Sacred Heart Canossian College
S6 Mock Examination 2021-2022
Mathematics (Compulsory Part) Paper 1 Solution

Solution	Marks
SECTION A(1) (35 marks)	
1. $\dfrac{(xy^2)^{-1}}{-5x^4}$ $=\dfrac{x^{-1}y^{-2}}{-5x^4}$ $=\dfrac{1}{-5x^{4+1}y^2}$ $=-\dfrac{1}{5x^5y^2}$	----------------(3)
2. $x-2=\dfrac{y+3}{4y}$ $4xy-8y=y+3$ $4xy-9y=3$ $y(4x-9)=3$ $y=\dfrac{3}{4x-9}$	----------------(3)
3. (a) $4x^2-28xy+49y^2$ $=(2x-7y)^2$ (b) $81z^2-4x^2+28xy-49y^2$ $=81z^2-(2x-7y)^2$ $=[9z-(2x-7y)][9z+(2x-7y)]$ $=(9z-2x+7y)(9z+2x-7y)$	----------------(3)
4. (a) The selling price $=2800\times(1-20\%)$ $=\$2240$ (b) The cost $=\dfrac{2240}{1+40\%}$ $=\$1600$	----------------(4)

Solution	Marks
5. (a) 2030 (b) 2022.613 (c) 2023	----------------(3)
6. (a) $\frac{5-2x}{-3} < \frac{3x+5}{2}$ $10-4x > -9x-15$ $5x > -25$ $x > -5$ $3x-10 \le 0$ $x \le \frac{10}{3}$ $\therefore\ -5 < x \le \frac{10}{3}$ (b) 8	----------------(4)
7. (a) $\Delta = 0$ $12^2 - 4(p+2)(9) = 0$ $p = 2$ (b) Put $y = 0$ and $p = 2$, $0 = (2+2)x^2 + 12x + 9 - 4$ $4x^2 + 12 + 5 = 0$ $x = -\frac{1}{2}$ or $x = -\frac{5}{2}$ $\therefore$ The x-intercepts are $-\frac{1}{2}$ and $-\frac{5}{2}$.	----------------(5)
8. (a) $\angle AOS = \frac{160-40}{2}$ $= 60°$ (b) $r = 6\cos 60°$ $= 3$ $\theta = 40° + 60°$ $= 100°$ (c) $\angle BOS = \angle OSR = 60°$ (alt. $\angle$ s, $OB // RS$) $\therefore$ ΔOSR is an equil. Δ . $\therefore$ The number of axes of symmetry of ΔROS is 3 .	----------------(5)

Solution	Marks
9. (a) $\angle ACB = \angle ABD$ (given)	
$\angle BAC = \angle DAB$ (common)	
$\angle ABC = \angle ADB$ ($\angle$ sum of Δ)	
$\therefore\ \Delta ABC \sim \Delta ADB$ (AAA)	

Marking Scheme:	
Case 1 Any correct proof with correct reasons.	
Case 2 Any correct proof without reasons.	

(b) $\dfrac{DB}{BC} = \dfrac{AD}{AB}$ (corr. sides, $\sim \Delta$s)

$\dfrac{DB}{255} = \dfrac{64}{136}$

$DB = 120$

$AD^2 + BD^2 = 64^2 + 120^2$
$= 18496$

$AB^2 = 136^2$
$= 18496$
$= AD^2 + BD^2$

$\therefore\ \angle ADB = 90°$ (converse of Pyth. theorem)
$\therefore\ \angle BDC = 180° - 90°$ (adj. $\angle$s on st. line)
$= 90°$
$\therefore\ \Delta BCD$ is a right-angled triangle.

