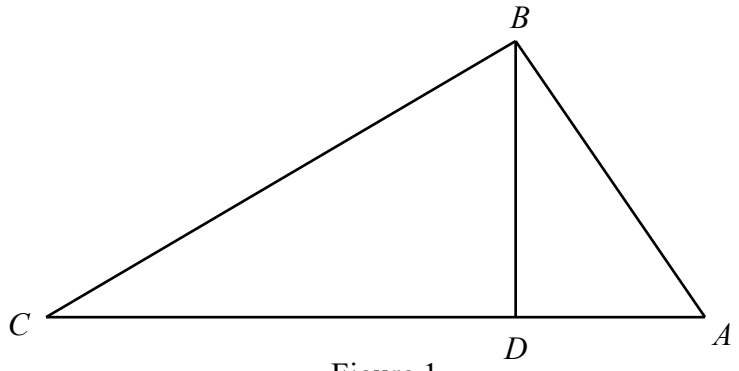

Figure 1

SECTION A(2) (35 marks)

10. AE is the diameter of the semi-circle $ABCDE$. Denote the centre of the semi-circle by O. It is given that $AB:BC:CE=1:1:4$ and $AC//OD$.

(a) Find $\angle COE$ and $\angle DOE$. (3 marks)

(b) Hence, or otherwise, find $\frac{BD}{CE}$. (4 marks)

11. Let $p(x) = ax^3 - 23x^2 + bx - 10$, where a and b are constants. When $p(x)$ is divided by $x-2$, the remainder is -120 . It is given that $p(x)$ is divisible by $2x+1$.

(a) Find a and b . (4 marks)

(b) Anson claims that the equation $p(x)+120=0$ has at least one irrational root. Do you agree? Explain your answer. (3 marks)

$P(x) = (x-2)Q(x) + (-120)$

$Ax^3 + 120 = (x-2)Q(x)$

12. The stem-and-leaf diagram below shows the distribution of the number of bags of rubbish disposed by 17 households in February.

Stem (tens)	Leaf (units)
1	0 4
2	4 x y 6 6 8
3	1 1 1 5 7 8
4	0 2

(a) Find the range of the values of

(i) $x+y$,

(ii) the inter-quartile range of the distribution.

(2 marks)

(b) It is given that the inter-quartile range of the distribution is 11 . Find x and y .

(4 marks)

(c) It is given that the standard deviation of the distribtuion is σ . The garbage levy charged by the district council to each household consists of a fixed amount each month $\$250$ and an additional charge of $\$28$ per bag. Write down the standard deviation of the garbage levy of the 17 households in February in terms of σ .

(1 mark)

13. In Company A, the rental fee $\$F$ of a taxi in a day is partly constant and partly varies as the square of the distance travelled d km. When $d = 250$, $F = 1300$ and when $d = 500$, $F = 3550$.

(a) If a taxi travelled 200 km in a day, find the rental fee of the taxi on that day.

(4 marks)

(b) Ian rented a taxi from Company A and travelled x km in a day. On the next day, he continued to rent the taxi from Company A and travelled 50 km more than that of the previous day. It is given that the total rental fee is $\$5450$. Find x.

(3 marks)

14. An empty right circular conical container of base radius 20 cm and height 40 cm is held vertically on a horizontal surface.

(a) Find the capacity of the container in terms of π. (1 mark)

(b) When $V\text{ cm}^3$ of water is poured into the container, the depth of water is $h\text{ cm}$.

Show that $V = \frac{\pi}{12}\left[64000 - (40-h)^3\right]$. (2 marks)

(c) A right circular cylindrical vessel of base radius 8 cm and height 78 cm is fully filled with water. The water in the vessel is now poured into the container.

(i) Find the depth of water in the container.

(ii) Find the area of the wet surface in the container in terms of π.