----------------(5)

SECTION A(2) (35 marks)

10. (a) $\angle COE = 180° \times \dfrac{4}{1+1+4}$
$= 120°$

$\angle DOE = \angle CAE$
$= \dfrac{1}{2}\angle COE$
$= \dfrac{1}{2} \times 120°$
$= 60°$

----------------(3)

Solution	Marks
(b) $\angle BOD = \angle BOC + \angle COD$ $= 180^\circ \times \dfrac{1}{1+1+4} + (120^\circ - 60^\circ)$ $= 90^\circ$ Let r be the radius of the semi-circle $ABCDE$. $\dfrac{BD}{CE} = \dfrac{\sqrt{r^2 + r^2}}{2 \times r \sin 60^\circ}$ $= \dfrac{\sqrt{2}r}{\sqrt{3}r}$ $= \dfrac{\sqrt{2}}{\sqrt{3}}$ (or $\dfrac{\sqrt{6}}{3}$)	----------------(4)
11. (a) $a(2)^3 - 23(2)^2 + b(2) - 10 = -120$ $4a + b = -9 \quad \ldots$(i) $a(-\frac{1}{2})^3 - 23(-\frac{1}{2})^2 + b(-\frac{1}{2}) - 10 = 0$ $a + 4b = -126 \quad \ldots$(ii) Solving (i) and (ii), $a = 6$ and $b = -33$	----------------(4)
(b) $\text{p}(2) + 120$ $= -120 + 120$ $= 0$ $\text{p}(x) + 120 = 0$ $(6)x^3 - 23x^2 + (-33)x - 10 + 120 = 0$ $6x^3 - 23x^2 - 33x + 110 = 0$ $(x-2)(6x^2 - 11x - 55) = 0$ $x - 2 = 0$ or $6x^2 - 11x - 55 = 0$ $x = \dfrac{-(-11) \pm \sqrt{(-11)^2 - 4(6)(-55)}}{2(6)}$ $x = \dfrac{11 \pm \sqrt{1441}}{12}$ $\because \dfrac{11 \pm \sqrt{1441}}{12}$ are irrational roots. $\therefore$ The claim is agreed.	----------------(3)

Solution	Marks
12. (a) (i) $\because\ 4 \le x, y \le 6$ $\therefore\ 4+4 \le x+y \le 6+6$ $8 \le x+y \le 12$	
(ii) The inter-quartile range $= \frac{35+37}{2} - \frac{(20+x)+(20+y)}{2}$ $= 16 - \frac{x+y}{2}$ $8 \le x+y \le 12$ $-4 \ge -\frac{x+y}{2} \ge -6$ $12 \ge 16 - \frac{x+y}{2} \ge 10$ $12 \ge$ The inter-quartile range ≥ 10	----------------(2)
(b) $16 - \frac{x+y}{2} = 11$ $x+y=10$ $\begin{cases} x=4 \\ y=6 \end{cases}$ or $\begin{cases} x=5 \\ y=5 \end{cases}$	----------------(4)
(c) 28σ	----------------(1)
13. (a) $F = k_1 + k_2 d^2$, where k_1 and k_2 are non-zero constants. $\begin{cases} 1300 = k_1 + k_2(250)^2 & \text{...(i)} \\ 3550 = k_1 + k_2(500)^2 & \text{...(ii)} \end{cases}$ Solving (i) and (ii), $k_1 = 550$ and $k_2 = \frac{3}{250}$ The required rental fee $= 550 + \frac{3}{250}(200)^2$ $= \$1030$	----------------(4)
(b) $550 + \frac{3}{250}x^2 + 550 + \frac{3}{250}(x+50)^2 = 5450$ $x^2 + 50x - 180000 = 0$ $x = 400$ or $x = -450$ (rej.)	----------------(3)

Solution	Marks
14. (a) The capacity of the container $= \frac{1}{3}\pi(20)^2(40)$ $= \frac{16000}{3}\pi \text{ cm}^3$	----------------(1)
(b) $V = \frac{16000}{3}\pi\left[\frac{40^3-(40-h)^3}{40^3}\right]$ $= \frac{\pi}{12}\left[64000-(40-h)^3\right]$	----------------(2)
(c) (i) $\pi(8)^2(78) = \frac{\pi}{12}\left[64000-(40-h)^3\right]$ $(40-h)^3 = 4096$ $40-h = 16$ $h = 24$ $\therefore$ The depth of water is 24 cm . (ii) The area of the wet surface in the container $= \pi(20)^2 + \pi(20)(\sqrt{20^2+40^2})\left[\frac{40^2-(40-24)^2}{40^2}\right]$ $= 400\pi + 400\sqrt{5}\pi\left(\frac{21}{25}\right)$ $= (400+336\sqrt{5})\pi \text{ cm}^2$	----------------(4)
SECTION B (35 marks)	
15. The probability that the two balls are red $= \frac{n}{56}\times\frac{n-1}{55}$ The probability that the two balls are blue $= \frac{56-n}{56}\times\frac{(56-n)-1}{55}$ $= \frac{56-n}{56}\times\frac{55-n}{55}$ $\frac{n}{56}\times\frac{n-1}{55}+\frac{56-n}{56}\times\frac{55-n}{55} = \frac{193}{385}$ $n(n-1)+(56-n)(55-n) = 1544$ $n^2-56n+768=0$ $n = 24$ or $n = 32$ (rej.)	----------------(4)

Solution	Marks

16. (a) $\begin{cases} y \le -2x+13 \\ 5x+6y-15 \ge 0 \\ y \le 5 \end{cases}$

----------------(2)

(b) Note that the vertices of R are $(-3,5)$, $(4,5)$ and $(9,-5)$.
When $x=-3$ and $y=5$, $5x-4y=-35$.
When $x=4$ and $y=5$, $5x-4y=0$.
When $x=9$ and $y=-5$, $5x-4y=65$.
Thus, the least value of $5x-4y$ is -35 .

----------------(2)

(c) -35

----------------(1)

17. (a) Let a and d be the first term and the common difference of the arithmetic sequence respectively.

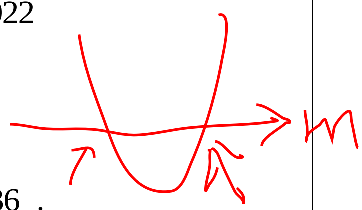
$$\begin{cases} a+(29-1)d=-838 \\ a+(66-1)d=-616 \end{cases}$$

$\therefore\ a=-1006$ and $d=6$

$$\mathrm{A}(1)+\mathrm{A}(2)+\mathrm{A}(3)+\ldots+\mathrm{A}(n)$$
$$=\frac{n}{2}[2(-1006)+(n-1)(6)]$$
$$=3n^2-1009n$$

----------------(4)

(b)
$$\log\left(\mathrm{B}(1)\mathrm{B}(2)\mathrm{B}(3)\ldots\mathrm{B}(m)\right)<2022$$
$$\log\left(10^{\mathrm{A}(1)+6}\cdot10^{\mathrm{A}(2)+6}\cdot10^{\mathrm{A}(3)+6}\ldots10^{\mathrm{A}(m)+6}\right)<2022$$
$$\log\left(10^{\mathrm{A}(1)+\mathrm{A}(2)+\mathrm{A}(3)+\ldots+\mathrm{A}(m)+6m}\right)<2022$$
$$\mathrm{A}(1)+\mathrm{A}(2)+\mathrm{A}(3)+\ldots+\mathrm{A}(m)+6m<2022$$
$$3m^2-1009m+6m<2022$$
$$3m^2-1003m-2022<0$$
$$-2.003940841<m<336.3372742$$



$\therefore$ The greatest integral value of m is 336 .

----------------(4)