(4 marks)

SECTION B (35 marks)

15. There are 56 balls in an urn, where n balls are red and the rest are blue. It is given that there are more blue balls in the urn. If two balls are randomly drawn from the urn at the same time, then the probability that the two balls are of the same colour is $\frac{193}{385}$. Find n. (4 marks)

16. Let R be the region (including the boundary) bounded by the straight lines $y=-2x+13$, $5x+6y-15=0$ and $y=5$.

(a) It is given that R represents the solution of a system of inequalities. Find the system of inequalities. (2 marks)

(b) Find the least value of $5x-4y$, where (x, y) is a point lying in R. (2 marks)

(c) Let S be the region (including the boundary) bounded by the straight lines $y=-2x+13$, $5x+6y-15=0$, $y=5$ and the x-axis. Write down the least value of $5x-4y$, where (x, y) is a point lying in S. (1 mark)

17. For any positive integer n, let $A(n)$ be the nth term of an arithmetic sequence. It is given that $A(29) = -838$ and $A(66) = -616$.

(a) Express $A(1) + A(2) + A(3) + ... + A(n)$ in terms of n. (4 marks)

(b) For any positive integer n, $B(n) = 10^{A(n)+6}$. Find the greatest integral value of m such that $\log\left(B(1)B(2)B(3)...B(m)\right) < 2022$.

(4 marks)

18. Figure 2(a) shows a solid model $VABC$ in the form of tetrahedron. It is given that $VA = VB = 19\,\text{cm}$, $CA = CB = 16\,\text{cm}$, $\angle AVB = 40°$ and $\angle VCB = 69°$.

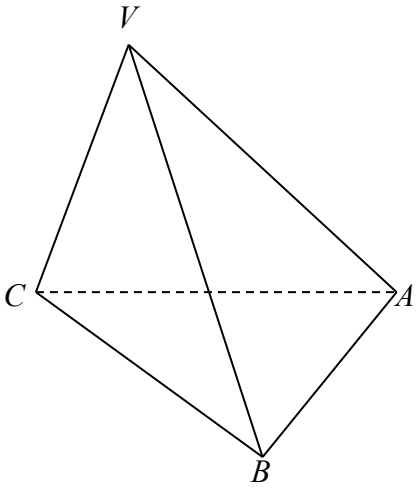

Figure 2(a)

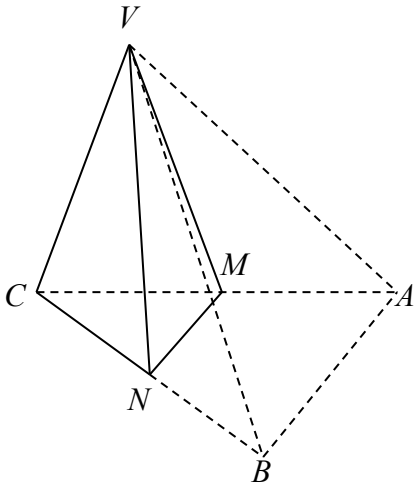

Figure 2(b)

(a) Find VC. (2 marks)

(b) Find the shortest distance from V to the plane ABC. (3 marks)

(c) M and N are the mid-points of AC and BC respectively. A geometric model is made by cutting off the $VABNM$ from $VABC$ as shown in Figure 2(b). Stanley claims that the volume of $VMNC$ is less than $130\,\text{cm}^3$. Do you agree? Explain your answer. (3 marks)

19. Let $f(x) = x^2 - 2kx + 6k - 10$, where k is a real constant.

(a) For any real number k , prove that the graph of $y = f(x)$ cuts the x-axis at two distinct points. (3 marks)

(b) It is given that $M(m, 0)$ and $N(n, 0)$ are points lying on the graph of $y = f(x)$.

(i) Find $(m-n)^2$ in terms of k .

(ii) It is given that $MN = 2$.

(1) Find the value of k .

(2) Let $g(x) = f(\frac{x}{2})$. On the same rectangular coordinate system, the graphs of $y = g(x)$ and $y = -g(x) + t$ intersect at X and Y . Denote the vertices of the graphs of $y = g(x)$ and $y = -g(x) + t$ by U and V respectively. If $UXVY$ is a square and the area of $UXVY$ is 32 square units, find t .

(7 marks)

END OF PAPER