Solution	Marks
18. (a) $\frac{BC}{\sin \angle BVC} = \frac{BV}{\sin \angle BCV}$ $\frac{16}{\sin \angle BVC} = \frac{19}{\sin 69^\circ}$ $\angle BVC \approx 51.82929608^\circ$ $\frac{VC}{\sin \angle BVC} = \frac{BV}{\sin \angle BCV}$ $\frac{VC}{\sin(180^\circ - 69^\circ - 51.82929608^\circ)} \approx \frac{19}{\sin 69^\circ}$ $VC \approx 17.47601061$ cm $VC \approx 17.5$ cm	----------------(2)
(b) Let Q be the foot of $\perp$ from V to AB such that $AQ = QB$. $VQ = 19\cos 20^\circ$ $BQ = 19\sin 20^\circ$ $CQ = \sqrt{16^2 - (19\sin 20^\circ)^2} \approx 14.62091044$ $\cos \angle VCQ = \frac{VC^2 + CQ^2 - VQ^2}{2(VC)(CQ)}$ $\cos \angle VCQ \approx \frac{17.47601061^2 + 14.62091044^2 - (19\cos 20^\circ)^2}{2(17.47601061)(14.62091044)}$ $\angle VCQ \approx 66.91038909^\circ$ $\angle VCQ \approx 66.9^\circ$ The shortest distance $= VC \sin \angle VCQ$ $\approx 17.47601061 \sin 66.91038909^\circ$ ≈ 16.07605323 ≈ 16.1 cm	----------------(3)
(c) The volume of $VMNC$ $= \frac{1}{3} \times \text{Area of } \Delta CMN \times 16.07605323$ $= \frac{1}{3} \times \boxed{\left(\frac{1}{2}\right)^2} \times \boxed{\frac{(AB)(CQ)}{2}} \times 16.07605323$ $\approx \frac{1}{3} \times \left(\frac{1}{2}\right)^2 \times \frac{(2 \times 19\sin 20^\circ)(14.62091044)}{2} \times 16.07605323$ ≈ 127.2851948 < 130 $\therefore$ The claim is agreed.	----------------(3)

Solution	Marks

19. (a) $\Delta = (-2k)^2 - 4(1)(6k-10)$

$= 4k^2 - 24k + 40$

$= 4(k^2 - 6k) + 40$

$= 4(k^2 - 6k + 3^2 - 3^2) + 40$

$= 4(k-3)^2 + 4$

> 0

$\therefore$ For any real number k, the graph of $y = \text{f}(x)$ cuts the x-axis at two distinct points.

----------------(3)

(b) (i) $\because$ m and n are the roots of $x^2 - 2kx + 6k - 10 = 0$.

$\therefore$ $m + n = 2k$ and $mn = 6k - 10$

$(m-n)^2$

$= (m+n)^2 - 4mn$

$= (2k)^2 - 4(6k-10)$

$= 4k^2 - 24k + 40$

(ii) (1) $\because$ $MN = 2$

$\therefore$ $(m-n)^2 = 2^2 = 4$

$4k^2 - 24k + 40 = 4$

$(k-3)^2 = 0$

$k = 3$ (repeated)

(2) $\text{g}(x)$

$= \text{f}\left(\frac{x}{2}\right)$

$= \left(\frac{x}{2}\right)^2 - 2(3)\left(\frac{x}{2}\right) + 6(3) - 10$

$= \frac{1}{4}x^2 - 3x + 8$

$= \frac{1}{4}(x^2 - 12x + 6^2 - 6^2) + 8$

$= \frac{1}{4}(x-6)^2 - 1$

$\therefore$ $U(6, -1)$

$-\text{g}(x) + t$

$= -\left[\frac{1}{4}(x-6)^2 - 1\right] + t$

$= -\frac{1}{4}(x-6)^2 + 1 + t$

$\therefore$ $V(6, 1+t)$

Solution	Marks
$\because$ The area of $UXVY = 32$ $\therefore \dfrac{[(1+t)-(-1)]^2}{2} = 32$ $(t+2)^2 = 64$ $t = 6$ or $t = -10$ (rej.)	----------------(7)

Mathematics (Compulsory Part) Paper 2 Answer Key

1.	B	11.	B	21.	C	31.	A	41.	A
2.	D	12.	D	22.	B	32.	B	42.	C
3.	D	13.	B	23.	A	33.	B	43.	B
4.	C	14.	B	24.	A	34.	A	44.	C
5.	D	15.	C	25.	A	35.	D	45.	C
6.	A	16.	C	26.	B	36.	C		
7.	A	17.	D	27.	B	37.	A		
8.	D	18.	C	28.	A	38.	B		
9.	D	19.	D	29.	C	39.	C		
10.	B	20.	D	30.	A	40.	D		